M5 – Holonomic Coherence and Geometric Quantization on a Scalar–Conformal Manifold

Preprint, Version 1.0[*]

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Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.
- The dimensionless scalar diagnostic is

$$\mathcal{A}(x) \coloneqq \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

Notation convention. We reserve distinct symbols for fundamental fields and derived quantities. The scalar field Λ represents structural availability, while $\mathcal{A}$ denotes a derived normalized response field. These should not be conflated.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-,+,+,+)$.

Abstract

This paper studies coherent exchange cycles on a scalar–conformal manifold within the NUVO framework. Building on the exchange sector introduced in M4 [1], we examine closed exchange configurations for which circulating exchange structure remains coherent over a complete cycle.

We introduce a diagnostic phase-return variable for exchange cycles and show that coherent circulation imposes a global holonomic admissibility constraint on the cycle as a whole. This constraint restricts the family of exchange configurations that can persist without radiative decay.

The purpose of the present paper is not to impose an external quantization rule, but to identify the geometric and structural origin of restricted admissible exchange states within the scalar–conformal framework. In this way, M5 provides the coherence-based bridge from the exchange sector to the later quantization program.

1 Introduction

The preceding papers in the NUVO series established the scalar–conformal geometric framework and the two principal dynamical sectors that arise within it. The scalar field equation introduced in M1 [2] governs the structural modulation of the physical metric. M2 [3] interpreted this scalar structure in terms of structural capacity and availability. M3 [4] then identified localized structural depletion as the source of the gravitational sector, while M4 [1] introduced the exchange sector associated with open-loop exchange structure and the radiative propagation of dynamic-loop configurations.

The present paper addresses a further structural feature of exchange systems: the existence of coherent exchange cycles. When exchange processes occur in a configuration that allows exchange structure to circulate through a closed path while maintaining coherence with the surrounding scalar environment, global constraints arise on the admissible configurations of the system. These constraints are holonomic in nature and restrict the possible exchange states that may persist in equilibrium.

The appearance of such constraints restricts the admissible exchange configurations to a limited family compatible with global coherence. Within the present framework, this restricted admissible family is taken as the structural precursor of discrete exchange states. In the NUVO framework these discrete configurations are interpreted as structurally stable exchange states arising from coherent transport on the scalar–conformal manifold.

The goal of the present work is therefore to identify the structural conditions under which coherent exchange cycles can exist and to show how these conditions restrict the admissible family of exchange states and prepare the emergence of discrete state structure. No reference to external quantum formalisms is required for this construction; the discrete structure emerges purely from the internal coherence constraints of the exchange sector.

The organization of the paper is as follows. Section 2 introduces the concept of exchange cycles and describes how closed transport paths arise within systems composed of open-loop exchange structures. Section 3 examines the coherence conditions required for such cycles to persist. Section 4 studies the global holonomic constraints that follow from coherent transport. Section 5 shows how these constraints restrict the admissible exchange configurations to a coherent family that is interpreted as discrete within the NUVO framework. Finally, Section 6 summarizes the resulting structural mechanism for geometric quantization within the NUVO framework.

1.1 Scope of the present paper

The present paper isolates a coherence-level structural feature of the exchange sector. It does not introduce a new field equation, and it does not attempt to recover the full phenomenology of quantum theory within the present manuscript. Its purpose is more limited: to identify the global return constraints associated with coherent exchange cycles and to show how these constraints restrict the admissible family of exchange configurations.

2 Exchange Cycles

In the exchange sector described in M4 [1], exchange structure is organized through open-loop configurations whose local behavior is described by the exchange current J^{μ}_{ex}. These currents serve as a diagnostic representation of directed exchange structure between source and sink structures embedded in the scalar–conformal manifold $(\mathcal{M}, g)$.

In many physical systems, however, exchange transport does not occur as a purely open flow between isolated structures. Instead, multiple exchange elements may be arranged in such a way that the exchange configuration follows a closed path through the system. When this occurs, The exchange configuration associated with one element may return to its point of origin after circulating through the surrounding exchange network.

Such configurations define *exchange cycles*. An exchange cycle is a closed exchange path γ on the manifold along which exchange structure circulates through a sequence of exchange interactions.

Formally, an exchange cycle may be represented by a closed curve

$$\gamma : S^1 \to \mathcal{M}$$

along which the exchange current remains tangent to the path,

$$J^{\mu}_{\text{ex}} \propto \dot{\gamma}^{\mu}.$$

The exchange configuration along such a path produces a circulating exchange pattern within the system. Because the path is closed, the cycle may be internally sustained over one complete traversal without requiring a net external exchange imbalance.

Exchange cycles therefore represent internally sustained transport configurations within the exchange sector. Their persistence depends on the compatibility of the circulating transport with the surrounding capacity field. When this compatibility is satisfied, the exchange cycle can operate as a dynamically stable structure embedded in the manifold.

The existence of exchange cycles provides the structural setting in which coherent transport can arise. The conditions under which such coherence is maintained will be examined in the following section.

3 Coherence Conditions for Exchange Cycles

The exchange cycles introduced in the previous section describe closed paths along which exchange structure circulates through a sequence of exchange interactions. The mere existence of a closed transport path, however, does not guarantee that the corresponding exchange flow can persist as a stable configuration. For sustained circulation to occur, the circulating exchange configuration must remain compatible with the surrounding scalar environment throughout the entire cycle.

This compatibility requirement is expressed through a coherence condition. As capacity is transported along a cycle γ, the local exchange process must remain synchronized with the ambient scalar structure defined by the scalar field. If this synchronization is lost, the circulating configuration cannot be maintained and the exchange cycle will dissipate through radiative exchange with the surrounding manifold.

Let $\gamma : S^1 \to \mathcal{M}$ denote a closed exchange cycle. Along the path γ, the exchange current J^μ_{ex} defines a directed transport of capacity through the manifold. Associated with this cycle is a local exchange phase Φ, introduced here as a diagnostic return variable that tracks the relative state of the circulating exchange configuration with respect to the ambient scalar structure. The present paper does not require Φ to be specified as an independent dynamical field; it serves only to encode the global phase-return condition associated with coherent exchange cycles.

For the exchange cycle to persist, this phase must return to its initial value after one complete traversal of the cycle. If a mismatch occurs, the circulating exchange configuration will no longer remain synchronized with the exchange structures through which it propagates, and the cycle will break down.

The requirement of phase return defines the coherence condition for a stable exchange cycle. Denoting the accumulated phase mismatch along the cycle by $\Delta\Phi(\gamma)$, coherence requires

$$\Delta\Phi(\gamma) = 0.$$

Exchange cycles satisfying this condition may operate as persistent transport configurations within the exchange sector. Cycles that fail to satisfy the coherence condition cannot maintain stable circulation and will decay through radiative exchange processes.

The global consequences of this coherence requirement will be examined in the following section, where the phase constraint associated with coherent exchange cycles will be shown to produce holonomic restrictions on admissible configurations of the system.

4 Holonomic Constraints from Coherent Exchange

The coherence condition introduced in the previous section ensures that circulating exchange structure remains synchronized with the surrounding capacity field throughout a complete traversal of an exchange cycle. When this condition is satisfied, the circulating exchange cycle forms a persistent coherent structure embedded in the exchange sector.

Definition 4.1 (Exchange holonomy)**.** *Let A denote the exchange connection 1-form associated with a coherent exchange cycle γ. The accumulated return of the cycle is defined by*

$$\Theta(\gamma) := \oint_\gamma A.$$

Because the exchange cycle is closed, the coherence condition imposes a global restriction on the admissible configurations of the system. The state of the circulating exchange configuration cannot be specified independently at each point of the cycle; instead, the configuration must satisfy a single compatibility condition over the entire closed path. Such a restriction constitutes a holonomic admissibility constraint on the exchange dynamics.

Let $\gamma : S^1 \to \mathcal{M}$ denote a closed exchange cycle and let Φ denote the exchange phase introduced in the previous section. The coherence condition requires that the accumulated phase change along the cycle vanish,

$$\Theta(\gamma) \in \mathcal{H}_{\mathrm{adm}}, \tag{1}$$

Here $\mathcal{H}_{\mathrm{adm}}$ denotes the admissible return class determined by the coherence structure of the exchange cycle. This condition expresses holonomic closure without assuming a universal phase periodicity. The specific form of $\mathcal{H}_{\mathrm{adm}}$ may depend on the structural setting and is left open at the present stage. In particular, the holonomic constraint is imposed modulo a coherence scale intrinsic to the cycle, rather than modulo a universal external phase unit.

Remark. The holonomic admissibility condition is imposed on the connection 1-form A, not on the differential of a globally defined scalar. Accordingly, the coherence constraint is nontrivial: it expresses a restriction on the exchange holonomy of the cycle rather than an exactness identity.

In particular, the condition does not assume a universal phase periodicity, but instead constrains the admissible return class of the cycle through its holonomy.

This relation expresses the fact that the transported capacity must return to its original exchange state after one complete traversal of the cycle.

Holonomic constraints of this type restrict the family of admissible transport configurations available to the system. Only those configurations that satisfy the global compatibility condition can persist as coherent exchange cycles. Configurations that violate the constraint cannot maintain stable circulation and therefore decay through radiative exchange with the surrounding manifold.

The presence of such constraints implies that the set of stable exchange configurations is restricted by global coherence. In the NUVO setting, this restriction is taken to select a special admissible family of persistent exchange structures, rather than an arbitrary continuous class of exchange configurations. The resulting family of admissible configurations will be examined in the next section.

Interpretive note. The connection 1-form A is introduced as the geometric object encoding exchange transport along the cycle. The phase variable Φ may be viewed as a local diagnostic parameter along the path, but need not be globally well-defined. Accordingly, the holonomy condition is formulated in terms of A rather than $d\Phi$.

5 Restricted Exchange States

The holonomic constraint derived in the previous section restricts the set of exchange cycles that can exist as persistent coherent configurations.

Let γ denote a closed exchange cycle satisfying the holonomic coherence condition

$$\Theta(\gamma) = \oint_\gamma A \in \mathcal{H}_{\mathrm{adm}}.$$

This requirement is not satisfied by arbitrary transport flows. Only specific configurations of circulating exchange structure produce a globally compatible phase return.

Consequently, the family of admissible exchange cycles forms a restricted subset of all possible transport configurations. Each member of this subset corresponds to a stable configuration in which exchange structure circulates through the system while maintaining coherence with the surrounding scalar environment.

Exchange cycles that fail to satisfy the holonomic condition cannot persist and instead decay through radiative exchange with the environment.

The resulting set of coherent exchange cycles therefore defines a restricted family of admissible configurations embedded within the continuous space of exchange configurations. In the present structural setting, this admissible family is interpreted as the origin of discrete exchange-state structure within the NUVO framework.

6 Structural Quantization

The preceding sections established that exchange transport within the NUVO framework can organize into closed exchange cycles embedded in the scalar–conformal manifold. When such cycles maintain coherence with the surrounding scalar environment, the circulating exchange transport must satisfy a global compatibility condition along the closed path.

This requirement introduces a holonomic constraint on the admissible exchange configurations of the system. Only those exchange cycles for which the circulating configuration returns to its initial exchange state after a complete traversal of the cycle can persist as stable structures.

The presence of this constraint restricts the family of admissible exchange cycles to a special set of coherent configurations, which in the present framework is interpreted as giving rise to discrete exchange-state structure. Each such configuration corresponds to a stable exchange state in which exchange structure circulates through the system in a globally compatible manner.

In the NUVO framework, the appearance of discrete exchange states is therefore not imposed through external quantization rules. Instead, the restricted state structure arises from the coherence requirements of circulating exchange cycles on the scalar–conformal manifold.

This mechanism provides the structural basis for quantization within the NUVO program. Subsequent work [5] will examine how these coherent exchange states manifest in specific physical systems and how their admissible configurations determine observable spectral structure.

References

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