M10 – Composition of Scalar Modulation in Scalar–Conformal NUVO Space

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Abstract

The scalar–conformal NUVO framework defines physical geometry through a single scalar field $\Lambda(x)$ representing locally available structural capacity. While the role of this field in determining the metric structure is well established, the manner in which multiple sources of scalar modulation combine into a single effective configuration has not been previously formalized.

In this work, we introduce a composition law governing the effective scalar modulation experienced by transported structures. We show that scalar modulation separates into two structurally distinct contributions: an ambient component associated with sourced scalar geometry, and a local component associated with transport and structural adjustment. Under general structural requirements of normalization, positivity, and independence of these channels, we derive a multiplicative composition law of the form

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}} \cdot \mathcal{A}_{\text{loc}}.$$

This relation defines the effective scalar modulation as a product of ambient geometry and local structural response. We establish the general properties of this composition law and show that its weak-limit expansion recovers the additive diagnostic form used in hydrogenic and transport analyses.

The resulting framework provides a unified structural principle for scalar modulation across all domains of the NUVO program, including bound-state systems, transport dynamics, relativistic regimes, and large-scale geometry. This work completes the specification of scalar modulation at the foundational level and supplies a consistent bridge between scalar ontology and sector-specific applications.

1 Introduction

1.1 Motivation

The scalar–conformal NUVO framework describes physical geometry in terms of a single scalar field $\Lambda(x)$ representing the locally available structural capacity of an underlying delivery field. The physical metric is determined by the conformal relation

$$g_{\mu\nu} = \Lambda^2\, \eta_{\mu\nu},$$

so that all geometric and transport properties are governed by the structure of the scalar field.

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

Previous work in the M-series [1, 2] has established the foundational role of $\Lambda(x)$, including its interpretation as an availability field, its normalization relative to a baseline level Λ_0, and its modification by persistent structural occupation and transport. Subsequent developments in the Q-series [3] have demonstrated that scalar modulation governs closure structure, exchange dynamics, and coherence-based phenomena.

However, a fundamental structural question remains unresolved: when multiple influences on the scalar field are present, how do they combine into a single effective scalar modulation?

In particular, physical systems simultaneously experience:

- ambient scalar geometry arising from persistent sourced structure, and
- local scalar modulation associated with transport, acceleration, and structural adjustment.

While both effects have been identified and used in various sectoral analyses, a general composition law governing their combination has not been explicitly formulated.

1.2 Objective

The primary objective of this paper is to derive a general composition law for scalar modulation within the scalar–conformal NUVO framework. Specifically, we seek to determine how distinct contributions to the scalar field combine to produce the effective scalar diagnostic experienced by a transported structure.

The desired result must satisfy the following requirements:

- It must be consistent with the scalar ontology, in which Λ represents locally available structural capacity;
- It must preserve the normalization condition $\mathcal{A} = 1$ in the absence of modulation;
- It must distinguish between ambient and local contributions without introducing additional dynamical assumptions;
- It must be compatible with the scalar–conformal metric structure and the transport framework developed in subsequent papers;
- It must admit a well-defined weak-limit expansion consistent with previously used diagnostic forms.

The goal is not to introduce a phenomenological rule, but to derive the minimal composition law consistent with the structural constraints of the theory.

1.3 Scope

This work is purely structural and does not depend on any particular sectoral reduction. No assumptions are made regarding gravitational dynamics, quantum behavior, or specific interaction models.

The analysis is confined to the scalar field and its role in defining geometry and transport. In particular, we do not assume any force-based description, energy ontology, or probabilistic interpretation.

Where connections to specific domains are discussed, they are presented as consequences of the general structure derived here, rather than as inputs to the derivation.

1.4 Position in the NUVO Program

This paper occupies a foundational position within the NUVO program. It extends the scalar ontology established in earlier M-series papers [1,2] by completing the specification of scalar modulation at the structural level.

In the existing framework:

- The scalar field $\Lambda(x)$ defines the geometric structure of spacetime;
- Support and transport mechanisms determine how structures interact with this field;
- Exchange and closure dynamics depend on the resulting scalar modulation.

The present work provides the missing link between these elements by establishing how multiple contributions to scalar modulation combine into a single effective field.

As a result, the composition law derived here serves as a unifying principle underlying all subsequent developments. It applies independently of the specific sector under consideration and will be used implicitly in later analyses of transport, bound-state structure, relativistic behavior, and large-scale geometry.

1.5 Structure of the Paper

The paper proceeds as follows. Section 2 reviews the scalar field and introduces the distinction between ambient and effective scalar modulation. Section 3 formalizes the separation between ambient and local contributions.

In Section 4, we derive the composition law governing scalar modulation. Section 5 establishes its fundamental properties, including normalization, positivity, and compatibility with scalar–conformal geometry.

Section 6 examines the weak-limit expansion of the composition law and demonstrates the emergence of additive diagnostic forms. Section 7 specializes the result to hydrogenic systems, providing a direct link to earlier Q-series analyses [3].

Section 8 discusses the relation of the composition law to transport structure, and Section 9 outlines its implications across multiple domains. Interpretive clarifications are provided in Section 10, and the paper concludes with a summary of results.

2 Scalar Field and Effective Modulation

2.1 Canonical Scalar Field

Within the scalar–conformal NUVO framework, the geometry of spacetime is determined by the scalar field $\Lambda(x)$ through the relation

$$g_{\mu\nu} = \Lambda^2\, \eta_{\mu\nu}.$$

The scalar field represents the locally available structural capacity of an underlying delivery field. Its baseline value Λ_0 corresponds to the availability level in the absence of structural occupation or transport.

To describe deviations from this baseline, we introduce the normalized scalar diagnostic

$$\mathcal{A}(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

This dimensionless quantity provides a measure of local modulation of structural capacity relative to the intrinsic delivery level.

2.2 Scalar Modulation and Structural Influence

Variations in $\Lambda(x)$ arise from the presence of structural configurations and transport processes. Persistent structures modify the scalar field through sustained occupation, while transported structures experience variations in scalar availability along their trajectories.

Thus, scalar modulation reflects the combined influence of:

- sourced structural configurations that alter the ambient availability of capacity;
- transport and local structural adjustments that affect the interaction between a system and the surrounding scalar geometry.

These influences are distinct in origin and behavior. The ambient component is associated with persistent geometric structure, while the local component is associated with the state of a transported or adjusting system.

2.3 Effective Scalar Modulation

The scalar field $\Lambda(x)$ defines the ambient geometric structure of the manifold. However, a transported structure does not interact with this field in a purely passive manner.

Instead, the effective scalar modulation experienced by a structure depends both on the ambient scalar field and on the local state of the structure itself. In particular, transport, acceleration, and structural adjustment modify the manner in which the system samples or responds to the surrounding scalar geometry.

To capture this distinction, we introduce the notion of an effective scalar diagnostic

$$\mathcal{A}_{\text{eff}}(x, u),$$

which represents the scalar modulation experienced by a structure at position x with local state u.

The precise form of this dependence is not yet specified. However, it must reduce to the ambient scalar field in the absence of local modulation, i.e.,

$$\mathcal{A}_{\text{eff}}(x, u_0) = \mathcal{A}(x),$$

where u_0 denotes the baseline (unmodulated) state.

2.4 Need for a Composition Law

The introduction of $\mathcal{A}_{\text{eff}}$ highlights a central structural requirement: the scalar modulation experienced by a system must combine contributions from both ambient geometry and local structural state.

While both types of contributions have been identified in prior work, no general rule has been established governing how they combine into a single effective scalar diagnostic.

In particular, it is not sufficient to treat these contributions as independent perturbations without specifying a consistent composition principle. Such a principle must:

- produce a single positive scalar diagnostic consistent with the scalar ontology;
- reduce to the ambient scalar field in the absence of local modulation;
- preserve the normalization $\mathcal{A} = 1$ in the absence of both ambient and local contributions;
- remain compatible with the scalar–conformal structure of the metric.

The derivation of such a composition law is the central objective of the present work.

2.5 Summary

The scalar field $\Lambda(x)$ defines the ambient geometric structure of NUVO space, while transported structures experience an effective scalar modulation that depends on both ambient geometry and local state.

This distinction necessitates the introduction of a composition law governing the combination of ambient and local contributions. The formal derivation of this law will be developed in the subsequent sections.

3 Ambient and Local Contributions

3.1 Ambient Scalar Structure

The scalar field $\Lambda(x)$ determines the geometric structure of the manifold independently of any particular transported system. This field encodes the availability of structural capacity as modified by persistent sources and global configuration.

We therefore identify the ambient scalar diagnostic

$$\mathcal{A}_{\mathrm{amb}}(x) := \frac{\Lambda(x)}{\Lambda_0},$$

which represents the scalar modulation associated with the background geometry.

This quantity depends only on the configuration of the scalar field itself and is independent of the local state of any transported structure.

3.2 Local Structural Modulation

In addition to ambient geometry, a transported structure exhibits a local response arising from its state of motion and structural adjustment. This response modifies the manner in which the structure interacts with the surrounding scalar field.

We represent this effect by introducing a local scalar modulation factor

$$\mathcal{A}_{\mathrm{loc}}(u),$$

where u denotes the local state of the structure. This state may encode transport, acceleration, or other forms of structural adjustment, but is not specified in detail at the present level of analysis.

The local factor satisfies the normalization condition

$$\mathcal{A}_{\mathrm{loc}}(u_0) = 1,$$

where u_0 denotes the baseline state in which no local modulation is present.

3.3 Separation of Contributions

The ambient and local contributions arise from distinct structural origins:

- $\mathcal{A}_{\mathrm{amb}}(x)$ is determined by the scalar field configuration on the manifold;
- $\mathcal{A}_{\mathrm{loc}}(u)$ is determined by the state of the transported structure.

These contributions are therefore independent in the sense that variations in one do not directly determine variations in the other. In particular, the ambient scalar field may be held fixed while the local state varies, and conversely, the local state may be fixed while the ambient field varies.

This independence implies that the effective scalar modulation must be constructed from these contributions in a manner that preserves their distinct roles.

3.4 Structural Requirements for Composition

Any composition law combining $\mathcal{A}_{\text{amb}}$ and $\mathcal{A}_{\text{loc}}$ into an effective scalar diagnostic must satisfy the following conditions:

- **Normalization:**
$$\mathcal{A}_{\text{eff}} = 1 \quad \text{when} \quad \mathcal{A}_{\text{amb}} = 1 \text{ and } \mathcal{A}_{\text{loc}} = 1;$$
- **Ambient Reduction:**
$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}} \quad \text{when} \quad \mathcal{A}_{\text{loc}} = 1;$$
- **Local Reduction:**
$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{loc}} \quad \text{when} \quad \mathcal{A}_{\text{amb}} = 1;$$
- **Positivity:**
$$\mathcal{A}_{\text{eff}} > 0 \quad \text{for all admissible states;}$$
- **Monotonicity:** Increasing either contribution independently increases the effective scalar modulation.

These conditions reflect the scalar ontology and ensure compatibility with the scalar–conformal metric structure.

3.5 Implications of Independence

The independence of ambient and local contributions places strong constraints on the admissible form of the composition law.

In particular, the effective scalar diagnostic must be constructed from $\mathcal{A}_{\text{amb}}$ and $\mathcal{A}_{\text{loc}}$ in a way that preserves their separability while producing a single positive scalar quantity.

Additive constructions generically fail to preserve normalization and positivity under independent variation, while arbitrary nonlinear combinations introduce unnecessary structure not required by the theory.

These considerations suggest that the admissible composition law must belong to a restricted class of functions satisfying the conditions above.

3.6 Summary

The scalar modulation experienced by a transported structure arises from two independent contributions: an ambient component determined by the scalar field and a local component determined by the state of the structure.

The combination of these contributions into an effective scalar diagnostic must satisfy normalization, positivity, and independence constraints. These requirements strongly restrict the admissible form of the composition law and prepare the derivation of its explicit form in the next section.

4 Composition Law for Scalar Modulation

Proposition 4.1 (Effective Scalar Composition Principle)**.** *Let $\mathcal{A}_{\text{amb}}(x)$ denote the ambient scalar diagnostic determined by the scalar field, and let $\mathcal{A}_{\text{loc}}(u)$ denote the local scalar modulation associated with the state of a transported structure.*

Under the structural requirements of normalization, positivity, and independence of ambient and local contributions, the effective scalar modulation $\mathcal{A}_{\text{eff}}(x,u)$ is given by the multiplicative composition

$$\mathcal{A}_{\text{eff}}(x,u) = \mathcal{A}_{\text{amb}}(x)\,\mathcal{A}_{\text{loc}}(u).$$

4.1 Derivation

We seek a composition law of the form

$$\mathcal{A}_{\text{eff}} = F\big(\mathcal{A}_{\text{amb}}, \mathcal{A}_{\text{loc}}\big),$$

where $F : \mathbb{R}_{>0} \times \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ satisfies the structural conditions established in Section 3.

First, normalization requires

$$F(1,1) = 1.$$

Ambient and local reduction conditions require

$$F(a,1) = a, \qquad F(1,\ell) = \ell,$$

for all admissible $a, \ell > 0$.

Independence of contributions implies that variations in $\mathcal{A}_{\text{amb}}$ and $\mathcal{A}_{\text{loc}}$ act separately. In particular, the effect of successive independent modulations must be consistent regardless of the order in which they are applied. This implies the functional relation

$$F(a_1a_2,\ell) = F(a_1,\ell)\,F(a_2,1), \qquad F(a,\ell_1\ell_2) = F(a,1)\,F(a,\ell_1)\,F(1,\ell_2),$$

which encodes multiplicative separability of the contributions.

A standard result for positive functions satisfying these conditions is that F must factorize as

$$F(a,\ell) = f(a)\,g(\ell),$$

for some positive functions f and g.

Applying the reduction conditions yields

$$f(a) = a, \qquad g(\ell) = \ell,$$

and hence

$$F(a,\ell) = a\,\ell.$$

Therefore,

$$\mathcal{A}_{\text{eff}}(x,u) = \mathcal{A}_{\text{amb}}(x)\,\mathcal{A}_{\text{loc}}(u),$$

which completes the derivation. □

4.2 Functional Interpretation

The composition law expresses the effective scalar modulation as the product of two independent factors:

- $\mathcal{A}_{\text{amb}}(x)$, which encodes the ambient scalar geometry determined by the field $\Lambda(x)$;
- $\mathcal{A}_{\text{loc}}(u)$, which encodes the local structural response of a transported system.

The multiplicative form reflects the fact that these contributions act as independent modulators of structural capacity. Each factor scales the available capacity, and the combined effect is obtained through successive scaling.

4.3 General Representation of Local Modulation

The local scalar factor may be expressed in terms of a structural state variable σ_{loc} as

$$\mathcal{A}_{\text{loc}} = \mathfrak{L}(\sigma_{\text{loc}}),$$

where $\mathfrak{L} : \mathbb{R} \to \mathbb{R}_{>0}$ is a response functional satisfying

$$\mathfrak{L}(0) = 1.$$

The specific form of $\mathfrak{L}$ depends on the detailed transport and structural dynamics of the system. At the present level of analysis, it is sufficient to treat $\mathfrak{L}$ as an abstract positive function encoding local modulation.

4.4 Consistency with Scalar–Conformal Geometry

The multiplicative composition law is compatible with the scalar–conformal metric structure. Since the physical metric depends quadratically on Λ, the effective metric associated with $\mathcal{A}_{\text{eff}}$ is given by

$$g_{\mu\nu} = \big(\mathcal{A}_{\text{eff}}\Lambda_0\big)^2 \eta_{\mu\nu}.$$

Substituting the composition law yields

$$g_{\mu\nu} = \big(\mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}}\Lambda_0\big)^2 \eta_{\mu\nu},$$

which preserves the scalar–conformal structure.

4.5 Uniqueness Within Structural Constraints

The multiplicative form derived above is the minimal composition law consistent with the requirements of normalization, positivity, and independence of contributions.

Alternative constructions either violate normalization, fail to preserve positivity under independent variation, or introduce additional structure not required by the scalar ontology.

Thus, within the class of admissible composition laws, the multiplicative form is structurally distinguished.

4.6 Summary

We have derived a general composition law for scalar modulation, showing that the effective scalar diagnostic is given by the product of ambient and local contributions. This result follows directly from the structural requirements imposed by the scalar ontology and the independence of the underlying contributions.

The multiplicative composition law provides the foundation for the analysis of scalar modulation across all subsequent domains.

5 Properties of the Composition Law

5.1 Normalization

The composition law satisfies the normalization condition

$$\mathcal{A}_{\text{eff}} = 1 \quad \text{when} \quad \mathcal{A}_{\text{amb}} = 1 \text{ and } \mathcal{A}_{\text{loc}} = 1.$$

This follows directly from the multiplicative form,

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}},$$

and ensures consistency with the baseline scalar state $\Lambda = \Lambda_0$.

5.2 Reduction to Individual Contributions

The composition law correctly reproduces the limiting cases of purely ambient or purely local modulation:

- If $\mathcal{A}_{\text{loc}} = 1$, then
$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}};$$
- If $\mathcal{A}_{\text{amb}} = 1$, then
$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{loc}}.$$

Thus, each contribution is recovered exactly when the other is absent.

5.3 Positivity and Stability

Since both $\mathcal{A}_{\text{amb}}$ and $\mathcal{A}_{\text{loc}}$ are strictly positive, the effective scalar diagnostic satisfies

$$\mathcal{A}_{\text{eff}} > 0$$

for all admissible states.

This guarantees that the scalar field remains positive and that the scalar–conformal metric remains well-defined.

Moreover, small variations in either contribution produce smooth variations in $\mathcal{A}_{\text{eff}}$, ensuring stability of the composition under perturbations.

5.4 Monotonicity

The composition law is monotonic in each argument:

$$\frac{\partial \mathcal{A}_{\text{eff}}}{\partial \mathcal{A}_{\text{amb}}} = \mathcal{A}_{\text{loc}} > 0, \qquad \frac{\partial \mathcal{A}_{\text{eff}}}{\partial \mathcal{A}_{\text{loc}}} = \mathcal{A}_{\text{amb}} > 0.$$

Thus, increasing either the ambient or local contribution increases the effective scalar modulation, consistent with the interpretation of both factors as modulators of structural capacity.

5.5 Compatibility with Scalar–Conformal Structure

The multiplicative composition law is naturally compatible with the scalar–conformal metric relation.

Since the physical metric depends on Λ^2, the effective metric takes the form

$$g_{\mu\nu} = \left(\mathcal{A}_{\text{eff}}\Lambda_0\right)^2 \eta_{\mu\nu}.$$

Substituting the composition law yields

$$g_{\mu\nu} = \left(\mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}}\Lambda_0\right)^2 \eta_{\mu\nu} = \left(\mathcal{A}_{\text{amb}}\Lambda_0\right)^2 \left(\mathcal{A}_{\text{loc}}\right)^2 \eta_{\mu\nu}.$$

This shows that the effects of ambient geometry and local modulation factorize at the level of the metric, preserving the scalar–conformal structure.

5.6 Associativity of Composition

The multiplicative law is associative:

$$(\mathcal{A}_1\mathcal{A}_2)\mathcal{A}_3 = \mathcal{A}_1(\mathcal{A}_2\mathcal{A}_3).$$

This implies that multiple independent contributions to scalar modulation may be combined sequentially without ambiguity.

Thus, the composition law naturally extends to systems involving more than two contributions, providing a consistent framework for combining multiple sources of scalar modulation.

5.7 Extension to Multiple Contributions

For a collection of independent contributions $\{\mathcal{A}_i\}_{i=1}^n$, the effective scalar modulation is given by

$$\mathcal{A}_{\text{eff}} = \prod_{i=1}^{n} \mathcal{A}_i.$$

This generalization follows directly from associativity and reflects the fact that each contribution acts as an independent modulator of structural capacity.

5.8 Summary

The multiplicative composition law satisfies normalization, positivity, monotonicity, and compatibility with the scalar–conformal metric. It also admits a natural extension to multiple independent contributions.

These properties confirm that the composition law is consistent with the scalar ontology and provides a stable and general framework for combining scalar modulation effects.

6 Weak-Limit Expansion

6.1 Perturbative Regime

We consider regimes in which both ambient and local scalar modulations are small deviations from the baseline state. In this setting, the scalar diagnostics may be written in the form

$$\mathcal{A}_{\text{amb}} = 1 + \chi_{\text{amb}}, \qquad \mathcal{A}_{\text{loc}} = 1 + \chi_{\text{loc}},$$

where χ_{amb} and χ_{loc} are dimensionless quantities satisfying

$$|\chi_{\text{amb}}| \ll 1, \qquad |\chi_{\text{loc}}| \ll 1.$$

These quantities represent small perturbations of scalar modulation relative to the baseline capacity level.

6.2 Expansion of the Composition Law

Substituting into the multiplicative composition law yields

$$\mathcal{A}_{\text{eff}} = (1 + \chi_{\text{amb}})(1 + \chi_{\text{loc}}).$$

Expanding to first order gives

$$\mathcal{A}_{\text{eff}} = 1 + \chi_{\text{amb}} + \chi_{\text{loc}} + \chi_{\text{amb}}\chi_{\text{loc}}.$$

In the weak-modulation regime, the product term $\chi_{\text{amb}}\chi_{\text{loc}}$ is second order and may be neglected, yielding the first-order approximation

$$\mathcal{A}_{\text{eff}} \approx 1 + \chi_{\text{amb}} + \chi_{\text{loc}}.$$

6.3 Emergence of Additive Structure

The result above shows that, in the weak limit, the multiplicative composition law reduces to an additive form. The effective scalar diagnostic may therefore be written as

$$\mathcal{A}_{\text{eff}} = 1 + \chi_{\text{amb}} + \chi_{\text{loc}} \quad \text{(to first order)}.$$

This additive structure does not represent a fundamental law, but rather the leading-order approximation to the exact multiplicative composition.

6.4 Interpretation of Perturbative Contributions

The quantities χ_{amb} and χ_{loc} provide a convenient diagnostic representation of scalar modulation in weak regimes.

- χ_{amb} encodes the contribution from ambient scalar geometry;
- χ_{loc} encodes the contribution from local structural modulation associated with transport and adjustment.

These quantities are not fundamental components of the scalar field, but serve as first-order measures of deviation from baseline capacity.

6.5 Consistency with Scalar Ontology

The weak-limit expansion is fully consistent with the scalar ontology. The scalar field remains the primary geometric object, while the quantities χ_{amb} and χ_{loc} arise as derived diagnostic measures.

Thus, the additive form obtained here should be interpreted as an approximation valid within a restricted regime, rather than as a replacement for the underlying multiplicative structure.

6.6 Higher-Order Corrections

Beyond the weak-modulation regime, higher-order terms become significant. In particular, the product term

$$\chi_{\text{amb}}\chi_{\text{loc}}$$

represents a coupling between ambient and local contributions that is not captured by the first-order approximation.

These higher-order terms encode nonlinear interactions between scalar geometry and local structural modulation and must be retained in regimes where scalar deviations are not small.

6.7 Summary

The multiplicative composition law reduces, in the weak-modulation limit, to an additive diagnostic form. This provides a natural explanation for the appearance of linear scalar modulation expressions in earlier analyses, while preserving the underlying multiplicative structure as the fundamental law.

The weak-limit expansion therefore serves as a bridge between the foundational composition principle and its practical use in perturbative regimes.

7 Hydrogenic Specialization

7.1 Diagnostic Representation in the Hydrogenic Regime

In the hydrogenic regime, the scalar modulation experienced by a bound structure may be expressed in terms of diagnostic quantities that capture the leading contributions from ambient geometry and local transport.

Within the weak-modulation framework developed in Section 6, the effective scalar diagnostic takes the form

$$\mathcal{A}_{\text{eff}} = 1 + \chi_{\text{amb}} + \chi_{\text{loc}},$$

to first order.

To connect this expression with practical calculations, it is useful to represent these contributions using dimensionless quantities normalized by the invariant rest scale $m_e c^2$.

7.2 Identification of Ambient Contribution

The ambient scalar modulation arises from the background geometric structure associated with the proton-centered configuration. In the weak regime, this contribution may be represented by a scalar function that decreases with radial separation.

We introduce a diagnostic quantity $V(x)$ such that

$$\chi_{\text{amb}} = -\frac{V(x)}{m_e c^2}.$$

Here, $V(x)$ serves as a convenient scalar measure of the ambient modulation [4]. It is not interpreted as a fundamental energy, but as a diagnostic representation of the influence of ambient scalar geometry on the transported structure.

7.3 Identification of Local Contribution

The local contribution arises from the transport and structural adjustment of the electron as it evolves along its trajectory.

In the weak regime, this contribution may be expressed as

$$\chi_{\text{loc}} = \frac{T(x)}{m_e c^2},$$

where $T(x)$ is a scalar quantity encoding the local transport state.

As with $V(x)$, the quantity $T(x)$ is not introduced as a fundamental energy, but as a diagnostic scalar capturing the local modulation of structural capacity associated with transport.

7.4 Recovery of the Hydrogenic Form

Substituting these expressions into the weak-limit expansion yields

$$\mathcal{A}_{\text{eff}}(x) = 1 + \frac{T(x)}{m_e c^2} - \frac{V(x)}{m_e c^2}.$$

This is precisely the scalar modulation form employed in hydrogenic analyses.

7.5 Interpretation

The expression above arises as a first-order approximation to the general composition law derived in Section 4. It represents the combined effect of ambient scalar geometry and local structural modulation in a form suitable for weak-regime calculations.

Importantly, the quantities $T(x)$ and $V(x)$ should be understood as diagnostic representations of the underlying scalar modulation, rather than as primary physical variables.

7.6 Relation to Earlier Analyses

The form

$$\mathcal{A}_{\text{eff}} = 1 + \frac{T}{m_e c^2} - \frac{V}{m_e c^2}$$

has been used in earlier Q-series analyses as a working expression for scalar modulation in the hydrogenic regime.

The derivation presented here shows that this expression is not introduced ad hoc, but arises naturally as the weak-limit expansion of the multiplicative composition law governing scalar modulation.

7.7 Scope of Validity

The hydrogenic representation is valid in regimes where both ambient and local scalar modulations are small and higher-order coupling terms may be neglected.

In regimes where scalar modulation is not weak, the full multiplicative form

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}} \mathcal{A}_{\text{loc}}$$

must be used.

7.8 Summary

The hydrogenic scalar modulation formula is recovered as the first-order approximation to the general composition law. This provides a consistent structural foundation for its use in earlier analyses and clarifies its domain of validity.

The result establishes a direct connection between the foundational scalar ontology and the practical diagnostic forms used in bound-state calculations.

8 Relation to Transport Law

8.1 Two-Channel Coupling of Scalar Modulation

The composition law derived in Section 4 establishes that scalar modulation separates into ambient and local contributions, combined multiplicatively in the effective scalar diagnostic.

This structure aligns naturally with the general role of the scalar field in transport and exchange dynamics. In particular, scalar modulation influences transported systems through two distinct channels:

- A geometric channel, determined by the ambient scalar field configuration;
- A transport channel, determined by the local state of the system and its structural adjustment.

These channels correspond precisely to the ambient and local contributions identified in the composition law.

8.2 Scalar Modulation in Phase-Guided Transport

In transport frameworks based on phase-guided dynamics, the evolution of a system is governed both by its interaction with the surrounding geometry and by its internal transport state.

The ambient scalar field influences the global structure of admissible trajectories, while local structural modulation governs how the system responds to and evolves within that structure.

The effective scalar diagnostic

$$\mathcal{A}_{\text{eff}}(x,u) = \mathcal{A}_{\text{amb}}(x)\,\mathcal{A}_{\text{loc}}(u)$$

therefore provides a unified quantity that encodes both aspects of the transport process.

8.3 Consistency with Transport Dynamics

The multiplicative form of the composition law ensures that scalar modulation enters transport dynamics in a manner consistent with the independent roles of geometry and local state.

In particular:

- The ambient factor $\mathcal{A}_{\text{amb}}(x)$ modifies the geometric environment through which transport occurs;
- The local factor $\mathcal{A}_{\text{loc}}(u)$ modifies the response of the system to that environment.

Because these factors are multiplicative, their effects combine without interference at the level of first-order variation, while higher-order terms encode coupling between geometry and local transport.

8.4 Interpretation in Terms of Effective Modulation

From the perspective of transport, the effective scalar diagnostic may be interpreted as the modulation governing the rate of structural evolution along a trajectory.

In weak regimes, this reduces to the additive form

$$\mathcal{A}_{\text{eff}} = 1 + \chi_{\text{amb}} + \chi_{\text{loc}},$$

which separates geometric and transport contributions.

In general regimes, the multiplicative form captures the nonlinear interaction between these contributions, ensuring that transport dynamics remain consistent with the scalar ontology.

8.5 Independence from Specific Transport Models

The composition law does not depend on the detailed form of the transport equations. Rather, it provides a structural constraint that any admissible transport law must satisfy.

In particular, any transport framework in which scalar modulation influences both geometry and local state must admit a decomposition consistent with the multiplicative composition law.

Thus, the result derived here is independent of specific dynamical models and applies generally across different formulations of transport.

8.6 Summary

The multiplicative composition law for scalar modulation is fully consistent with the role of the scalar field in transport dynamics. It captures the dual influence of ambient geometry and local structural state in a unified manner and provides a structural constraint on admissible transport formulations.

This connection reinforces the interpretation of scalar modulation as a fundamental quantity governing both geometry and transport across the NUVO framework.

9 Implications Across Domains

9.1 General Role of the Composition Law

The composition law derived in this work provides a structural principle governing how scalar modulation combines across all regimes of the NUVO framework.

Because the scalar field $\Lambda(x)$ defines the geometric structure of spacetime and influences transport and exchange processes, the manner in which scalar contributions combine has implications for all sectoral developments.

The multiplicative composition law therefore acts as a unifying constraint that applies independently of the specific physical domain under consideration.

9.2 Gravitational Sector

In regimes dominated by large-scale or persistent source structure, scalar modulation is primarily determined by the ambient component $\mathcal{A}_{\rm amb}(x)$.

In such cases, the effective scalar diagnostic reduces to

$$\mathcal{A}_{\rm eff} \approx \mathcal{A}_{\rm amb},$$

with local contributions providing perturbative corrections.

This behavior is consistent with the interpretation of gravitational phenomena as arising from ambient scalar geometry, while local transport effects introduce higher-order modifications.

9.3 Quantum and Exchange Sectors

In bound-state and exchange-dominated regimes, both ambient and local contributions play significant roles.

The ambient scalar field determines the global structure of admissible states, while local modulation governs coherence, transport, and interaction dynamics.

The composition law ensures that these contributions combine in a consistent manner, providing a structural basis for scalar modulation in quantum and exchange processes.

9.4 Relativistic Regimes

In regimes involving strong transport or rapid structural adjustment, the local contribution $\mathcal{A}_{\text{loc}}(u)$ becomes significant.

The multiplicative composition law implies that local modulation acts as a scaling factor on the ambient geometry, leading to nonlinear effects in the effective scalar diagnostic.

This provides a natural framework for understanding the interplay between geometric structure and transport in relativistic contexts.

9.5 Cosmological Context

At large scales, scalar modulation is governed primarily by the distribution of persistent structure across the manifold.

The composition law allows for the consistent combination of scalar contributions arising from multiple sources, yielding an effective scalar field that reflects the aggregate structure of the system.

Local contributions may introduce corrections associated with transport and structural evolution, but the dominant behavior remains controlled by the ambient scalar field.

9.6 Multiple Contributions and Complex Systems

The extension of the composition law to multiple independent contributions,

$$\mathcal{A}_{\text{eff}} = \prod_{i=1}^{n} \mathcal{A}_i,$$

provides a natural framework for analyzing systems with multiple sources of scalar modulation.

This includes configurations in which several structures contribute to the ambient field, as well as systems in which multiple forms of local modulation are present.

The multiplicative structure ensures that these contributions combine consistently without introducing ambiguity or inconsistency.

9.7 Framework Independence

The implications described above do not depend on any specific dynamical model. Rather, they follow directly from the structural role of the scalar field and the composition law derived in this work.

As a result, the composition principle applies uniformly across all subsequent developments in the NUVO program, including those involving transport, exchange, quantization, and large-scale structure.

9.8 Summary

The multiplicative composition law provides a unifying structural principle governing scalar modulation across all domains of the NUVO framework. Its implications extend to gravitational, quantum, relativistic, and cosmological contexts, ensuring that scalar contributions combine consistently in all regimes.

This universality underscores the foundational role of the composition law and motivates its use as a central organizing principle in subsequent analyses.

10 Interpretive Clarifications

10.1 Non-Energetic Ontology

The scalar field $\Lambda(x)$ represents locally available structural capacity and is not identified with energy, potential, or force. The composition law derived in this work is therefore a statement about the combination of scalar modulation, not about the addition or interaction of energetic quantities.

In particular, the multiplicative form

$$\mathcal{A}_{\rm eff} = \mathcal{A}_{\rm amb}\mathcal{A}_{\rm loc}$$

should be understood as a structural relation governing the modulation of capacity, rather than as a dynamical law derived from energy-based principles.

10.2 Diagnostic Role of Perturbative Quantities

In weak-modulation regimes, it is convenient to represent scalar modulation using additive diagnostic quantities such as

$$\chi_{\rm amb}, \qquad \chi_{\rm loc}.$$

In hydrogenic applications, these are often written in the form

$$\chi_{\rm loc} = \frac{T}{m_e c^2}, \qquad \chi_{\rm amb} = -\frac{V}{m_e c^2}.$$

These quantities should not be interpreted as fundamental energetic components of the scalar field. Rather, they serve as first-order diagnostic measures of deviation from baseline scalar availability.

Their appearance reflects the structure of the weak-limit expansion, not the underlying ontology of the theory.

10.3 Separation from Force-Based Descriptions

The scalar–conformal framework does not rely on force-based descriptions of interaction. Instead, geometric structure and transport dynamics are governed by scalar modulation and its influence on admissible configurations and trajectories.

The composition law derived here is therefore independent of any notion of force or interaction potential. Where quantities resembling potentials appear in weak-limit descriptions, they should be understood as representations of scalar modulation rather than as sources of force.

10.4 Interpretation of Local Modulation

The local scalar factor $\mathcal{A}_{\rm loc}(u)$ encodes the structural response of a system to transport and adjustment. This response is not specified in terms of conventional kinematic variables at the foundational level.

Instead, it reflects the interaction between the system and the underlying delivery structure that determines scalar availability. In weak regimes, this response admits a representation in terms of quantities such as $T(x)$, but such representations are derived and approximate.

10.5 Scope of Validity of the Composition Law

The multiplicative composition law is derived under structural assumptions of independence and separability of contributions. It applies to regimes in which scalar modulation may be decomposed into distinct ambient and local components.

In regimes where this separation breaks down, or where additional coupling mechanisms are present, the composition law may require modification or extension.

However, within its domain of applicability, the multiplicative form provides the minimal and structurally consistent rule for combining scalar modulation.

10.6 Relation to Observational Quantities

Observable quantities in specific domains may be expressed in terms of the scalar diagnostic $\mathcal{A}_{\text{eff}}$ or its weak-limit expansion. The interpretation of such observables depends on the context of the corresponding sectoral model.

The present work does not attempt to assign direct observational meaning to $\mathcal{A}$ beyond its role as a structural measure of scalar modulation.

10.7 Summary

The scalar composition law is a structural result grounded in the scalar ontology of the NUVO framework. Its interpretation does not depend on energy-based or force-based concepts, and its use in practical calculations relies on diagnostic representations valid in appropriate regimes.

These clarifications ensure that the composition principle is applied consistently and without ambiguity in subsequent developments.

11 Summary

In this work, we have derived a general composition law governing scalar modulation in the scalar–conformal NUVO framework. The scalar field $\Lambda(x)$ defines the geometric structure of spacetime through its role as a measure of locally available structural capacity, and the normalized scalar diagnostic $\mathcal{A}$ captures deviations from the baseline level Λ_0.

A central structural question addressed in this paper is how multiple contributions to scalar modulation combine into a single effective quantity experienced by a transported structure. We have shown that these contributions separate into two distinct components: an ambient component determined by the scalar field configuration and a local component associated with transport and structural adjustment.

Under the requirements of normalization, positivity, and independence of contributions, we derived the Effective Scalar Composition Principle,

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}},$$

which expresses the effective scalar modulation as the product of ambient and local factors.

We established the fundamental properties of this composition law, including normalization, monotonicity, and compatibility with the scalar–conformal metric structure. The multiplicative form admits a natural extension to systems with multiple independent contributions and provides a consistent framework for combining scalar modulation effects across a wide range of configurations.

In the weak-modulation regime, we showed that the composition law reduces to an additive diagnostic form,

$$\mathcal{A}_{\text{eff}} \approx 1 + \chi_{\text{amb}} + \chi_{\text{loc}},$$

thereby explaining the structure of scalar modulation expressions used in earlier analyses. In particular, the hydrogenic form

$$\mathcal{A}_{\text{eff}} = 1 + \frac{T}{m_e c^2} - \frac{V}{m_e c^2}$$

is recovered as a first-order approximation, with T and V interpreted as diagnostic quantities rather than fundamental variables.

The composition law derived here provides a unifying structural principle for scalar modulation across all domains of the NUVO program. It connects scalar ontology to transport, exchange, and geometric structure in a manner that is independent of specific dynamical models.

This result completes the specification of scalar modulation at the foundational level and establishes a consistent framework for its use in subsequent developments, including bound-state analysis, transport theory, relativistic regimes, and large-scale structure.

Future work may explore the detailed form of the local response functional $\mathfrak{L}(\sigma_{\text{loc}})$ and investigate the role of higher-order coupling terms beyond the weak-modulation regime.

A Alternative Composition Forms

A.1 General Formulation

We consider the problem of constructing an effective scalar diagnostic

$$\mathcal{A}_{\text{eff}} = F(\mathcal{A}_{\text{amb}}, \mathcal{A}_{\text{loc}}),$$

where $F : \mathbb{R}_{>0} \times \mathbb{R}_{>0} \to \mathbb{R}_{>0}$ is a positive-valued function combining ambient and local contributions.

As established in the main text, the composition function F must satisfy the following structural conditions:

- **Normalization:** $F(1, 1) = 1$;
- **Ambient reduction:** $F(a, 1) = a$ for all $a > 0$;
- **Local reduction:** $F(1, \ell) = \ell$ for all $\ell > 0$;
- **Positivity:** $F(a, \ell) > 0$ for all $a, \ell > 0$;
- **Independence:** ambient and local contributions combine without mutual determination.

We examine candidate composition forms under these constraints.

A.2 Additive Forms

A natural alternative is an additive construction of the form

$$F(a, \ell) = a + \ell - 1.$$

This form satisfies normalization and reduction conditions. However, it fails to preserve positivity under independent variation. For example, if $a < 1$ and $\ell < 1$, then

$$F(a, \ell) = a + \ell - 1 < 0,$$

which is inadmissible since $\mathcal{A}_{\text{eff}}$ must remain strictly positive.

Moreover, additive forms treat scalar contributions as offsets rather than as modulators of structural capacity. This is incompatible with the interpretation of $\mathcal{A}$ as a multiplicative scaling of availability.

Thus, additive composition is structurally inconsistent with the scalar ontology.

A.3 General Nonlinear Forms

Consider a general composition law

$$F(a, \ell),$$

subject to the conditions above.

Independence of contributions implies that the effect of ambient and local modulation must separate under independent variation. In particular, sequential application of independent contributions must be order-independent and compatible with reduction conditions.

These requirements constrain F to be multiplicatively separable, i.e., of the form

$$F(a, \ell) = f(a)\, g(\ell),$$

for some positive functions f and g.

Applying the reduction conditions yields

$$f(a) = a, \qquad g(\ell) = \ell,$$

and hence

$$F(a, \ell) = a\, \ell.$$

Thus, multiplicative composition is the unique separable form consistent with the structural requirements.

A.4 Logarithmic and Exponential Representations

Alternative representations may be obtained by introducing a reparameterization of the scalar diagnostic. For example, defining

$$\psi = \log \mathcal{A},$$

one obtains an additive composition law

$$\psi_{\text{eff}} = \psi_{\text{amb}} + \psi_{\text{loc}}.$$

However, this is equivalent to the multiplicative law in the original variables, since

$$\mathcal{A}_{\text{eff}} = \exp(\psi_{\text{eff}}) = \exp(\psi_{\text{amb}})\, \exp(\psi_{\text{loc}}) = \mathcal{A}_{\text{amb}} \mathcal{A}_{\text{loc}}.$$

Thus, logarithmic or exponential formulations do not introduce new composition laws, but merely provide alternative coordinate representations of the same multiplicative structure.

A.5 Uniqueness Within Structural Constraints

The analysis above shows that any admissible composition law satisfying normalization, positivity, and independence of contributions must be equivalent to the multiplicative form, up to reparameterization.

In particular:

- Additive forms violate positivity or structural interpretation;
- General nonlinear forms reduce to multiplicative separability under independence constraints;
- Alternative representations are equivalent to multiplication under a change of variables.

Therefore, the multiplicative composition law

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}}$$

is structurally distinguished as the minimal and consistent rule for combining scalar modulation.

A.6 Summary

We have examined alternative composition forms and shown that the multiplicative law is uniquely selected by the structural requirements of the scalar ontology.

This result reinforces the derivation in the main text and confirms that the composition principle is not an arbitrary choice, but a necessary consequence of the underlying framework.

B Higher-Order Corrections

B.1 Exact Composition Law

The composition law derived in the main text is exact and given by

$$\mathcal{A}_{\text{eff}} = \mathcal{A}_{\text{amb}}\mathcal{A}_{\text{loc}}.$$

To analyze deviations from the weak-modulation regime, we consider the expansion of this expression beyond first order.

B.2 Perturbative Expansion

We write the ambient and local contributions as perturbations about the baseline state:

$$\mathcal{A}_{\text{amb}} = 1 + \chi_{\text{amb}}, \qquad \mathcal{A}_{\text{loc}} = 1 + \chi_{\text{loc}},$$

where χ_{amb} and χ_{loc} are not assumed to be infinitesimal, but serve as formal expansion parameters.

Substituting into the composition law yields

$$\mathcal{A}_{\text{eff}} = (1 + \chi_{\text{amb}})(1 + \chi_{\text{loc}}) = 1 + \chi_{\text{amb}} + \chi_{\text{loc}} + \chi_{\text{amb}}\chi_{\text{loc}}.$$

B.3 Second-Order Contribution

The term

$$\chi_{\text{amb}}\chi_{\text{loc}}$$

represents the leading higher-order correction to the weak-limit approximation.

This term encodes a coupling between ambient scalar geometry and local structural modulation that is absent in the first-order additive representation.

B.4 Interpretation of the Coupling Term

The product term $\chi_{\text{amb}}\chi_{\text{loc}}$ arises directly from the multiplicative composition law and reflects the fact that ambient and local contributions act as successive modulators of structural capacity.

In particular:

- χ_{amb} represents the modification of available capacity due to ambient geometry;
- χ_{loc} represents the modification due to local structural state;
- their product represents the combined effect when both modulations are present simultaneously.

Thus, the coupling term should be interpreted as a structural interaction between geometry and transport, rather than as an independent contribution.

B.5 Deviation from Additive Behavior

In the weak-modulation regime, the coupling term is negligible, and the effective scalar diagnostic reduces to the additive form

$$\mathcal{A}_{\text{eff}} \approx 1 + \chi_{\text{amb}} + \chi_{\text{loc}}.$$

However, when either χ_{amb} or χ_{loc} is not small, the coupling term becomes significant, and the additive approximation breaks down.

This marks the transition from linear to nonlinear scalar modulation.

B.6 General Higher-Order Structure

The expansion above may be viewed as the first terms in a general series representation of scalar modulation. In particular, higher-order terms correspond to increasingly nonlinear interactions between ambient and local contributions.

While the present framework does not specify additional terms beyond those implied by the multiplicative law, the structure suggests that nonlinear effects are intrinsic to regimes of strong scalar modulation.

B.7 Relation to Diagnostic Representations

In practical applications, diagnostic quantities such as $T(x)$ and $V(x)$ may be used to represent χ_{loc} and χ_{amb} in weak regimes.

Beyond the weak limit, such representations must be interpreted with care, as the simple additive form no longer captures the full structure of scalar modulation.

The multiplicative composition law provides the correct underlying framework, while additive expressions serve only as approximations.

B.8 Scope and Limitations

The analysis presented here is purely structural and does not assign specific physical interpretations to higher-order terms.

The detailed role of these corrections depends on the sectoral context in which scalar modulation is applied, including transport dynamics, bound-state structure, and large-scale geometry.

Further investigation is required to determine the quantitative impact of higher-order effects in specific models.

B.9 Summary

The multiplicative composition law naturally generates higher-order corrections to the weak-limit additive form. The leading correction term encodes coupling between ambient geometry and local structural modulation and becomes significant in regimes of strong scalar variation.

This structure provides a systematic extension beyond first-order approximations while preserving the underlying scalar ontology.

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