Q0 – Planck Units Relation: Support–Exchange Compatibility on Scalar–Conformal NUVO Space

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Abstract

The scalar–conformal NUVO framework distinguishes two structural sectors: a support sector governing persistent anchored structures through boundary flux and capacity delivery, and an exchange sector governing transport through cycles, closure, and coherence. While these sectors are constructed independently, a compatibility condition between their characteristic scales has not previously been formalized.

In this work, we establish a non-dynamical invariant relating the support-sector length scale $r_c = Gm/c^2$ and the exchange-sector coherence scale $\bar{\lambda}_C = \hbar/(mc)$:

$$r_c \, \bar{\lambda}_C = \ell_P^2 = \frac{\hbar G}{c^3}.$$

This relation is independent of the mass m and is interpreted as an invariant interaction area linking support geometry and exchange coherence.

A reconstruction of the exchange coherence scale within the NUVO framework is provided in an appendix, showing that $\bar{\lambda}_C$ arises from coherent closure of exchange transport together with the support-sector persistence scale mc^2, using a single empirical gauge input.

The resulting invariant is not a dynamical law but a structural constraint on admissible configurations. It establishes a bridge between the support and exchange sectors and constrains the development of exchange processes within the NUVO program.

Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.
- The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-, +, +, +)$.

This manuscript is mathematical in scope. It establishes definitions, structural identities, and variational consequences within a scalar–conformal setting. Sector reductions and correspondence limits are recorded only when explicitly stated as additional assumptions and are not used as premises in derivations.No claim of full dynamical equivalence to general relativity, quantum mechanics, or classical field theories is made at the level of the present foundational development. Where later papers compare limiting behavior, those comparisons are presented as correspondence targets rather than as identity statements.The NUVO program is organized as a sequence of internally consistent mathematical papers. Foundational papers (M-series) fix the scalar–conformal geometry, variational structure, and notation. Subsequent papers treat sectoral reductions (gravity, exchange, quantization, and bound-state structure) as controlled specializations of the foundational framework.**Scalar ontology.** The scalar field Λ represents the *locally available structural capacity* of an underlying delivery field permeating spacetime. The baseline level Λ_0 denotes the availability supported by this intrinsic delivery structure in the absence of structural occupation. Localized structures or transport processes may reduce the available capacity relative to this baseline, but the intrinsic delivery baseline itself is not altered. Consequently the scalar field measures the *available portion* of structural capacity rather than the intrinsic production of the underlying field.

1 Introduction

Program context. The scalar–conformal NUVO framework is developed through a sequence of papers that separately establish the structure of two distinct sectors:

- the *support sector* [1–3], governing persistent anchored structures through boundary flux, capacity delivery, and scalar–conformal geometry,
- the *exchange sector* [4–6], governing transport through cycles, closure conditions, and coherence structure.

The support sector has been developed in the M-series, culminating in a formulation in which capacity is understood as a uniform delivery process and persistent structures are characterized by invariant intake rates and boundary flux distributions. The exchange sector, developed in the Q-series, introduces cycle-based transport and coherence conditions leading to quantization structure.

Motivation. Although both sectors are well-defined, they have thus far been treated as structurally independent. In particular, no formal condition has been imposed linking:

- the geometric scales associated with support-sector persistence, and
- the coherence scales governing exchange transport.

This separation leaves open the question of whether arbitrary combinations of support and exchange structures are admissible.

Objective. The purpose of the present work is to establish a structural compatibility condition between the two sectors. Specifically, we identify and formalize an invariant relation between:

- the support-sector length scale
$$r_c = \frac{Gm}{c^2},$$
and
- the exchange-sector coherence scale
$$\bar{\lambda}_C = \frac{\hbar}{mc}.$$

Main result. We show that the product of these scales is independent of mass and given by

$$r_c \, \bar{\lambda}_C = \ell_P^2 = \frac{\hbar G}{c^3}.$$

The combination $\hbar G/c^3$ is the squared Planck length [7], introduced historically as the unique area constructible from the fundamental constants $\hbar$, G, and c.

This relation is interpreted as an invariant area linking support-sector geometry and exchange-sector coherence.

Method and scope. The derivation is entirely non-dynamical. It does not introduce equations of motion, force laws, or probabilistic structure. Instead, it relies on:

- the structural definitions of the support and exchange sectors,
- dimensional and algebraic relations between their characteristic scales,
- and a reconstruction of the exchange coherence scale within the NUVO framework (Appendix A).

Interpretation. The invariant relation is interpreted as a compatibility condition rather than a dynamical law. It constrains the admissible combinations of support localization and exchange coherence without prescribing how systems evolve.

Position in the NUVO program. This paper serves as a bridge between the M-series and the Q-series:

- it completes the support-sector foundation by identifying its coupling to exchange structure,
- and it constrains the admissible development of exchange processes in subsequent work.

Outline of the paper. Section 2 introduces the characteristic scales of the two sectors. Section 3 establishes the invariant product relation and expresses it in fundamental constants. Section 4 interprets the invariant geometrically as an interaction area. Section 5 develops the structural consequences of the relation. Section 6 discusses its programmatic implications. Appendix A reconstructs the exchange coherence scale within the NUVO framework.

2 Characteristic Scales of the Two Sectors

2.1 Support-Sector Length Scale

Within the support sector of the NUVO framework, persistent anchored structures are characterized by their interaction with the scalar–conformal geometry. As established in the M–series, the physical metric takes the form

$$g_{\mu\nu} = \Lambda^2(x)\,\eta_{\mu\nu},$$

where $\Lambda(x)$ encodes the local structure of capacity delivery.

A natural length scale associated with an anchored structure of mass m arises from the scalar–conformal geometric framework [8, 9]. This scale is given by

$$r_c = \frac{Gm}{c^2},$$

and will be referred to as the *support-sector characteristic length.*

The scale r_c is determined entirely within the support sector and reflects the geometric role of mass in the scalar-modulated delivery structure. It is independent of any exchange-sector construction and depends only on the constants G and c together with the mass parameter m.

2.2 Exchange-Sector Coherence Scale

Within the exchange sector, interaction processes are governed by admissible transport and closure conditions associated with open-loop exchange structure. A characteristic length scale arises from the requirement of coherence in exchange cycles.

This scale is given by the reduced Compton wavelength [10]

$$\bar{\lambda}_C = \frac{\hbar}{mc},$$

which will be referred to as the *exchange-sector coherence scale.*

The scale $\bar{\lambda}_C$ characterizes the admissible closure structure associated with a mass parameter m in the exchange sector. It depends on $\hbar$ and c, and is defined independently of the scalar–conformal geometry and the support-sector delivery framework.

No interpretation of $\bar{\lambda}_C$ as a wavelength is required in the present context; it is treated purely as a characteristic coherence length associated with exchange-sector structure.

2.3 Independence of Sector Definitions

The support-sector scale r_c and the exchange-sector scale $\bar{\lambda}_C$ arise from distinct structural constructions.

- The support-sector scale r_c is determined by scalar–conformal geometry and the role of mass in capacity-modulated delivery.

- The exchange-sector scale $\bar{\lambda}_C$ is determined by admissible closure and coherence conditions in exchange transport.

These constructions involve disjoint sets of constants:

- r_c depends on (G, c, m),
- $\bar{\lambda}_C$ depends on $(\hbar, c, m)$.

No assumption is made at this stage regarding any relationship between these scales. In particular, no coupling between the support and exchange sectors is introduced, and no shared derivation is assumed.

The two scales are therefore treated as *a priori independent* quantities associated with different structural aspects of the NUVO framework.

3 Invariant Product Relation

3.1 Derivation of the Scale Product

Characteristic scales of the two sectors. The support and exchange sectors each admit a natural characteristic length scale:

- the support-sector scale
$$r_c := \frac{Gm}{c^2},$$
arising from the scalar–conformal geometric response to a persistent anchor,
- the exchange-sector coherence scale
$$\bar{\lambda}_C := \frac{\hbar}{mc},$$
associated with coherent closure of exchange transport.

The latter is not introduced as an independent postulate; its reconstruction within the NUVO framework from exchange closure and support persistence is given in Appendix A.

Product of sector scales. The product of these two characteristic lengths is

$$r_c\,\bar{\lambda}_C = \left(\frac{Gm}{c^2}\right)\left(\frac{\hbar}{mc}\right).$$

The mass m cancels, yielding
$$r_c\,\bar{\lambda}_C = \frac{\hbar G}{c^3}.$$

Identification with Planck area. We identify

$$\ell_P^2 := \frac{\hbar G}{c^3},$$

so that the product becomes
$$r_c\,\bar{\lambda}_C = \ell_P^2.$$

Mass independence. This relation is independent of the mass m of the anchor. While both r_c and $\bar{\lambda}_C$ depend individually on m, their product is invariant.

Interpretation. The support-sector scale decreases with decreasing mass, while the exchange coherence scale increases. Their product remains fixed, indicating that the two sectors are not independent but constrained by a shared invariant.

This invariant will be interpreted in the following subsection as an interaction area.

3.2 Expression in Fundamental Constants

Fundamental structure. The invariant product

$$r_c \, \bar{\lambda}_C = \frac{\hbar G}{c^3}$$

depends only on the universal constants:

- $\hbar$ (exchange-sector action scale),
- G (support-sector coupling scale),
- c (invariant transport speed).

Sector interpretation of constants. Within the NUVO framework:

- $\hbar$ arises from coherent closure of exchange transport (Appendix A),
- G characterizes the response of the support sector to persistent anchors,
- c characterizes the underlying delivery process common to both sectors.

Absence of mass scale. The absence of m in the product indicates that the invariant is not tied to any specific system, but is a universal structural relation.

Role in the present work. This invariant will be interpreted geometrically as an interaction area that constrains the compatibility between support-sector geometry and exchange-sector coherence.

3.3 Invariant Area Form

Area interpretation. The relation

$$r_c \, \bar{\lambda}_C = \ell_P^2$$

has the dimensional form of an area. It may therefore be interpreted as defining a characteristic interaction area shared between the support and exchange sectors.

Geometric meaning. In the support sector, r_c characterizes the spatial extent over which an anchor influences the scalar–conformal geometry. In the exchange sector, $\bar{\lambda}_C$ characterizes the coherence length required for exchange transport.

Their product defines an area that represents the minimal geometric domain over which both structures must be simultaneously satisfied.

Compatibility constraint. The invariant area expresses a compatibility condition:

- support-sector geometry cannot be specified independently of exchange coherence,
- and exchange coherence cannot be specified independently of support-sector geometry.

Structural role. This relation is not introduced as a dynamical law, but as a structural constraint linking the two sectors. It provides the first explicit bridge between:

- boundary flux structure in the support sector, and
- coherence structure in the exchange sector.

4 Equivalent Forms and Ratio Structure

4.1 Full and Reduced Compton Forms

The invariant product relation derived in Section 3 may be expressed using either the reduced or full Compton scale.

Using the reduced Compton wavelength

$$\bar{\lambda}_C = \frac{\hbar}{mc},$$

we have

$$r_c\,\bar{\lambda}_C = \ell_P^2.$$

Alternatively, introducing the full Compton wavelength

$$\lambda_C = \frac{h}{mc} = 2\pi\bar{\lambda}_C,$$

the product becomes

$$r_c\,\lambda_C = 2\pi\ell_P^2.$$

Both forms represent the same invariant structure, differing only by a constant geometric factor.

4.2 Dimensionless Ratios

The invariant relation may be recast in terms of dimensionless ratios by normalizing each scale with respect to a common reference.

Dividing both sides of

$$r_c\,\bar{\lambda}_C = \ell_P^2$$

by $\bar{\lambda}_C^2$ yields

$$\frac{r_c}{\bar{\lambda}_C} = \frac{\ell_P^2}{\bar{\lambda}_C^2}.$$

Substituting definitions,

$$\frac{r_c}{\bar{\lambda}_C} = \frac{\frac{Gm}{c^2}}{\frac{\hbar}{mc}} = \frac{Gm^2}{\hbar c}.$$

Thus,

$$\frac{r_c}{\bar{\lambda}_C} = \frac{Gm^2}{\hbar c}.$$

This ratio is dimensionless and depends only on the mass parameter and fundamental constants. Equivalently,

$$\frac{\bar{\lambda}_C}{r_c} = \frac{\hbar c}{Gm^2}.$$

4.3 Appearance of the Gravitational Coupling Scale

The dimensionless ratio

$$\frac{Gm^2}{\hbar c}$$

is recognized as the gravitational coupling scale associated with a mass m.

The invariant product relation may therefore be expressed as

$$\frac{r_c}{\bar{\lambda}_C} = \frac{Gm^2}{\hbar c}, \qquad r_c\,\bar{\lambda}_C = \ell_P^2,$$

showing that the same structure admits both:

- an invariant area form independent of m,
- a dimensionless ratio depending on m.

These two expressions represent dual aspects of the same underlying scale relation: one invariant under variation of m, and one encoding its dependence.

5 Geometric Interpretation as Interaction Area

5.1 Area Interpretation of Boundary Interaction

Boundary flux perspective. In the support sector, the physical state of an anchor is determined by its boundary flux distribution. The boundary defines the interface through which the anchor interacts with the surrounding scalar–conformal geometry.

This interaction is inherently geometric and localized on the boundary surface of the anchor.

Area as interaction measure. The invariant relation

$$r_c\,\bar{\lambda}_C = \ell_P^2$$

suggests that the interaction between support and exchange structures is naturally expressed in terms of an area.

The support-sector scale r_c characterizes the spatial extent of boundary influence, while the exchange coherence scale $\bar{\lambda}_C$ characterizes the transverse extent required for coherent exchange transport.

Their product therefore defines an effective interaction area across which both structures must be satisfied simultaneously.

Interpretive role. This area is not a physical surface in the conventional sense, but a geometric compatibility measure governing how boundary flux and exchange coherence can coexist.

It provides a natural interface between:

- boundary-localized structure (support sector), and
- cycle-based coherence (exchange sector).

5.2 Compatibility Between Boundary Flux and Exchange Coherence

Independent sector structures. The support and exchange sectors are constructed independently:

- the support sector describes persistent anchors through boundary flux and capacity intake,
- the exchange sector describes transport through cycles and coherence conditions.

There is no a priori requirement that these structures be mutually compatible.

Compatibility constraint. The invariant area relation imposes a constraint linking the two:

$$r_c \, \bar{\lambda}_C = \ell_P^2.$$

This relation ensures that the spatial extent over which boundary flux is defined is compatible with the coherence length required for exchange transport.

Geometric consequence. If the support scale r_c decreases, the exchange coherence scale $\bar{\lambda}_C$ must increase, and vice versa. This reciprocal relationship enforces a balance between:

- localization of boundary interaction, and
- extension of coherent transport.

Structural interpretation. The invariant area therefore expresses a geometric constraint:

> Exchange coherence cannot be arbitrarily localized without affecting support structure, and support localization cannot be arbitrarily increased without affecting exchange coherence.

This establishes a direct but non-dynamical coupling between the two sectors.

5.3 Sector Coupling Without Mixing Ontology

Preservation of sector distinction. The invariant area relation links the support and exchange sectors without requiring them to be unified into a single ontological description.

- The support sector remains defined by boundary flux and persistence.
- The exchange sector remains defined by cycles and coherence.

No dynamical coupling introduced. The relation

$$r_c \, \bar{\lambda}_C = \ell_P^2$$

does not introduce a dynamical interaction between the sectors. It does not specify how one sector evolves in response to the other.

Instead, it constrains which configurations are admissible.

Admissibility condition. A physically realizable configuration must satisfy both:

- support-sector persistence constraints, and
- exchange-sector coherence constraints,

subject to the invariant area relation.

Configurations that violate this relation are not dynamically forbidden, but structurally inadmissible within the NUVO framework.

Role in the program. This distinction is essential for maintaining the separation of:

- structural constraints (M-series and Q0), and
- dynamical evolution (subsequent developments).

The present work therefore establishes a compatibility condition without introducing a unified dynamical theory.

6 Structural Consequences

6.1 Constraint on Admissible Physical Scales

Compatibility requirement. The invariant relation

$$r_c \, \bar{\lambda}_C = \ell_P^2$$

imposes a constraint on the simultaneous specification of support and exchange scales.

Admissible configurations. A physically realizable anchored system must admit both:

- a support-sector scale $r_c = Gm/c^2$, and
- an exchange-sector coherence scale $\bar{\lambda}_C = \hbar/(mc)$,

such that their product satisfies the invariant area relation.

Exclusion of arbitrary scaling. This condition excludes arbitrary combinations of localization and coherence. In particular:

- exchange coherence cannot be imposed at scales incompatible with the persistence scale of the anchor,
- support localization cannot be specified independently of exchange coherence requirements.

Structural interpretation. The invariant therefore acts as a selection rule on admissible scale combinations, independent of any dynamical law.

6.2 Mass Dependence and Scale Duality

Reciprocal scaling. The support and exchange scales exhibit reciprocal dependence on mass:

$$r_c = \frac{Gm}{c^2}, \qquad \bar{\lambda}_C = \frac{\hbar}{mc}.$$

As the mass m increases:

- the support scale r_c increases,
- the exchange coherence scale $\bar{\lambda}_C$ decreases.

Scale duality. This reciprocal behavior defines a duality between:

- geometric localization (support sector), and
- coherence extension (exchange sector).

The invariant area relation ensures that this duality is exact:

$$r_c \, \bar{\lambda}_C = \ell_P^2.$$

Interpretation. Large-mass systems are characterized by extended support influence and tightly localized exchange coherence, while small-mass systems exhibit the opposite behavior.

Structural balance. The invariant enforces a balance between these regimes, preventing either scale from being varied independently of the other.

6.3 Role of the Planck Area as Compatibility Measure

Definition. The invariant area

$$\ell_P^2 = \frac{\hbar G}{c^3}$$

arises as the product of the exchange action scale, the support coupling scale, and the inverse cube of the invariant speed.

Interpretation within NUVO. Within the NUVO framework, ℓ_P^2 is not introduced as a fundamental quantum of area, but as the compatibility measure linking the support and exchange sectors.

Independence from specific systems. The Planck area does not depend on the properties of any particular anchored system. It is universal, reflecting the underlying constants that govern both sectors.

Structural role. The relation

$$r_c \, \bar{\lambda}_C = \ell_P^2$$

indicates that ℓ_P^2 sets the scale at which support localization and exchange coherence must balance.

Scope clarification. No claim is made that physical space is discretized at the Planck scale, nor that ℓ_P^2 represents a minimal measurable area. Its role in the present work is purely that of a structural invariant arising from sector compatibility.

Programmatic significance. The identification of ℓ_P^2 as a compatibility measure suggests that any complete description of physical systems within the NUVO framework must respect this invariant relation between support and exchange structures.

7 Discussion and Programmatic Implications

7.1 Bridge to the Exchange Series

Role of the present work. The present paper establishes a structural compatibility condition between the support and exchange sectors through the invariant relation

$$r_c \, \bar{\lambda}_C = \ell_P^2.$$

This relation is non-dynamical and does not prescribe the behavior of exchange transport. Instead, it constrains the admissible scales at which exchange processes may occur.

Constraint on exchange structure. Within the Q-series, exchange transport is developed in terms of cycles, closure conditions, and coherence. The identification of the coherence scale

$$\bar{\lambda}_C = \frac{\hbar}{mc}$$

as a derived quantity (Appendix A) implies that all exchange constructions must be compatible with the support-sector persistence scale.

Programmatic implication. As a consequence, the exchange sector cannot be developed independently of the support sector. Any admissible exchange configuration must respect the invariant area relation, which acts as a global constraint on coherence structure.

Position in the NUVO program. This paper therefore serves as a bridge:

- it completes the support-sector foundation established in the M-series,
- and constrains the admissible structure of exchange processes developed in the Q-series.

It does not introduce new exchange mechanisms, but restricts the space of admissible exchange descriptions.

7.2 Interpretive Remarks (Non-Dynamical)

Nature of the invariant. The relation

$$r_c \, \bar{\lambda}_C = \ell_P^2$$

is not a dynamical law. It does not describe evolution, force, or interaction in time.

Instead, it expresses a compatibility condition between two independently defined structures:

- support-sector persistence and boundary flux,
- exchange-sector coherence and cycle closure.

Geometric interpretation. The invariant is naturally interpreted as an area, representing the geometric domain over which both structures must be simultaneously satisfied.

This interpretation does not imply the existence of a physical surface or a discretized spatial element, but rather a structural measure of compatibility.

Absence of dynamical coupling. No mechanism is proposed by which the support and exchange sectors influence each other dynamically. The invariant does not specify how one sector responds to changes in the other.

Instead, it defines which combined configurations are admissible.

Relation to established theories. Although the invariant involves the constants $\hbar$, G, and c, no claim is made that it reproduces or replaces existing quantum or gravitational theories.

Its role is internal to the NUVO framework, providing a structural link between two sectors that are otherwise independently defined.

Interpretive discipline. All conclusions drawn in this work are restricted to structural and geometric considerations. No probabilistic, wave-based, or field-theoretic interpretations are introduced.

7.3 Limitations and Scope

Use of empirical input. The reconstruction of the exchange coherence scale in Appendix A relies on a single empirical input: the hydrogen ground-state binding energy.

This input serves only to fix the overall scale of the system. The structural relations derived in this work are otherwise independent of empirical calibration.

Weak-limit correspondence. Certain relations used in the appendix, including energy–scale correspondences and characteristic transport scales, reflect known weak-limit behavior of bound systems.

These are not introduced as foundational assumptions, but as calibrated correspondences consistent with the NUVO framework.

Absence of dynamics. The present work does not introduce a dynamical law governing exchange transport or support-sector evolution. No equations of motion or interaction laws are derived.

The invariant relation is purely structural and applies only as a constraint on admissible configurations.

Restricted domain. The analysis is limited to:

- scalar–conformal NUVO space,
- persistent anchored structures,
- and exchange processes exhibiting coherent closure.

Extensions to more general configurations, including non-closure transport or multi-body systems, are not addressed.

Future development. The consequences of the invariant relation for specific exchange processes, including closure conditions, transport quantization, and interaction structure, will be developed in subsequent Q-series papers.

Summary of scope. The present paper establishes a compatibility condition between support and exchange sectors. It does not constitute a complete physical theory, but provides a constraint that all subsequent developments within the NUVO framework must satisfy.

A NUVO Reconstruction of the Exchange Coherence Scale

A.1 Compact Derivation of the Coherence Length (Theorem Form)

Statement. Within the scalar–conformal NUVO framework, the exchange coherence length associated with a persistent anchored structure of mass m is uniquely given by

$$\bar{\lambda}_C = \frac{\hbar}{mc},$$

where $\hbar$ is the invariant closure action scale emerging from exchange-cycle coherence and mc^2 is the invariant persistence (capacity intake) scale of the anchor.

Derivation.

1. **Geometric closure defect.** Exchange transport along a bound trajectory exhibits a cycle defect of invariant magnitude [9]
$$\Delta s = 2\pi r_e,$$
where r_e is the characteristic geometric length associated with the electron anchor. This defect is independent of orbital radius to leading order and arises from scalar–conformal geometric transport.

2. **Closure count.** Coherent return occurs when accumulated defect matches a full geometric circumference, yielding a closure count
$$N_{\text{coh}} = \frac{a_0}{r_e}.$$

3. **Hydrogen gauge.** Using the empirical binding energy E_1 of the hydrogen ground state,
$$N_{\text{coh}} = \frac{m_e c^2}{2|E_1|}.$$

4. **Total transport over a closure cycle.** The total exchange transport accumulated over one coherent cycle is
$$T_{\text{tot}} = 2\pi m_e v a_0,$$
which is invariant under closure.

5. **Emergence of invariant action scale.** Evaluation at the hydrogen gauge yields
$$T_{\text{tot}} = h,$$
establishing the invariant action scale h, and hence $\hbar = h/(2\pi)$.

6. **Persistence scale from support sector.** From the support-sector formulation [2], a persistent anchored structure of mass m satisfies

$$\dot{C}_S = mc^2,$$

defining the invariant persistence scale.

7. **Construction of coherence length.** The unique length scale formed from the invariant action scale $\hbar$, persistence scale mc^2, and invariant speed c is

$$\bar{\lambda}_C := \frac{\hbar}{mc}.$$

8. **Scale hierarchy.** For the electron,

$$\bar{\lambda}_C = \frac{r_e}{\alpha}, \qquad a_0 = \frac{\bar{\lambda}_C}{\alpha} = \frac{r_e}{\alpha^2},$$

establishing a coherence ladder relating geometric defect, exchange coherence, and orbital scale.

Conclusion. The reduced Compton scale arises as the unique exchange coherence length compatible with:

- geometric closure of exchange cycles,
- invariant action scale from coherent transport,
- and invariant persistence scale from the support sector.

It is therefore not an independently assumed quantity, but a derived compatibility scale within the NUVO framework.

A.2 Geometric Closure Defect and Hydrogen Gauge

Geometric defect under scalar–conformal transport. Within the scalar–conformal NUVO geometry, transport along a closed orbital trajectory does not return exactly to its initial geometric configuration after a single revolution. Instead, a residual advancement (defect) is accumulated.

To leading order, this defect is independent of orbital radius and is given by

$$\Delta s = 2\pi r_e,$$

where r_e is the characteristic geometric length associated with the electron anchor.

This result is structurally analogous to perihelion-type advance in curved geometries [9], arising here from scalar modulation of the conformal transport metric rather than from curvature of spacetime in the relativistic sense. The defect is therefore a geometric property of transport on scalar–conformal NUVO space.

Interpretation as cycle advancement. The defect Δs represents a persistent mismatch between geometric transport and exact closure. Each completed orbit contributes an identical advancement, so that after N cycles the total accumulated defect is

$$N \cdot \Delta s = 2\pi N r_e.$$

Coherent return occurs when this accumulated defect matches a full geometric circumference of the trajectory.

Hydrogen gauge and closure scale. For the hydrogen ground state, the orbital scale is denoted by a_0. Closure therefore requires

$$2\pi N_{\mathrm{coh}} r_e = 2\pi a_0,$$

yielding the coherence count

$$N_{\mathrm{coh}} = \frac{a_0}{r_e}.$$

The physical value of a_0 is not determined purely geometrically, but is fixed by the empirical binding energy of the hydrogen ground state, denoted E_1 [11].

Using this single empirical input as a gauge, the closure condition yields

$$N_{\mathrm{coh}} = \frac{m_e c^2}{2|E_1|},$$

which sets the numerical scale for coherent exchange cycles.

Role of the hydrogen gauge. The binding energy E_1 serves only to fix the overall physical scale of the system. All subsequent relations follow from geometric closure and transport structure.

In this sense, E_1 acts as a gauge input, while the relations between r_e, a_0, and the coherence count are determined structurally within the NUVO framework.

A.3 Closure Count and Coherent Return

Coherent return condition. As established in the preceding subsection, scalar–conformal transport introduces a fixed geometric defect per cycle,

$$\Delta s = 2\pi r_e.$$

Coherent return occurs when the accumulated defect over N cycles matches a full geometric circumference at the orbital scale r, yielding the condition

$$N \cdot \Delta s = 2\pi r.$$

This gives the coherence count

$$N(r) = \frac{r}{r_e}.$$

In particular, for the hydrogen ground-state scale $r = a_0$,

$$N_{\mathrm{coh}} = \frac{a_0}{r_e}.$$

Energy scaling at the closure radius. The hydrogen ground state provides a calibrated relation between orbital scale and interaction energy [12]. At $r = a_0$, the potential energy magnitude satisfies

$$|U(a_0)| = \alpha^2 m_e c^2,$$

while the total binding energy is

$$|E_1| = \frac{1}{2}\alpha^2 m_e c^2.$$

These relations are not introduced as fundamental laws, but as the empirically calibrated energy–scale correspondence at the closure radius.

Coherence–energy relation. Combining the closure count with the energy scale at $r = a_0$, we obtain

$$N_{\text{coh}}\,|U(a_0)| = \left(\frac{a_0}{r_e}\right)\left(\alpha^2 m_e c^2\right).$$

Using the scale relation

$$\frac{r_e}{a_0} = \alpha^2,$$

this simplifies to

$$N_{\text{coh}}\,|U(a_0)| = m_e c^2.$$

Interpretation. This relation shows that the product of:

- the number of exchange cycles required for coherent return, and
- the interaction energy scale at the closure radius,

is equal to the persistence scale $m_e c^2$ of the anchor.

Thus, coherence over N_{coh} cycles corresponds precisely to the restoration of the full persistence scale.

Structural role. The identity

$$N_{\text{coh}}\,|U(a_0)| = m_e c^2$$

is not introduced as a dynamical law, but as a structural compatibility relation between:

- exchange coherence (cycle count),
- interaction scale (potential energy), and
- support-sector persistence (rest-energy scale).

This relation will serve as a key bridge in the reconstruction of the invariant action scale and the associated coherence length.

A.4 Emergence of the Invariant Action Scale

Defect transport per traversal. The closure count N_{coh} does not count repeated accumulations of the full orbital circumference. Rather, each traversal contributes the scalar–conformal defect length

$$\Delta s = 2\pi r_e.$$

Coherent return occurs when these defect increments accumulate to the background circumference at the hydrogen closure scale:

$$N_{\text{coh}}\Delta s = 2\pi a_0.$$

Since $\Delta s = 2\pi r_e$, this gives

$$N_{\text{coh}} = \frac{a_0}{r_e}.$$

Total defect transport to coherent return. Let

$$p_H := m_e v_H$$

denote the characteristic exchange momentum scale at the hydrogen gauge. The total transport accumulated through the defect channel over one coherent return is then

$$T_{\text{tot}} = N_{\text{coh}}\, p_H\, \Delta s.$$

Substituting the closure relations gives

$$T_{\text{tot}} = \left(\frac{a_0}{r_e}\right)(m_e v_H)(2\pi r_e) = 2\pi m_e v_H a_0.$$

Hydrogen gauge reduction. At the hydrogen gauge the weak-limit velocity scale satisfies

$$v_H = \alpha c,$$

and the calibrated scale hierarchy gives

$$a_0 = \frac{\bar{\lambda}_C}{\alpha}.$$

Therefore

$$T_{\text{tot}} = 2\pi m_e(\alpha c)\left(\frac{\bar{\lambda}_C}{\alpha}\right) = 2\pi m_e c\, \bar{\lambda}_C.$$

Definition of the invariant action scale. The coherent-return transport scale is therefore independent of the closure count once the accumulated defect condition is imposed. We define

$$h_{\text{NUVO}} := T_{\text{tot}} = 2\pi m_e c\, \bar{\lambda}_C,$$

and hence

$$\hbar_{\text{NUVO}} := \frac{h_{\text{NUVO}}}{2\pi} = m_e c\, \bar{\lambda}_C.$$

Equivalently,

$$\bar{\lambda}_C = \frac{\hbar_{\text{NUVO}}}{m_e c}.$$

Interpretation. The invariant action scale is not obtained by imposing the Bohr condition $m_e v r = \hbar$. It arises from defect accumulation:

$$N_{\text{coh}} \Delta s = 2\pi a_0,$$

together with the hydrogen gauge velocity scale. The apparent single-orbit form

$$T_{\text{tot}} = 2\pi m_e v_H a_0$$

appears only after the defect increments have coherently accumulated to the closure circumference.

A.5 Persistence Scale and Rest-Energy Normalization

Support-sector persistence. Within the support-sector formulation of the NUVO framework, persistent anchored structures are characterized by a constant capacity intake rate. For an anchor of mass m, this is given by

$$\dot{C}_S = mc^2.$$

This relation does not arise from dynamical considerations, but from the definition of persistence: an anchored structure must continuously draw capacity from the underlying delivery process in order to maintain its existence.

Interpretation of the rest scale. The quantity mc^2 therefore represents the invariant persistence scale of the anchor. It is not introduced as kinetic or potential energy, but as the rate at which capacity must be supplied to sustain the structure.

In this sense, mass m serves as the normalization of required intake relative to the invariant delivery speed c.

Independence from exchange structure. The persistence scale mc^2 is defined entirely within the support sector and does not depend on the existence of exchange cycles, coherence conditions, or interaction structure.

Thus:

- exchange coherence determines how transport closes,
- while support persistence determines the scale of sustained existence.

These two structures are defined independently and will be related only through compatibility conditions.

Compatibility requirement. For a physically realizable anchored system participating in exchange processes, the coherence structure of exchange transport must be compatible with the persistence requirement of the support sector.

In particular, the invariant action scale arising from exchange coherence must combine with the persistence scale to produce a consistent length scale governing interaction.

Preparation for scale construction. We therefore consider the combination of:

- the invariant action scale $\hbar$ (from exchange coherence),
- the persistence scale mc^2 (from support structure),
- and the invariant speed c,

from which a natural length scale may be constructed in the following subsection.

A.6 Reconstruction of the Reduced Compton Scale

Available invariant quantities. From the preceding development, we have established two independent invariant scales:

- the exchange-sector invariant action scale $\hbar$, arising from coherent closure of exchange cycles,

- the support-sector persistence scale mc^2, arising from the invariant capacity intake required to sustain an anchored structure.

In addition, the scalar–conformal framework admits a universal invariant speed c, associated with the underlying delivery process.

Construction of a compatibility length. We seek a length scale that is compatible with both:

- exchange coherence (action scale $\hbar$),
- and support persistence (scale mc^2).

The unique length scale that can be formed from $\hbar$, m, and c is

$$\ell_{\text{coh}} := \frac{\hbar}{mc}.$$

Interpretation as coherence scale. The length ℓ_{coh} represents the characteristic scale at which:

- exchange transport accumulates one unit of invariant action, and
- support persistence provides the corresponding normalization through mc^2.

It therefore defines the natural spatial scale at which exchange coherence is compatible with the persistence requirements of the anchor.

Identification with the reduced Compton scale. We identify

$$\bar{\lambda}_C := \frac{\hbar}{mc},$$

and interpret this not as an independently introduced quantity, but as the exchange coherence length arising from compatibility between the two sectors.

Consistency with the hydrogen scale hierarchy. For the electron, this scale satisfies

$$\bar{\lambda}_C = \frac{r_e}{\alpha}, \qquad a_0 = \frac{\bar{\lambda}_C}{\alpha} = \frac{r_e}{\alpha^2},$$

which reproduces the established hierarchy between geometric defect, exchange coherence, and orbital closure scale.

Structural role. The reduced Compton scale is therefore not a fundamental input, but a derived compatibility scale determined by:

- geometric closure of exchange cycles,
- invariant action accumulation,
- and support-sector persistence normalization.

Conclusion. The exchange coherence length $\bar{\lambda}_C$ arises uniquely from the requirement that exchange transport and support persistence be simultaneously satisfied. It will therefore serve as the canonical exchange-sector length scale in the main development of this work.

A.7 Scale Ladder and Coherence Hierarchy

Hierarchy of characteristic scales. The preceding construction establishes a hierarchy of characteristic lengths associated with a bound electron–proton system:

$$r_e, \qquad \bar{\lambda}_C = \frac{\hbar}{m_e c}, \qquad a_0.$$

These scales are related through the dimensionless parameter α [11] by

$$\frac{r_e}{\bar{\lambda}_C} = \alpha, \qquad \frac{\bar{\lambda}_C}{a_0} = \alpha, \qquad \frac{r_e}{a_0} = \alpha^2.$$

Interpretation as a coherence ladder. This hierarchy may be interpreted as a ladder of coherence scales:

- r_e represents the geometric defect scale governing exchange advancement per cycle,
- $\bar{\lambda}_C$ represents the coherence length associated with one unit of invariant action,
- a_0 represents the closure scale at which full coherent return is achieved.

Structural significance. The existence of this ladder demonstrates that:

- geometric transport (defect),
- exchange coherence (action),
- and orbital closure (scale)

are not independent structures, but are related through a single dimensionless ratio.

Role in the present work. In the main development of this paper, $\bar{\lambda}_C$ will serve as the exchange-sector coherence scale, while $r_c = Gm/c^2$ characterizes the support sector. Their product defines the invariant area relation central to this work.

A.8 Remarks on Weak-Limit Energy–Velocity Correspondence

Use of hydrogen energy relations. In the preceding derivation, relations of the form

$$|U(a_0)| = \alpha^2 m_e c^2, \qquad |E_1| = \frac{1}{2}\alpha^2 m_e c^2$$

have been employed.

These relations are not introduced as fundamental laws within the NUVO framework, but as empirically calibrated correspondences at the hydrogen ground state.

Status of velocity relations. Similarly, the use of a characteristic transport scale

$$v \sim \alpha c$$

should be understood as part of the same weak-limit correspondence.

Within the NUVO framework:

- velocity is not a primitive quantity,
- but an emergent descriptor of boundary flux asymmetry in the support sector.

Interpretive clarification. The appearance of expressions analogous to classical relations such as

$$E \sim \frac{1}{2}mv^2$$

does not imply that such relations are assumed at the foundational level.

Rather, they arise as effective descriptions of:

- exchange interaction scales, and
- their compatibility with support-sector persistence,

in regimes where the NUVO structure reproduces known weak-limit behavior.

Scope discipline. The present appendix does not introduce a dynamical law relating energy and velocity. Its purpose is limited to establishing the structural compatibility of:

- exchange coherence (through invariant action),
- and support persistence (through mc^2),

from which the coherence length $\bar{\lambda}_C$ follows.

Conclusion. The reconstruction of the reduced Compton scale therefore rests only on:

- geometric closure,
- a single empirical gauge input,
- and support-sector normalization,

without requiring the introduction of independent dynamical assumptions.

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