

# M1 – Scalar–Conformal Geometry and the Canonical NUVO Equation

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## Abstract

We construct the scalar–conformal geometric framework underlying the NUVO program. A Lorentzian manifold  $(\mathcal{M}, \eta)$  is equipped with a smooth positive scalar field  $\Lambda(x)$  which generates the physical metric

$$g_{\mu\nu} = \Lambda^2(x)\eta_{\mu\nu}.$$

Within this geometry we introduce a scalar variational principle whose Euler–Lagrange equation defines the canonical scalar field equation (TNE). The present paper is strictly foundational: it establishes the scalar–conformal manifold, derives the variational field equation, and records its structural mathematical properties. Sector reductions and physical correspondence limits are deferred to later work.

## Notation and Conventions

- $\mathcal{M}$  denotes the spacetime manifold.
- $\eta$  denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- $g$  denotes the physical metric.
- The scalar field  $\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$  is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$  denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies  $\Lambda(x) = \Lambda_0$ .
- The dimensionless scalar diagnostic is

$$\mathcal{A}(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

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\*Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline  $\Lambda_0$  remains fixed.
- Greek indices  $\mu, \nu, \dots$  range over spacetime coordinates  $0, 1, 2, 3$ .
- We use the Einstein summation convention unless explicitly stated otherwise.

**Notation convention.** We reserve distinct symbols for fundamental fields and derived quantities. The scalar field  $\Lambda$  represents structural availability, while  $\mathcal{A}$  denotes a derived normalized response field. These should not be conflated.

**Remark 0.1.** *Unless otherwise stated, the background signature is  $(-, +, +, +)$ .*

This manuscript is mathematical in scope. It establishes definitions, structural identities, and variational consequences within a scalar–conformal setting. Sector reductions and correspondence limits are recorded only when explicitly stated as additional assumptions and are not used as premises in derivations.

No claim of full dynamical equivalence to general relativity, quantum mechanics, or classical field theories is made at the level of the present foundational development. Where later papers compare limiting behavior, those comparisons are presented as correspondence targets rather than as identity statements.

The NUVO program is organized as a sequence of internally consistent mathematical papers. Foundational papers (M-series) fix the scalar–conformal geometry, variational structure, and notation. Subsequent papers treat sectoral reductions (gravity, exchange, quantization, and bound-state structure) as controlled specializations of the foundational framework.

**Scalar ontology.** The scalar field  $\Lambda$  represents the *locally available structural capacity* of an underlying delivery field permeating spacetime. The baseline level  $\Lambda_0$  denotes the availability supported by this intrinsic delivery structure in the absence of structural occupation. Localized structures or transport processes may reduce the available capacity relative to this baseline, but the intrinsic delivery baseline itself is not altered. Consequently the scalar field measures the *available portion* of structural capacity rather than the intrinsic production of the underlying field.

## 1 Introduction

Many physical theories describe dynamics through field equations defined on geometric structures. In general relativity the geometry itself becomes dynamical, while in classical field theory the geometry is typically fixed and fields evolve on that background.

The NUVO program adopts a different starting point: a scalar–conformal modulation of spacetime geometry governed by a single scalar variational law. Rather than introducing multiple independent field equations for different physical sectors, the framework begins with a scalar field defined on a Lorentzian manifold and studies the admissible geometric configurations that arise from a variational principle.

Scalar fields modulating spacetime geometry have a long history in gravitational physics [1]; the present framework differs in treating the conformal factor as a diagnostic of substrate availability rather than as a dynamical companion to a tensorial gravitational field.

At this stage,  $\Lambda$  is introduced purely as a geometric scalar field defined on the manifold. No physical or ontological interpretation is imposed at this level. In particular, statements regarding capacity, delivery, or structural depletion are deferred to later developments in the series.

The constant reference level  $\Lambda_0$  denotes a uniform scalar state, characterized by  $\Lambda = \Lambda_0$ , and serves as a baseline configuration for the geometric analysis that follows.

Because the scalar field determines the physical metric through a conformal relation, variations of the locally available capacity directly modulate the effective unit structure of spacetime.

Let  $(\mathcal{M}, \eta)$  denote a Lorentzian manifold with reference metric  $\eta$ . A smooth positive scalar field

$$\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$$

generates the physical metric

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

The scalar field therefore modulates the local unit structure of spacetime while preserving the causal structure of the background metric.

The central mathematical object of the framework is a scalar variational principle defined for  $\Lambda$ . The Euler–Lagrange equation obtained from this variational principle defines the canonical scalar field equation of the NUVO program, referred to as the NUVO Equation (TNE).

The purpose of the present paper is strictly foundational. We construct the scalar–conformal geometric setting, introduce the scalar action functional, derive the associated Euler–Lagrange equation, and record the principal structural properties of the resulting field equation. No sector reductions or phenomenological interpretations are used as premises in the derivation.

Subsequent papers in the series treat controlled reductions of the canonical equation, including gravitational structure, exchange dynamics, and coherent closure phenomena. These developments are not required for the mathematical results established here.

## 1.1 Scope and Limitations

This work establishes a geometric framework only. No dynamical laws, force relations, or transport mechanisms are introduced here. The results should be interpreted as structural constraints on admissible geometries, not as governing equations of motion.

## 1.2 Forward Structure

In later works, the scalar field  $\Lambda$  will be related to structural notions such as availability, intake, and response. The conformal factor  $\Lambda$  will then serve as a local geometric indicator of these structures. Such interpretations, however, are not assumed here and play no role in the present derivation.

This manuscript is the first of a sequence of mathematical papers establishing the scalar–conformal framework underlying the NUVO program.

# 2 Mathematical Setting

## 2.1 Lorentzian Background

Let  $(\mathcal{M}, \eta)$  denote a smooth four–dimensional Lorentzian manifold equipped with reference metric  $\eta_{\mu\nu}$ .

## 2.2 Scalar–Conformal Metric

**Definition 2.1.** *Let  $\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$  be a smooth scalar field. The physical metric is defined by*

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu} \tag{1}$$

The pair  $(\mathcal{M}, g)$  defines a scalar–conformal spacetime geometry.

### 2.3 Normalized Scalar Response Field

**Remark (positivity of the scalar field).** The scalar field is required to satisfy  $\Lambda(x) > 0$  in order for the scalar–conformal metric

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}$$

to remain well-defined.

The scalar potential  $U(\Lambda)$  introduced in the canonical formulation diverges as  $\Lambda \rightarrow 0^+$ , providing an effective barrier that prevents the field from reaching zero for finite-energy configurations. Accordingly, the admissible solution space is restricted to strictly positive scalar fields.

**Definition (Normalized scalar response field).** Fix a reference value  $\Lambda_0 > 0$ . Define

$$\mathcal{A}(x) := \frac{\Lambda(x)}{\Lambda_0}. \tag{2}$$

The field  $\mathcal{A}(x)$  is a derived scalar field encoding the geometric response of the scalar–conformal structure to local availability.

### 2.4 Field and Response Distinction

We emphasize a strict distinction between the scalar field  $\Lambda(x)$  and the normalized response field  $\mathcal{A}(x)$ .

- $\Lambda(x)$  denotes the underlying scalar field defined on the manifold.
- $\mathcal{A}(x)$  denotes the normalized response field derived from  $\Lambda(x)$ .

Thus  $\mathcal{A}(x)$  is not an independent field, but a derived scalar quantity encoding the geometric response of the conformal structure. This distinction will be essential in subsequent developments.

### 2.5 Depletion interpretation

Because the scalar field  $\Lambda(x)$  represents the locally available structural capacity of the underlying delivery field [2, 3], it is useful to measure local depletion relative to the baseline level  $\Lambda_0$ .

We therefore introduce the depletion field

$$\mathcal{D}(x) := \Lambda_0 - \Lambda(x). \tag{3}$$

The sign of  $\mathcal{D}$  distinguishes three local regimes:

- $\mathcal{D}(x) = 0$  corresponds to the baseline availability state  $\Lambda(x) = \Lambda_0$ ;
- $\mathcal{D}(x) > 0$  corresponds to local depletion,  $\Lambda(x) < \Lambda_0$ ;
- $\mathcal{D}(x) < 0$  corresponds to local excess availability,  $\Lambda(x) > \Lambda_0$ .

Equivalently, in terms of the normalized response field  $\mathcal{A}(x) = \Lambda(x)/\Lambda_0$ , one has

$$\frac{\mathcal{D}(x)}{\Lambda_0} = 1 - \mathcal{A}(x). \tag{4}$$

The depletion field is introduced as a diagnostic quantity only. The fundamental dynamical variable of the present framework remains the positive scalar field  $\Lambda$ , and the physical metric continues to be determined by the scalar–conformal relation

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

### 3 Differential Geometry

In this section we record the geometric structures associated with the scalar-conformal metric

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

#### 3.1 Levi-Civita connection

Let  $\nabla^{(\eta)}$  denote the Levi-Civita connection associated with the background metric  $\eta$ , and let  $\nabla^{(g)}$  denote the connection associated with the physical metric  $g$ .

The two connections are related by a standard conformal transformation rule [4]. Writing

$$\Omega(x) := \Lambda(x),$$

the Christoffel symbols satisfy

$$\Gamma_{\mu\nu}^{\rho}(g) = \Gamma_{\mu\nu}^{\rho}(\eta) + \delta_{\mu}^{\rho} \partial_{\nu}(\ln \Omega) + \delta_{\nu}^{\rho} \partial_{\mu}(\ln \Omega) - \eta_{\mu\nu} \eta^{\rho\sigma} \partial_{\sigma}(\ln \Omega). \quad (5)$$

Thus the scalar field introduces additional connection terms determined entirely by the gradient of  $\ln \Lambda$ .

#### 3.2 Conformal identities

Because the physical metric differs from the background metric only by a scalar factor, several differential identities follow immediately.

The determinant transforms according to

$$\sqrt{-g} = \Lambda^4 \sqrt{-\eta}. \quad (6)$$

Gradients of the scalar field may be expressed using either connection, since

$$\nabla_{\mu}^{(\eta)} \Lambda = \nabla_{\mu}^{(g)} \Lambda.$$

The d'Alembertian operators are related by

$$\square_g \Lambda = \Lambda^{-2} (\square_{\eta} \Lambda + 2\eta^{\mu\nu} \partial_{\mu}(\ln \Lambda) \partial_{\nu} \Lambda). \quad (7)$$

These identities will be used when expressing the variational field equation in both physical and background representations.

#### 3.3 Curvature relations

The curvature tensors of the conformally related metrics are connected by well-known transformation formulas. In four spacetime dimensions the Ricci scalar satisfies

$$R(g) = \Lambda^{-2} [R(\eta) - 6 \square_{\eta}(\ln \Lambda) - 6 \eta^{\mu\nu} \partial_{\mu}(\ln \Lambda) \partial_{\nu}(\ln \Lambda)]. \quad (8)$$

In many applications of the NUVO framework the background metric is taken to be flat so that  $R(\eta) = 0$ , but no such restriction is required for the geometric identities recorded here.

These relations show that curvature in the conformal metric can arise purely from spatial variation of the scalar field. Consequently the scalar field  $\Lambda$  provides the only independent geometric degree of freedom within the present framework.

## 4 Scalar Variational Principle

We now introduce the scalar variational principle that governs the modulation field  $\Lambda$ . The physical metric is determined by the scalar–conformal relation

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu},$$

so that admissible spacetime configurations correspond to admissible configurations of the scalar field.

### 4.1 Scalar action functional

Let  $\mathcal{L}_{\text{scalar}}$  denote a scalar Lagrangian density depending on  $\Lambda$  and its first derivatives. The scalar action is defined by

$$S_{\text{scalar}}[\Lambda] = \int_{\mathcal{M}} \mathcal{L}_{\text{scalar}}(\Lambda, \partial\Lambda) \sqrt{-g} d^4x. \quad (9)$$

A minimal form compatible with the scalar–conformal geometry is

$$\mathcal{L}_{\text{scalar}} = -\frac{\beta}{2} g^{\mu\nu} \nabla_\mu \Lambda \nabla_\nu \Lambda - U(\Lambda), \quad (10)$$

where

- $\beta > 0$  is a kinetic normalization constant,
- $U(\Lambda)$  is a scalar self–potential governing the admissible availability states of the scalar field relative to the baseline level  $\Lambda_0$ .

The kinetic term measures variation of the scalar modulation across the manifold, while the potential term determines the admissible scalar configuration space.

### 4.2 Admissible scalar potential

The scalar potential  $U(\Lambda)$  encodes the intrinsic response of the availability field to deviations from the baseline level  $\Lambda_0$ .

The NUVO framework requires the potential to satisfy the following structural conditions:

- $U'(\Lambda_0) = 0$  so that the baseline configuration  $\Lambda = \Lambda_0$  is stationary.
- $U''(\Lambda_0) > 0$  ensuring local stability of the baseline state.
- $U(\Lambda) \rightarrow +\infty$  as  $\Lambda \rightarrow 0^+$  so that the scalar field remains strictly positive.

A simple potential satisfying these conditions is

$$U(\Lambda) = \mu^2 \Lambda_0^2 \left( \frac{\Lambda}{\Lambda_0} - 1 - \ln \frac{\Lambda}{\Lambda_0} \right), \quad \Lambda > 0, \quad (11)$$

where  $\mu$  is a positive constant setting the stiffness scale of the scalar field.

This potential has a unique minimum at  $\Lambda = \Lambda_0$ , diverges as  $\Lambda \rightarrow 0^+$ , and permits arbitrarily large scalar values. Consequently the scalar field remains positive while allowing both depletion ( $\Lambda < \Lambda_0$ ) and excess ( $\Lambda > \Lambda_0$ ) availability relative to the baseline level.

### 4.3 Matter coupling

Additional physical sectors may couple to the scalar field through a matter Lagrangian density  $\mathcal{L}_{\text{matt}}$ . The total action is then

$$S[\Lambda] = \int_{\mathcal{M}} (\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{matt}}) \sqrt{-g} d^4x. \quad (12)$$

The precise form of  $\mathcal{L}_{\text{matt}}$  depends on the sector under consideration and is not specified here. For the purposes of the present derivation we require only that the matter Lagrangian depend smoothly on the scalar field so that the functional derivative

$$\frac{\partial \mathcal{L}_{\text{matt}}}{\partial \Lambda}$$

exists.

### 4.4 Functional setting

The scalar field is taken to lie in a function space appropriate for variational analysis. In particular we assume

$$\Lambda \in H_{\text{loc}}^1(\mathcal{M}),$$

so that the action functional is well defined and first variations exist. Variations are taken with compact support,

$$\Lambda \rightarrow \Lambda + \varepsilon \delta \Lambda,$$

with  $\delta \Lambda$  smooth and vanishing on the boundary of the integration domain.

Under these assumptions the action admits well-defined Euler–Lagrange equations obtained by the standard variational procedure.

### 4.5 Equivalence of covariant and background representations

The scalar field equation obtained from the variational principle may be written either in covariant form using the physical metric  $g$  or in background form using the reference metric  $\eta$ .

**Proposition 4.1.** *Let the physical metric be related to the reference metric by the scalar–conformal relation*

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

*Then the covariant scalar field equation*

$$\beta \square_g \Lambda = U'(\Lambda) + \frac{\partial L_{\text{matt}}}{\partial \Lambda} \quad (13)$$

*is mathematically equivalent to the background-form equation*

$$\beta (\Lambda^{-2} \square_\eta \Lambda + 2\Lambda^{-3} \eta^{\mu\nu} (\partial_\mu \Lambda)(\partial_\nu \Lambda)) = U'(\Lambda) + \frac{\partial L_{\text{matt}}}{\partial \Lambda}. \quad (14)$$

*Proof.* The result follows from the conformal transformation of the wave operator derived in the previous subsection. Substituting the explicit relation between  $\square_g$  and  $\square_\eta$  into the covariant scalar equation yields the background-form equation identically. Conversely, multiplying the background-form equation by  $\Lambda^2$  and reversing the conformal substitution recovers the covariant representation. The two forms therefore encode the same differential equation expressed in different metric variables.  $\square$

## 5 The NUVO Equation (TNE)

We now derive the field equation governing the scalar modulation field  $\Lambda$  from the variational principle introduced in the previous section.

### 5.1 Variation of the action

Consider the total action

$$S[\Lambda] = \int_{\mathcal{M}} (\mathcal{L}_{\text{scalar}} + \mathcal{L}_{\text{matt}}) \sqrt{-g} d^4x.$$

Let

$$\Lambda \rightarrow \Lambda + \varepsilon \delta\Lambda$$

with  $\delta\Lambda$  a compactly supported smooth variation. The first variation of the scalar part of the action is

$$\delta S_{\text{scalar}} = \int_{\mathcal{M}} [-\beta g^{\mu\nu} \nabla_{\mu} \Lambda \nabla_{\nu} (\delta\Lambda) - U'(\Lambda) \delta\Lambda] \sqrt{-g} d^4x. \quad (15)$$

Integrating the kinetic term by parts and discarding boundary terms yields

$$\delta S_{\text{scalar}} = \int_{\mathcal{M}} [\beta \square_g \Lambda - U'(\Lambda)] \delta\Lambda \sqrt{-g} d^4x. \quad (16)$$

The variation of the matter action contributes

$$\delta S_{\text{matt}} = \int_{\mathcal{M}} \frac{\partial \mathcal{L}_{\text{matt}}}{\partial \Lambda} \delta\Lambda \sqrt{-g} d^4x. \quad (17)$$

Combining these contributions gives

$$\delta S = \int_{\mathcal{M}} \left[ \beta \square_g \Lambda - U'(\Lambda) + \frac{\partial \mathcal{L}_{\text{matt}}}{\partial \Lambda} \right] \delta\Lambda \sqrt{-g} d^4x.$$

Since  $\delta\Lambda$  is arbitrary, the Euler–Lagrange equation is

$$\beta \square_g \Lambda = U'(\Lambda) - \frac{\partial \mathcal{L}_{\text{matt}}}{\partial \Lambda}. \quad (18)$$

Equation (23) is the canonical scalar field equation of the NUVO framework and will be referred to as the *NUVO Equation* (TNE).

### 5.2 Background representation of the scalar equation

For later analysis it is useful to express the scalar equation in terms of the reference metric  $\eta$ .

Recall that the physical metric is related to the reference metric by

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

The covariant wave operator associated with  $g$  acting on a scalar function  $f$  is

$$\square_g f = \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu} f). \quad (19)$$

For the conformal metric the determinant and inverse metric are

$$\sqrt{-g} = \Lambda^4 \sqrt{-\eta}, \quad g^{\mu\nu} = \Lambda^{-2} \eta^{\mu\nu}.$$

Substituting these relations gives

$$\square_g f = \frac{1}{\Lambda^4 \sqrt{-\eta}} \partial_\mu (\Lambda^2 \sqrt{-\eta} \eta^{\mu\nu} \partial_\nu f). \quad (20)$$

Applying this operator to the scalar field itself yields

$$\square_g \Lambda = \Lambda^{-2} \square_\eta \Lambda + 2\Lambda^{-3} \eta^{\mu\nu} (\partial_\mu \Lambda) (\partial_\nu \Lambda). \quad (21)$$

Substituting this relation into the covariant scalar field equation derived from the variational principle gives the exact background-form representation

$$\beta (\Lambda^{-2} \square_\eta \Lambda + 2\Lambda^{-3} \eta^{\mu\nu} (\partial_\mu \Lambda) (\partial_\nu \Lambda)) = U'(\Lambda) + \frac{\partial L_{\text{matt}}}{\partial \Lambda}. \quad (22)$$

Equation (22) is mathematically equivalent to the covariant form of the scalar equation but expressed entirely in terms of the reference metric  $\eta$ .

### 5.3 The canonical NUVO scalar equation

The scalar field equation derived from the variational principle plays a central structural role in the NUVO framework. We therefore record it explicitly as the canonical scalar equation governing the scalar–conformal geometry.

**Definition 5.1** (Canonical NUVO scalar equation). *Let  $(\mathcal{M}, \eta)$  be a Lorentzian manifold equipped with the physical metric*

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu},$$

where  $\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$  is smooth.

The NUVO equation (TNE) is the scalar field equation

$$\beta \square_g \Lambda = U'(\Lambda) - \frac{\partial L_{\text{matt}}}{\partial \Lambda}. \quad (23)$$

This equation determines the scalar modulation field and its coupling to matter through the scalar dependence of  $L_{\text{matt}}$ .

**Remark (source convention).** For later convenience, we define the effective scalar source density

$$\rho_\Lambda := -\frac{\partial L_{\text{matt}}}{\partial \Lambda}.$$

With this definition, the canonical NUVO equation may be written equivalently as

$$\beta \square_g \Lambda = U'(\Lambda) + \rho_\Lambda.$$

This sign convention is used consistently in subsequent papers.

Equation (23) may equivalently be written in the background representation

$$\beta (\Lambda^{-2} \square_\eta \Lambda + 2\Lambda^{-3} \eta^{\mu\nu} (\partial_\mu \Lambda) (\partial_\nu \Lambda)) = U'(\Lambda) - \frac{\partial L_{\text{matt}}}{\partial \Lambda}. \quad (24)$$

The covariant form emphasizes the geometric role of the scalar field, while the background form is often convenient for analytical calculations and weak-field analysis.

## 5.4 Stress–energy and scalar coupling

The matter sector is defined by an action of the form

$$S_{\text{matt}} = \int L_{\text{matt}}(\psi, g_{\mu\nu}) \sqrt{-g} d^4x,$$

where  $\psi$  denotes generic matter fields and  $g_{\mu\nu}$  is the scalar–conformal metric.

The associated stress–energy tensor is defined by variation with respect to the metric,

$$T_{\mu\nu} := -\frac{2}{\sqrt{-g}} \frac{\delta S_{\text{matt}}}{\delta g^{\mu\nu}}.$$

Because the physical metric depends on the scalar field through

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu},$$

variation of the matter action with respect to  $\Lambda$  induces a trace coupling to the stress–energy tensor.

A direct computation yields

$$\frac{\partial L_{\text{matt}}}{\partial \Lambda} = \frac{1}{2} \Lambda^{-1} T^\mu{}_\mu,$$

so that the scalar source term appearing in the canonical NUVO equation may be written in terms of the trace of the stress–energy tensor.

Accordingly, the scalar field equation takes the equivalent form

$$\beta \square_g \Lambda = U'(\Lambda) - \frac{1}{2} \Lambda^{-1} T^\mu{}_\mu.$$

This expression makes explicit that the scalar field couples to matter through the trace of the stress–energy tensor, consistent with the conformal structure of the theory.

In particular, traceless matter configurations do not directly source the scalar field in this formulation.

## 6 Structural Properties

We record several structural properties of the NUVO Equation (23). These properties follow directly from the variational structure and the Lorentzian background geometry.

### 6.1 Principal symbol and hyperbolicity

The principal part of the scalar equation is determined by the kinetic term

$$\beta g^{\mu\nu} \nabla_\mu \nabla_\nu \Lambda.$$

The principal symbol associated with the differential operator is therefore

$$\sigma(\xi) = \beta g^{\mu\nu} \xi_\mu \xi_\nu.$$

Because  $g$  is Lorentzian and  $\beta > 0$ , the scalar field equation is a second–order hyperbolic partial differential equation. Consequently the initial value problem is well posed on globally hyperbolic regions of  $(\mathcal{M}, g)$  [4].

## 6.2 Energy functional

The variational structure of the theory yields a conserved energy functional associated with time translation symmetry of the background metric.

Let  $\Sigma_t$  denote a spacelike hypersurface with unit normal  $n^\mu$ . The canonical energy density associated with the variational system is

$$\mathcal{E} = \frac{\beta}{2} ((\nabla_n \Lambda)^2 + g^{ij} \nabla_i \Lambda \nabla_j \Lambda) + U(\Lambda).$$

The total scalar energy on  $\Sigma_t$  is therefore

$$E[\Lambda] = \int_{\Sigma_t} \mathcal{E} d\Sigma. \quad (25)$$

Under appropriate boundary conditions this energy is conserved along the evolution generated by the field equation.

## 6.3 Stationary reduction

In stationary situations where  $\partial_t \Lambda = 0$ , the scalar field equation reduces to an elliptic equation on spatial slices.

Writing the background metric in static form and restricting to spatial coordinates gives

$$\beta \Delta_g \Lambda = U'(\Lambda) - \frac{\partial \mathcal{L}_{\text{matt}}}{\partial \Lambda}, \quad (26)$$

where  $\Delta_g$  denotes the spatial Laplace–Beltrami operator.

This stationary reduction plays an important role in later sectoral analyses, including the gravitational reduction and bound-state structure.

## 6.4 Admissible solution classes

Solutions of the NUVO equation are required to satisfy the following regularity conditions:

- $\Lambda$  is smooth and strictly positive on  $\mathcal{M}$ ,
- $\Lambda \in H_{\text{loc}}^1(\mathcal{M})$ ,
- the scalar energy functional is finite on spacelike hypersurfaces.

These conditions ensure that the conformal metric

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}$$

remains nondegenerate and that the scalar dynamics are well defined.

Solutions are required to satisfy regularity conditions ensuring the conformal metric remains nondegenerate.

**Definition 6.1** (NUVO Equation). *Equation (23) is called the canonical NUVO Equation (TNE). It governs the scalar modulation field determining the physical metric  $g = \Lambda^2 \eta$ .*

**Invariant capacity transport.** Capacity transport through the delivery substrate propagates along null directions of the scalar–conformal metric. In particular, directed capacity transport follows trajectories satisfying

$$g_{\mu\nu}k^\mu k^\nu = 0.$$

Consequently the invariant propagation speed associated with capacity transport coincides with the causal propagation scale  $c$  of the scalar–conformal geometry. This invariant transport scale governs the routing of capacity flow throughout the manifold and provides the fundamental upper bound on directed transport processes.

**Steady intake postulate.** Persistent anchored structures maintain their existence through a steady intake of capacity from the surrounding substrate. For a given bundle configuration the required intake rate is invariant under steady transport and characterizes the structural load imposed by the bundle on the capacity transport field.

This steady intake condition defines the admissible equilibrium state of an anchored configuration. Departures from steady intake correspond to structural adjustment of the bundle and manifest dynamically as forces acting on the structure.

## 7 Discussion

The scalar–conformal framework constructed here establishes the mathematical backbone of the NUVO program. The canonical scalar field equation (TNE) determines admissible scalar configurations on the conformal manifold.

Later work will examine sector reductions and correspondence limits that arise from particular classes of solutions.

## 8 Conclusion

We have constructed a scalar–conformal geometric framework and derived its governing scalar field equation from a variational principle. This equation provides the foundational mathematical structure from which subsequent developments of the NUVO program are derived.

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# A Overview of M1: Scalar–Conformal Geometry and the Canonical NUVO Equation

## A.1 Why This Paper Exists

The first paper of the M–series establishes the mathematical foundation of the NUVO framework. Rather than beginning with multiple independent physical fields (such as gravitational, electromagnetic, or quantum fields), the NUVO program starts with a single geometric idea: that spacetime contains a scalar quantity representing the *locally available structural capacity* of an underlying delivery field.

The purpose of this paper is to construct the geometric and mathematical setting in which that scalar quantity operates. The work introduces the basic geometry, defines how the scalar field modifies spacetime, and derives the equation that governs the field itself.

Importantly, the paper deliberately avoids introducing physical interpretations such as gravity, quantum behavior, or particle dynamics. Instead, it focuses strictly on building a clean mathematical framework from which those physical sectors can later emerge.

The result is a foundational structure consisting of a scalar–conformal spacetime geometry together with a canonical scalar field equation that governs it.

## A.2 The Core Idea

The central idea of the paper is simple but powerful: spacetime geometry is modulated by a single scalar field.

In standard relativity, spacetime geometry is determined directly by a metric tensor. In the NUVO framework, however, the physical metric is obtained by scaling a reference Lorentzian metric using a scalar function. In other words, the local “size” of spacetime units can change depending on the value of the scalar field.

Conceptually, the scalar field measures how much structural capacity is locally available in the underlying delivery field that permeates spacetime. When this capacity is reduced by localized structures or transport processes, the scalar value changes and the effective geometry of spacetime adjusts accordingly.

Thus geometry is not treated as an independent dynamical entity. Instead, geometry is determined by the state of a scalar capacity field.

## A.3 Scalar–Conformal Spacetime

To formalize this idea, the paper begins with an ordinary Lorentzian manifold, which can be thought of as a standard spacetime background. A smooth positive scalar field is then defined on this manifold.

The physical spacetime metric is obtained by scaling the background metric by the square of this scalar field. Because the transformation multiplies the metric by a scalar factor, the geometry is described as *scalar–conformal*.

A key consequence of this construction is that the causal structure of spacetime remains unchanged. Light cones and the fundamental propagation speed remain the same as those of the underlying background metric. What changes is the local scale of spacetime units.

The scalar field therefore modulates the effective geometric scale of spacetime without altering its causal structure.

## A.4 Interpreting the Scalar Field

Within the NUVO ontology, the scalar field represents the locally available structural capacity of an underlying delivery field.

A baseline level of availability exists everywhere in empty space. When structures occupy space or transport processes occur, the locally available capacity may be reduced relative to this baseline.

The scalar field therefore measures *available capacity*, not the intrinsic production of the underlying delivery field itself.

To help interpret this idea mathematically, the paper introduces the concept of *depletion*. Depletion measures how much the available capacity differs from the baseline level. Positive depletion corresponds to a reduction of available capacity, while negative depletion represents excess availability.

Although this depletion quantity is useful for interpretation, the fundamental dynamical variable of the theory remains the scalar field itself.

## A.5 Building the Geometry

Once the scalar-conformal metric is defined, the paper develops the geometric structure associated with it.

Because the physical metric differs from the background metric only by a scalar factor, many geometric quantities can be expressed in terms of derivatives of the scalar field. For example, the connection and curvature tensors depend on gradients of the scalar field.

An important consequence of this relationship is that curvature of the physical metric can arise purely from spatial variation of the scalar field. In other words, geometric curvature does not require additional dynamical degrees of freedom. The scalar field alone determines the geometry.

This property plays a central role in the NUVO program: the scalar field becomes the single geometric degree of freedom governing spacetime structure.

## A.6 The Variational Principle

To determine how the scalar field evolves, the paper introduces a variational principle.

In physics, many fundamental equations arise from the principle that a certain action quantity is stationary. By defining an action functional for the scalar field and applying the standard calculus of variations, the governing equation of the field can be derived.

The action introduced in this work contains two parts:

- a kinetic term measuring how the scalar field varies across spacetime,
- a potential term describing how the field responds to deviations from the baseline availability level.

Additional physical sectors can later couple to the scalar field through a matter Lagrangian, but the present paper keeps this coupling general rather than specifying particular forms.

This variational structure ensures that the resulting field equation possesses a well-defined mathematical foundation.

## A.7 The Canonical NUVO Equation

Applying the variational principle yields the central result of the paper: the canonical scalar field equation of the NUVO framework.

This equation governs the scalar modulation field and therefore determines the physical space-time metric itself. Because the metric is obtained directly from the scalar field, solving this equation determines the geometry of spacetime.

The equation has the mathematical structure of a second-order hyperbolic partial differential equation, meaning that it supports well-posed evolution problems similar to other relativistic field equations.

The authors refer to this governing relation as the *NUVO Equation* or the *TNE*. It becomes the fundamental dynamical equation underlying all later developments of the program.

## A.8 Structural Properties of the Equation

After deriving the scalar equation, the paper examines several of its mathematical properties.

Because the equation arises from a variational principle, it possesses a conserved energy functional associated with time translation symmetry. This ensures that the theory has a consistent energy structure.

The equation is also hyperbolic, which guarantees that initial data evolve in a predictable manner under suitable conditions.

In stationary situations the equation simplifies to an elliptic form on spatial slices, a property that becomes useful when studying equilibrium configurations in later papers.

Finally, the paper specifies the regularity conditions required for admissible solutions, ensuring that the scalar field remains positive and that the resulting metric remains well defined.

## A.9 Connection to the Overall NUVO Program

This paper establishes the mathematical backbone of the entire NUVO program.

Later papers in the series do not introduce new geometric variables. Instead, they examine particular classes of solutions of the scalar equation derived here and interpret them in different physical sectors.

Examples include:

- gravitational behavior emerging from scalar depletion patterns,
- exchange transport between source and sink structures,
- closure conditions governing bound configurations,
- quantization phenomena arising from coherence constraints.

Because all of these developments are built on the scalar–conformal geometry introduced here, the canonical NUVO equation serves as the single governing law from which the later sectors of the theory are derived.

In this sense, the present paper functions as the mathematical foundation upon which the entire NUVO framework is constructed.

## B The M-Series: Classical Field Foundations of the NUVO Framework

The M-series establishes the foundational classical structure of the NUVO framework. These papers develop the geometric, mechanical, and interaction principles governing scalar–conformal NUVO space before any quantum or correspondence arguments are introduced.

The central premise of the framework is that spacetime is described by a Lorentzian reference metric together with a scalar diagnostic field that measures the locally available structural capacity of an underlying delivery field. The physical metric takes the scalar-conformal form

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu},$$

where the scalar field  $\Lambda$  represents the locally available capacity of the delivery structure relative to a fixed baseline level.

The papers of the M-series progressively build the classical physical structure that emerges from this geometry.

## Scalar Geometry and Capacity Transport

The first papers of the series establish the geometric role of the scalar field and the transport law governing structural capacity. Capacity may be locally depleted by anchored structures or redirected through transport processes. The resulting distribution of capacity determines the scalar field  $\Lambda$  and therefore the effective spacetime geometry.

Because the scalar field modulates the metric conformally, variations in capacity availability produce curvature-like effects in the geometry. Test trajectories respond to this geometry through geodesic motion.

In the weak-field regime this produces the familiar inverse-square gravitational behavior associated with localized capacity depletion. Thus gravitational dynamics arise as a geometric response to the distribution of structural capacity.

## Anchored Bundles and Persistent Structure

Persistent material structures are modeled as anchored bundles formed from closed-loop configurations of transport pathways. These bundles occupy structural capacity within the delivery field and therefore act as localized sources of scalar modulation.

The finite loop structure of anchored bundles gives them a characteristic spatial extent and provides a stable mechanism for persistent matter. This internal structure plays an important role in later dynamical results, including the regularization of interaction fields near the source.

## Exchange Transport and Interaction Fields

In addition to scalar capacity transport, the framework admits a second type of transport process: directional exchange transport occurring along open-loop pathways linking anchored structures.

Exchange transport redistributes structural influence without consuming capacity. When described in the weak-continuum limit, the exchange sector naturally acquires a gauge structure associated with circulation observables.

The resulting field description introduces an antisymmetric exchange flux tensor whose dynamics are governed by the Maxwell equations in the weak limit. Thus classical electromagnetic-style interaction fields emerge directly from the structural properties of exchange transport.

Anchored bundles interact with these exchange fields through the Lorentz force, providing the classical force law governing interaction dynamics.

## Radiative Dynamics

When exchange sources accelerate, the exchange field develops propagating disturbances that transport energy and momentum through the scalar-conformal manifold.

These disturbances form radiative exchange waves traveling along the null structure of spacetime at invariant speed  $c$ . The energy carried by these waves is described by the stress–energy tensor of the exchange field.

For localized accelerating sources the resulting radiation power follows the familiar Larmor scaling proportional to the square of the acceleration.

Because anchored bundles possess finite structural cores, the exchange field remains finite within the source region. This finite-core structure naturally regularizes the near-field behavior of the exchange field and allows the radiation-reaction force acting on the source to be derived consistently.

In the weak-field limit the resulting dynamical equation reduces to the Landau–Lifshitz radiation-reaction form.

## Completion of the Classical Framework

Taken together, the M–series establishes the full classical field-theoretic foundation of the NUVO program.

Within this framework

- scalar capacity transport determines spacetime geometry,
- anchored bundles provide persistent material structure,
- exchange transport produces classical interaction fields,
- accelerating sources generate radiation,
- and radiation reaction governs the dynamical response of emitting structures.

The resulting structure reproduces the essential features of classical gravitational and electromagnetic field theory while arising from the scalar–capacity ontology of the NUVO framework.

Subsequent series of papers build on this foundation to examine relativistic correspondence, quantum coherence conditions, and bound-state structure within the same geometric setting.

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