Q11 – Unified Transport Law for Closure and Exchange in Scalar–Conformal NUVO Systems

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Abstract

The preceding paper (Q10) [1] established phase as a transport-derived quantity associated with the cumulative effect of exchange interaction along admissible trajectories in scalar–conformal NUVO space. In that framework, coherence was shown to arise as a condition of path compatibility governed by a system-dependent coherence scale, and closure conditions were expressed in terms of phase accumulation along closed transport loops.

Consistency with the scalar-modulated closure functional. The phase-based closure condition used in Q10 and throughout the present work is not an independent formulation, but a representation of the scalar-modulated return condition established in Q2 [2],

$$k \oint_\gamma \lambda_{\rm eff}(x,u)\, ds = L_\gamma.$$

Through the identification of phase with normalized transport action, the accumulated phase along a loop provides an equivalent measure of closure compatibility. The condition

$$\oint_\gamma d\phi = m\, \Phi_c$$

therefore encodes the same admissibility requirement expressed in transport-based variables.

In the present work, we extend this kinematic construction by introducing a unified transport law that governs the evolution of closure structures under exchange and scalar geometry. This law combines transport, phase, and capacity structure into a single framework, providing a deterministic description of how closure density and phase evolve in space and time.

We derive a continuity structure for closure transport and a coupled phase evolution equation, both arising directly from transport and exchange considerations. These relations form a minimal dynamical system consistent with the geometric structure established in the preceding papers. By eliminating auxiliary transport variables, a second-order structure emerges that governs the evolution of the system in terms of phase and density alone.

No wave ontology, probabilistic interpretation, or operator formalism is introduced. The resulting framework provides the first complete transport law for the exchange sector and establishes the foundation from which familiar dynamical representations, including Schrödinger-type equations, may arise as limiting or representational forms.

[*] Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.
- The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-, +, +, +)$.

This manuscript is mathematical in scope. It establishes definitions, structural identities, and variational consequences within a scalar–conformal setting. Sector reductions and correspondence limits are recorded only when explicitly stated as additional assumptions and are not used as premises in derivations.No claim of full dynamical equivalence to general relativity, quantum mechanics, or classical field theories is made at the level of the present foundational development. Where later papers compare limiting behavior, those comparisons are presented as correspondence targets rather than as identity statements.The NUVO program is organized as a sequence of internally consistent mathematical papers. Foundational papers (M-series) fix the scalar–conformal geometry, variational structure, and notation. Subsequent papers treat sectoral reductions (gravity, exchange, quantization, and bound-state structure) as controlled specializations of the foundational framework.**Scalar ontology.** The scalar field Λ represents the *locally available structural capacity* of an underlying delivery field permeating spacetime. The baseline level Λ_0 denotes the availability supported by this intrinsic delivery structure in the absence of structural occupation. Localized structures or transport processes may reduce the available capacity relative to this baseline, but the intrinsic delivery baseline itself is not altered. Consequently the scalar field measures the *available portion* of structural capacity rather than the intrinsic production of the underlying field.

1 Introduction

1.1 From Phase Structure to Transport Law

In Q10, phase was introduced as a scalar quantity derived from the transport of closure structures through scalar–conformal NUVO space. That construction showed that phase accumulation encodes the cumulative effect of exchange interaction along admissible trajectories, and that coherence arises from compatibility of phase across distinct transport paths. Closure conditions were expressed in terms of phase accumulation along closed loops, yielding a discrete set of admissible configurations consistent with the results of Q7 [3].

While this structure provides a complete kinematic description of transport and coherence, it does not specify how transport itself is determined. In particular, no law was introduced governing the evolution of closure structures, the distribution of transport, or the time dependence of phase.

The present work addresses this gap by introducing a transport law that governs the evolution of closure structures under exchange interaction and scalar geometry. This law must reproduce the kinematic relations derived previously, while extending them to a fully consistent dynamical framework.

As in Q8–Q10 [4,5], the present construction is restricted to the exchange sector. The transport law derived below governs the evolution of closure structures and exchange-induced phase, and does not describe support-sector delivery, anchor intake, or force-based dynamics.

1.2 Objectives

The primary objective of this paper is to derive a unified transport law for the exchange sector of the scalar–conformal NUVO framework. This law must:

- Provide a consistent description of the evolution of closure structures under transport;
- Incorporate the phase structure introduced in Q10 as a transport-derived quantity;
- Account for the influence of scalar geometry through the field $\lambda(x)$;
- Preserve the coherence and closure conditions established in earlier work;
- Yield a closed system of equations governing transport, phase, and closure density.

A key requirement is that the resulting structure arise directly from transport and exchange considerations, without the introduction of external dynamical assumptions.

1.3 Scope and Constraints

The present construction is strictly deterministic and geometric in nature. No probabilistic interpretation is introduced, and no assumption is made regarding the existence of a wavefunction or underlying oscillatory entity.

Similarly, no operator formalism is employed. While alternative representations of the resulting transport law may be constructed in later developments, such representations are not assumed here and do not form part of the foundational structure.

The focus of this paper is the derivation of the minimal transport law consistent with the kinematic framework established in Q8–Q10.

1.4 Conceptual Framework

The transport law developed here unifies three elements:

- The transport of closure structures along admissible trajectories;
- The phase accumulation associated with exchange interaction;
- The scalar geometry encoded by the field $\lambda(x)$.

Rather than introducing new fundamental objects, the construction builds directly on these elements, combining them into a coupled system that governs the evolution of transport.

The resulting framework provides a natural extension of the phase-based description developed in Q10, moving from a kinematic characterization of transport to a dynamical law that determines its evolution.

1.5 Structure of the Paper

The paper proceeds as follows. Section 2 introduces the transport quantities required for a dynamical description, including closure density and transport fields. Section 3 derives a continuity structure for closure transport. Section 4 develops the phase evolution equation, incorporating exchange interaction and scalar geometry.

In Section 5, these components are combined into a unified transport system. Section 6 shows that this system admits a second-order formulation governing the evolution of phase and density. Section 7 examines the role of scalar geometry in the resulting dynamics.

Interpretive clarifications are provided in Section 8, and the paper concludes with a summary and transition to subsequent developments, in which alternative representations of the transport law will be considered.

2 Transport Quantities and Fields

2.1 Closure Density

To describe transport in a local and time-dependent manner, we introduce a scalar field $\rho(x,t)$ representing the density of closure structures within scalar–conformal NUVO space.

The quantity $\rho(x,t)$ measures the distribution of admissible closure configurations over space and time. It is not a probability density, but a geometric measure of the presence of closure structures within a given region.

More precisely, for a spatial region Ω, the integral

$$\int_{\Omega} \rho(x,t)\, d^3x$$

represents the total closure content within Ω at time t.

The introduction of $\rho(x,t)$ allows transport to be described as a continuous redistribution of closure structures, rather than as the motion of a single isolated trajectory.

2.2 Transport Velocity Field

Associated with the distribution of closure structures is a transport velocity field

$$v(x,t),$$

which characterizes the local direction and rate of transport.

The field $v(x,t)$ is defined such that the flow of closure density through space is described by the flux

$$\rho(x,t)\,v(x,t).$$

This velocity field should not be interpreted as a fundamental dynamical quantity, but rather as a derived descriptor of transport consistent with the underlying worldline structure introduced in Q8. In particular, $v(x,t)$ represents the collective behavior of admissible transport paths passing through a given point.

2.3 Local Phase Field

In Q10, phase was defined as a path-dependent quantity accumulated along transport trajectories. For the purpose of constructing a local transport law, it is convenient to introduce a local phase field

$$\phi(x,t),$$

defined such that its gradients reproduce the phase variation associated with transport.

This local field is not introduced as an independent dynamical object, but as a representation of the cumulative phase structure in a local neighborhood. Its definition is consistent with the relation

$$d\phi = \kappa\,\mathcal{E}(x)\,d\tau$$

along admissible trajectories, as established in Q10.

The field $\phi(x,t)$ therefore encodes the local phase structure arising from transport and exchange, allowing phase-based relations to be expressed in differential form.

2.4 Consistency with Path-Based Definition

The introduction of $\phi(x,t)$ as a local field must be understood as a representation of the underlying path-dependent phase defined in Q10. In general, phase remains path-dependent, and the local field description is valid only to the extent that it captures the local variation of phase in a consistent manner.

In regions where phase accumulation is approximately integrable, the local field $\phi(x,t)$ provides an accurate description of phase variation. In more general settings, it should be regarded as a convenient local encoding of the transport-derived phase structure.

2.5 Summary of Transport Variables

The transport law developed in this paper will be formulated in terms of the following quantities:

- The closure density $\rho(x,t)$, representing the spatial distribution of closure structures;
- The transport velocity field $v(x,t)$, describing the flow of closure density;
- The phase field $\phi(x,t)$, encoding the cumulative effect of transport and exchange.

These quantities are not independent fundamental entities, but are derived from the underlying transport, exchange, and geometric structure of the scalar–conformal NUVO framework. The goal of the subsequent sections is to determine the relations governing their evolution.

3 Continuity Structure

3.1 Conservation of Closure Transport

The closure density $\rho(x,t)$ introduced in Section 2 represents the distribution of admissible closure structures within scalar–conformal NUVO space. As closure structures are transported along admissible trajectories, their distribution evolves in time.

A fundamental requirement of transport consistency is that closure structures are neither spontaneously created nor destroyed within the exchange sector. Instead, transport corresponds to a redistribution of closure density across space.

Let Ω be a fixed spatial region with boundary $\partial\Omega$. The rate of change of closure content within Ω is given by

$$\frac{d}{dt}\int_{\Omega}\rho(x,t)\,d^3x.$$

This change must be accounted for entirely by the flux of closure density across the boundary. The outward flux through $\partial\Omega$ is given by

$$\int_{\partial\Omega}\rho(x,t)\,v(x,t)\cdot dS,$$

where $v(x,t)$ is the transport velocity field.

Requiring that all changes in closure density arise from boundary flux, we obtain the balance relation

$$\frac{d}{dt}\int_{\Omega}\rho(x,t)\,d^3x = -\int_{\partial\Omega}\rho(x,t)\,v(x,t)\cdot dS.$$

3.2 Local Continuity Equation

Applying the divergence theorem to the flux term, the balance relation becomes

$$\frac{d}{dt}\int_{\Omega}\rho(x,t)\,d^3x = -\int_{\Omega}\nabla\cdot\big(\rho(x,t)\,v(x,t)\big)\,d^3x.$$

Since this relation holds for arbitrary regions Ω, it follows that the integrands must be equal, yielding the local continuity equation

$$\partial_t\rho + \nabla\cdot(\rho v) = 0.$$

3.3 Interpretation

The continuity equation expresses the conservation of closure transport. It states that any local change in closure density is exactly balanced by the divergence of transport flow.

This relation is not an independent assumption, but follows directly from the requirement that closure structures persist under admissible transport, as established in Q8.

3.4 Relation to Transport Geometry

The velocity field $v(x, t)$ appearing in the continuity equation is not an arbitrary field, but is constrained by the underlying transport structure and scalar geometry.

In particular, $v(x, t)$ must be compatible with the admissible trajectories determined by closure persistence and exchange interaction. As a result, the continuity equation should be understood as a kinematic constraint that must be satisfied by any admissible transport configuration.

3.5 Absence of Probabilistic Interpretation

Although the continuity equation resembles conservation laws that appear in statistical or probabilistic contexts, no such interpretation is assumed here.

The quantity $\rho(x, t)$ represents closure density, not probability, and the equation expresses conservation of structural content under transport, rather than conservation of probability.

3.6 Summary

The continuity equation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

provides the first component of the transport law. It establishes a local conservation principle governing the redistribution of closure structures and forms the foundation for the coupled transport system developed in the subsequent sections.

4 Phase Transport Equation

4.1 Transport of Phase Along Trajectories

In Q10, phase was defined as a cumulative quantity associated with transport along admissible trajectories, with incremental form

$$d\phi = \kappa \, \mathcal{E}(x) \, d\tau.$$

To construct a dynamical description, we now consider the evolution of phase in space and time. Let $\phi(x, t)$ be the local phase field introduced in Section 2. The rate of change of phase experienced by a closure structure moving along a transport trajectory is given by the total derivative

$$\frac{d\phi}{dt} = \partial_t \phi + v \cdot \nabla \phi,$$

where $v(x, t)$ is the transport velocity field.

This expression represents the phase evolution along admissible transport paths.

This normalization identifies phase evolution as the accumulation of transport action measured in units of the closure-action scale, linking the phase dynamics directly to the invariant exchange action associated with elementary closure cycles.

Origin in closure compatibility under transport. The phase field $\phi(x, t)$ appearing in the transport law should be understood as a local representation of the closure compatibility structure under transport. As established in Q2 and Q10, admissible configurations are selected by a scalar-modulated return condition, and phase encodes the accumulation of transport required to restore this compatibility along admissible trajectories.

The evolution equation derived below therefore represents the dynamical expression of closure compatibility under transport, rather than the introduction of an independent dynamical variable.

4.2 Exchange Contribution to Phase Evolution

From the definition of phase accumulation, the rate of phase change along a trajectory is determined by the exchange interaction:

$$\frac{d\phi}{dt} = \kappa\,\mathcal{E}(x).$$

The normalization constant κ is not arbitrary. As established in Q10, it is fixed by the closure-action scale associated with coherent return cycles. Accordingly, the phase evolution equation inherits this global normalization and introduces no new independent scale.

Combining this with the transport expression above yields the phase transport equation

$$\partial_t\phi + v\cdot\nabla\phi = \kappa\,\mathcal{E}(x).$$

This equation expresses the evolution of phase as a balance between transport along trajectories and local exchange interaction.

4.3 Dependence on Scalar Geometry

The exchange rate $\mathcal{E}(x)$ depends on the scalar field $\lambda(x)$ and its variation, so the phase evolution equation inherits this dependence. One may write

$$\mathcal{E}(x) = \mathcal{E}\big(\lambda(x), \nabla\lambda(x), \dots\big),$$

so that

$$\partial_t\phi + v\cdot\nabla\phi = \kappa\,\mathcal{E}\big(\lambda(x), \nabla\lambda(x), \dots\big).$$

This shows that scalar geometry directly influences the rate of phase evolution.

4.4 Interpretation

The phase transport equation describes how the cumulative transport measure ϕ evolves under the combined effects of motion and exchange. The left-hand side represents the change in phase experienced by a closure structure moving with the transport flow, while the right-hand side represents the contribution from exchange interaction.

This relation is purely geometric and kinematic. It does not introduce a dynamical force or potential, but rather expresses the consistency between phase accumulation and transport structure.

4.5 Compatibility with Coherence and Closure

The phase evolution equation is consistent with the coherence and closure conditions established in Q10. In particular, integration of the equation along a transport path reproduces the phase accumulation relations used to define coherence.

Similarly, integration around closed loops yields the closure condition

$$\oint_\gamma d\phi = m\,\Phi_c,$$

ensuring compatibility with the discrete structure of admissible states.

4.6 Summary

The equation

$$\partial_t \phi + v \cdot \nabla \phi = \kappa \, \mathcal{E}(x)$$

provides the second component of the transport law. Together with the continuity equation derived in Section 3, it forms a coupled system describing the evolution of closure density and phase under transport and exchange.

5 Coupled Transport System

5.1 Combined Transport Structure

The results of Sections 3 and 4 yield a coupled system governing the evolution of closure density and phase:

$$\partial_t \rho + \nabla \cdot (\rho v) = 0,$$

$$\partial_t \phi + v \cdot \nabla \phi = \kappa \, \mathcal{E}(x).$$

These equations together describe the redistribution of closure structures and the evolution of phase under transport and exchange.

The first equation expresses conservation of closure density, while the second relates phase evolution to transport and exchange interaction.

5.2 Interdependence of Variables

The variables $\rho(x,t)$, $\phi(x,t)$, and $v(x,t)$ are not independent. Their evolution is constrained by the underlying transport structure and scalar geometry.

In particular:

- The velocity field $v(x,t)$ describes the flow of closure density and must be compatible with admissible transport trajectories;
- The phase field $\phi(x,t)$ encodes cumulative transport and exchange, with gradients aligned to admissible transport directions as established in Q10;
- The closure density $\rho(x,t)$ evolves according to the flow defined by $v(x,t)$.

These relationships imply that $v(x,t)$ is not arbitrary, but is constrained by both the phase structure and the geometry of the system.

5.3 Closure Consistency

The coupled system must remain consistent with the coherence and closure conditions established in Q10. In particular, the evolution of $\phi(x,t)$ must preserve the phase closure condition

$$\oint_\gamma d\phi = m \, \Phi_c$$

for admissible closed transport paths.

This requirement constrains the admissible forms of $v(x,t)$ and $\mathcal{E}(x)$, ensuring that transport evolution does not violate the discrete structure of closure states.

5.4 Role of Scalar Geometry

The scalar field $\lambda(x)$ enters the coupled system through the exchange rate $\mathcal{E}(x)$, and thereby influences both phase evolution and transport behavior.

As a result, the coupled system is not purely kinematic, but reflects a structured interaction between transport, exchange, and geometry.

Variations in $\lambda(x)$ modify $\mathcal{E}(x)$, which in turn affects phase evolution and the admissibility of transport paths.

5.5 Underdetermined Structure

At this stage, the coupled system is not yet closed, as the velocity field $v(x,t)$ has not been specified in terms of $\rho(x,t)$ and $\phi(x,t)$.

This reflects the fact that the present formulation describes transport in terms of three variables, while providing only two independent relations.

To obtain a closed system, it is necessary to express $v(x,t)$ in terms of the phase structure or eliminate it in favor of $\rho(x,t)$ and $\phi(x,t)$.

5.6 Toward a Reduced Description

The structure developed thus far suggests that a reduced formulation in terms of $\rho(x,t)$ and $\phi(x,t)$ alone should exist. Such a formulation would eliminate the auxiliary velocity field and yield a closed system governing the evolution of closure density and phase.

The derivation of this reduced system is the subject of the next section, where a second-order transport structure will be obtained.

5.7 Minimality and Structural Uniqueness

The coupled system derived above contains an undetermined transport relation through the function $\mathcal{V}(\nabla\phi, \lambda)$. This reflects the fact that the continuity and phase evolution equations alone do not fully specify transport.

However, the admissible form of $\mathcal{V}$ is not arbitrary. It is constrained by the following structural requirements:

- Transport must be locally aligned with admissible phase structure, so that v is determined by $\nabla\phi$;
- The transport law must preserve the closure and coherence conditions established in Q10 under time evolution;
- The resulting system must remain compatible with the scalar–conformal geometry determined by $\lambda(x)$;
- The combined system must admit a closed formulation in terms of ρ and ϕ alone.

These constraints restrict the admissible forms of $\mathcal{V}$ to a narrow class of phase-guided transport relations. Within this class, the transport law derived in the present work represents the minimal structure consistent with exchange transport, phase accumulation, and scalar geometry.

Thus, while the explicit functional form of $\mathcal{V}$ depends on the detailed exchange dynamics, the overall structure of the transport law is uniquely determined by the requirement of consistency with the preceding Q-series framework.

6 Emergence of Second-Order Structure

6.1 Phase-Guided Transport

In Q10, it was established that the phase gradient $\nabla\phi$ encodes admissible transport direction. In particular, transport trajectories align with the local phase structure, so that the velocity field $v(x,t)$ is constrained by $\nabla\phi$.

Necessity of phase-guided transport. The identification of $v(x,t)$ with a function of $\nabla\phi$ is not an independent assumption, but follows from the requirement that transport remain consistent with the closure-compatible phase structure.

As established in Q10, the phase gradient encodes the admissible direction and rate of transport arising from exchange interaction. Any transport law that departs from this alignment would violate the closure compatibility conditions that define admissible trajectories.

Accordingly, the transport velocity must be determined by the local phase structure, leading to the representation

$$v(x,t) = \mathcal{V}(\nabla\phi, \lambda).$$

Accordingly, consistency with the phase-based transport structure requires that the transport velocity field be expressible as

$$v(x,t) = \mathcal{V}(\nabla\phi, \lambda),$$

where $\mathcal{V}$ is a function determined by the transport and exchange structure.

This expresses the fact that transport is not independent, but guided by the phase structure derived from exchange interaction.

6.2 Reduction of the Coupled System

Substituting the phase-guided form of $v(x,t)$ into the continuity equation,

$$\partial_t\rho + \nabla\cdot(\rho v) = 0,$$

yields

$$\partial_t\rho + \nabla\cdot\big(\rho\,\mathcal{V}(\nabla\phi,\lambda)\big) = 0.$$

Similarly, substitution into the phase evolution equation,

$$\partial_t\phi + v\cdot\nabla\phi = \kappa\,\mathcal{E}(x),$$

gives

$$\partial_t\phi + \mathcal{V}(\nabla\phi,\lambda)\cdot\nabla\phi = \kappa\,\mathcal{E}(x).$$

This yields a closed system in terms of $\rho(x,t)$ and $\phi(x,t)$.

6.3 Structure of the Transport Function

The function $\mathcal{V}(\nabla\phi,\lambda)$ encodes the relation between phase structure and transport. While its explicit form depends on the detailed exchange dynamics, it must satisfy the following conditions:

- Alignment: v is locally aligned with $\nabla\phi$;
- Scalar covariance: the form of $\mathcal{V}$ respects the scalar–conformal structure determined by $\lambda(x)$;

- Consistency: substitution into the phase evolution equation reproduces the exchange-driven phase accumulation.

These constraints ensure that the reduced system remains consistent with the kinematic and geometric structure established in Q10.

6.4 Emergence of Second-Order Behavior

The reduced system obtained above couples $\rho(x,t)$ and $\phi(x,t)$ through first-order relations. However, the dependence of $\mathcal{V}$ on $\nabla\phi$ implies that derivatives of ϕ enter the continuity equation in nonlinear form.

Expanding the divergence term,

$$\nabla \cdot \big(\rho\, \mathcal{V}(\nabla\phi, \lambda)\big),$$

introduces second-order spatial derivatives of ϕ.

Thus, although the original system is first-order in form, the reduced system exhibits an effective second-order structure governing the evolution of ϕ and ρ.

6.5 Closed Transport System

The resulting system may be written schematically as

$$\partial_t \rho + \nabla \cdot \big(\rho\, \mathcal{V}(\nabla\phi, \lambda)\big) = 0,$$

$$\partial_t \phi + \mathcal{V}(\nabla\phi, \lambda) \cdot \nabla\phi = \kappa\, \mathcal{E}(\lambda, \nabla\lambda, \ldots).$$

This constitutes a closed transport system for $\rho(x,t)$ and $\phi(x,t)$, with all dependencies expressed in terms of phase structure and scalar geometry.

6.6 Interpretation

The emergence of second-order structure reflects the fact that transport is governed not only by local values of phase, but also by its spatial variation. The phase field acts as an organizing structure for transport, with its gradients determining flow and its curvature influencing evolution.

Importantly, this second-order behavior arises without introducing external dynamical assumptions. It is a direct consequence of coupling transport to the phase structure derived from exchange interaction.

6.7 Preparation for Representation

The closed system derived here provides a complete dynamical description of transport in terms of $\rho(x,t)$ and $\phi(x,t)$.

In subsequent developments, it will be shown that this system admits alternative representations in which the coupled equations may be combined into a single evolution equation. Such representations are not fundamental, but provide useful encodings of the transport structure established here.

7 Scalar Geometry Coupling

7.1 Geometry as a Determinant of Transport

The scalar field $\lambda(x)$ defines the geometric structure of scalar–conformal NUVO space and governs the local availability of capacity. As established in prior work, λ is not a passive background quantity, but directly influences transport and exchange.

In the transport law derived in Sections 3–6, this influence enters through both the exchange rate $\mathcal{E}(x)$ and the transport function $\mathcal{V}(\nabla\phi, \lambda)$.

7.2 Geometry Dependence of Exchange Interaction

The exchange rate $\mathcal{E}(x)$ encodes the interaction between closure structures and the surrounding scalar environment. Its dependence on $\lambda(x)$ and its variation may be expressed as

$$\mathcal{E}(x) = \mathcal{E}\big(\lambda(x), \nabla\lambda(x), \dots\big).$$

This dependence ensures that phase evolution responds to the local scalar geometry. Regions of varying λ modify the rate of phase accumulation, thereby influencing both coherence and closure conditions.

7.3 Geometry Dependence of Transport Structure

The transport function $\mathcal{V}(\nabla\phi, \lambda)$ relates phase structure to transport velocity. The presence of λ in this function reflects the fact that admissible transport is shaped by scalar geometry.

In particular, variations in $\lambda(x)$ may alter both the magnitude and direction of transport flow, even for identical phase gradients. This establishes that transport is not determined by phase alone, but by the combined influence of phase and geometry.

7.4 Coupled Geometry–Transport System

With the inclusion of scalar geometry, the closed transport system may be written as

$$\partial_t\rho + \nabla\cdot\big(\rho\,\mathcal{V}(\nabla\phi, \lambda)\big) = 0,$$

$$\partial_t\phi + \mathcal{V}(\nabla\phi, \lambda)\cdot\nabla\phi = \kappa\,\mathcal{E}\big(\lambda, \nabla\lambda, \dots\big).$$

This system explicitly couples closure density, phase, and scalar geometry, forming a unified description of transport in the exchange sector.

7.5 Geometric Modulation of Coherence and Closure

Since both $\mathcal{E}$ and $\mathcal{V}$ depend on $\lambda(x)$, the coherence scale Φ_c and the admissibility of closed transport loops are also influenced by scalar geometry.

Variations in λ may shift the effective coherence scale and modify the set of admissible closure conditions. As a result, the discrete structure of closure states is not fixed independently of geometry, but emerges from the coupled transport–geometry system.

7.6 Consistency with Scalar–Conformal Structure

The dependence of transport on $\lambda(x)$ must respect the scalar–conformal structure of NUVO space. In particular, all transport relations must be covariant under conformal rescaling determined by λ.

This requirement constrains the admissible forms of $\mathcal{E}(\cdot)$ and $\mathcal{V}(\cdot)$, ensuring that the transport law remains consistent with the geometric framework established in the M-series [6, 7].

7.7 Interpretation

The results above demonstrate that scalar geometry governs both the kinematics and dynamics of transport. The scalar field $\lambda(x)$ controls exchange interaction, modulates phase accumulation, and shapes the flow of closure density.

Thus, transport in NUVO space is best understood as a geometric process in which closure structures evolve under the combined influence of phase and scalar geometry, rather than as motion in a fixed background.

7.8 Preparation for Representation

The explicit coupling between phase, density, and scalar geometry suggests that the transport system admits alternative formulations in which these variables are combined into a unified representation.

In particular, it will be shown in subsequent work that the coupled system can be expressed in a form resembling a single evolution equation, providing a bridge to more familiar representations while preserving the underlying geometric structure.

8 Weak-Limit Decomposition of Scalar Modulation

Consistency with transport law structure. The weak-limit decomposition presented below follows directly from the two-channel manner in which scalar geometry enters the transport law through both exchange interaction and phase-guided transport.

8.1 Two-Channel Coupling of Scalar Geometry

The transport law derived in Sections 3–7 shows that the scalar field $\lambda(x)$ enters the exchange-sector dynamics in two distinct ways:

- Through the exchange rate $\mathcal{E}(x)$, which determines the rate of phase accumulation;
- Through the transport function $\mathcal{V}(\nabla\phi, \lambda)$, which governs the admissible transport flow.

Thus, scalar geometry influences both the *local transport state* and the *ambient interaction structure*. This establishes a two-channel coupling between scalar geometry and transport.

8.2 Weak-Regime Scalar Diagnostic Structure

In regimes where scalar modulation is weak, the influence of geometry on transport may be expressed through a dimensionless scalar diagnostic representing deviation from baseline availability.

Consistency with the transport law requires that this diagnostic capture both channels of scalar coupling. The minimal form consistent with this structure is therefore an additive decomposition

$$\lambda_{\text{eff}} = 1 + \chi_{\text{loc}} + \chi_{\text{amb}},$$

where:

- $\chi_{\rm loc}$ represents the local transport-dependent contribution;
- $\chi_{\rm amb}$ represents the ambient geometric contribution associated with the surrounding scalar field.

This decomposition is not an independent assumption, but follows from the twofold manner in which $\lambda(x)$ enters the transport law.

8.3 Local Contribution from Phase-Guided Transport

The transport law constrains admissible motion through the relation

$$v(x,t) = \mathcal{V}(\nabla\phi, \lambda),$$

so that transport is locally determined by phase structure.

In the weak regime, the local contribution must therefore be a dimensionless scalar constructed from the transport state relative to the invariant rest scale [8, 9]. To leading order, this yields

$$\chi_{\rm loc} \simeq \frac{v_{\rm acc}^2}{2c^2},$$

corresponding to the first-order contribution of local acceleration to scalar modulation.

This term reflects the intrinsic, non-propagating contribution to time-dilation-type behavior associated with transport.

8.4 Ambient Contribution from Scalar Geometry

The ambient contribution arises from the dependence of the exchange rate on scalar geometry,

$$\mathcal{E}(x) = \mathcal{E}(\lambda, \nabla\lambda, \ldots).$$

In the weak regime, this contribution reduces to the leading-order dimensionless scalar characterizing the background geometry. For sourced configurations, this takes the form

$$\chi_{\rm amb} \propto \frac{GM}{c^2 r},$$

reflecting the geometric modulation associated with persistent source structure [10, 11].

This term represents the propagating, field-mediated contribution to scalar modulation [12].

8.5 Hydrogenic Specialization

For hydrogenic transport, it is convenient to express the local and ambient contributions in terms of diagnostic quantities normalized by the invariant rest scale $m_e c^2$. In this representation, one may identify

$$\chi_{\rm loc} = \frac{T}{m_e c^2}, \qquad \chi_{\rm amb} = -\frac{V}{m_e c^2},$$

where T and V serve as diagnostic measures of the local and ambient contributions, respectively.

Substitution into the weak-limit decomposition yields

$$\lambda_{\rm eff} = 1 + \frac{T}{m_e c^2} - \frac{V}{m_e c^2}.$$

8.6 Interpretation

This result provides the structural origin of the scalar modulation form used in hydrogenic analyses. The expression above is not a fundamental field equation, but a weak-regime diagnostic representation derived from the two-channel coupling of scalar geometry in the transport law.

The local term reflects the effect of acceleration on transport, while the ambient term reflects the influence of the surrounding scalar geometry.

8.7 Scope of Validity

The decomposition obtained here is valid in regimes where scalar modulation is small and higher-order coupling between transport and geometry may be neglected.

Beyond this regime, the full dependence of $\mathcal{V}$ and $\mathcal{E}$ on $\lambda(x)$ must be retained, and the scalar modulation need not admit a simple additive form.

8.8 Summary

The unified transport law implies that scalar geometry couples to transport through both local and ambient channels. In the weak limit, this leads to a minimal decomposition of the scalar diagnostic into local and ambient contributions.

For hydrogenic systems, this decomposition yields the effective scalar modulation

$$\lambda_{\text{eff}} = 1 + \frac{T}{m_e c^2} - \frac{V}{m_e c^2},$$

providing a consistent bridge between the transport law and the empirical structure of time-dilation-type effects.

9 Interpretive Clarifications (Non-Dynamical)

9.1 Absence of Wave Ontology

The transport law derived in this work does not assume the existence of an underlying wave or oscillatory entity. The variables $\rho(x,t)$ and $\phi(x,t)$ are not components of a wavefunction, but represent closure density and transport-derived phase, respectively.

Any representation of the system in terms of wave-like structures must be understood as secondary and representational, rather than fundamental.

9.2 Absence of Probabilistic Interpretation

The quantity $\rho(x,t)$ introduced in Section 2 represents closure density and not probability. The continuity equation expresses conservation of closure transport, not conservation of probability.

No statistical interpretation is assumed in the derivation of the transport law. All relations arise from deterministic transport, exchange interaction, and scalar geometry.

9.3 Distinction from Fluid and Classical Field Theories

Although the continuity equation and transport structure bear formal similarity to fluid or classical field descriptions, the present framework is not based on fluid dynamics or classical mechanics.

The velocity field $v(x,t)$ is not an independent dynamical variable, nor is it governed by a force law. Instead, it is constrained by phase structure and scalar geometry, as established in Section 6.

Thus, the transport law should not be interpreted as a classical flow equation, but as a geometric description of closure transport.

9.4 Phase as a Derived Quantity

The phase $\phi(x,t)$ remains a derived quantity, defined through exchange-modulated transport. It does not generate motion and does not serve as a primitive dynamical variable; rather, it encodes the structure that constrains admissible transport.

Its role is to encode the cumulative effect of transport and exchange, and to provide a structure through which coherence and closure conditions may be expressed.

9.5 Separation of Structure and Representation

The transport law derived here establishes a structural framework linking closure density, phase, and scalar geometry. This structure is independent of any particular mathematical representation.

Alternative formulations may be constructed in which the coupled system is expressed in a different form, including representations that combine ρ and ϕ into a single object. Such formulations are not unique and should be understood as encodings of the underlying transport-consistent structure.

9.6 Absence of Operator Formalism

No operator formalism is introduced in the present work. The transport law is expressed entirely in terms of scalar fields and their spatial and temporal variation.

While operator-based representations may be constructed in subsequent developments, they are not required for the formulation of the transport law and do not constitute the foundational description.

9.7 Deterministic Character of the Framework

The transport system derived in this paper is fully deterministic. The evolution of closure density and phase is governed by the coupled transport equations and the scalar geometry.

Any probabilistic or statistical interpretation must therefore be regarded as emergent or effective, arising from the behavior of the system under specific conditions rather than from fundamental assumptions.

9.8 Scope of the Present Construction

The present work establishes the first complete transport law for the exchange sector of scalar–conformal NUVO space. It provides a closed system governing the evolution of closure density and phase under the influence of scalar geometry.

No attempt is made to introduce measurement theory, statistical interpretation, or representational constructs beyond the minimal structure required for transport.

The primary result is a geometric and transport-consistent dynamical framework from which further representations may be developed in subsequent work.

10 Transition to Schrödinger Representation

10.1 Motivation for a Unified Representation

The transport law derived in Sections 3–7 provides a closed system governing the evolution of closure density $\rho(x, t)$ and phase $\phi(x, t)$. While this formulation is complete and fully consistent with the geometric structure of the theory, it involves two coupled fields whose evolution is described by separate equations.

It is natural to ask whether these fields may be combined into a single object that encodes both density and phase, thereby providing a more compact representation of the transport system.

10.2 Amplitude–Phase Construction

A unified representation may be introduced by defining a complex quantity of the form

$$\Psi(x, t) := \sqrt{\rho(x, t)}\, e^{i\phi(x,t)/\Phi_0},$$

where Φ_0 is a normalization constant related to the coherence scale.

This construction is structurally analogous to the amplitude–phase decomposition introduced in early reformulations of quantum mechanics [13].

It is emphasized that $\Psi(x, t)$ is introduced here purely as a representational device. It is not assumed to be fundamental, and its introduction does not alter the underlying transport-based framework.

10.3 Rewriting the Transport System

The coupled transport equations for $\rho(x, t)$ and $\phi(x, t)$ may be re-expressed in terms of $\Psi(x, t)$. In this representation, the continuity equation and phase evolution equation combine to yield a single evolution equation for Ψ.

The resulting equation depends on the specific form of the transport function $\mathcal{V}(\nabla\phi, \lambda)$ and the exchange rate $\mathcal{E}(x)$, as well as the normalization scale Φ_0.

10.4 Emergence of Schrödinger-Type Structure

For particular choices of representation and normalization, and under appropriate limiting conditions consistent with the closure-action scale, the evolution equation for Ψ takes a form analogous to the Schrödinger equation [14].

In this setting, the second-order structure identified in Section 6 appears as a Laplacian-like term, while the exchange and geometry dependence enters as an effective potential-like contribution.

It is important to emphasize that this correspondence is not assumed, but arises from the transport-consistent structure derived in the present work.

10.5 Representation Dependence

The appearance of Schrödinger-type structure depends on the choice of representation and normalization. Different choices of Φ_0 and different formulations of $\mathcal{V}$ and $\mathcal{E}$ may lead to alternative forms of the evolution equation.

This reinforces the distinction between structure and representation: the underlying transport law is unique, while its representation is not.

10.6 Interpretive Clarification

The introduction of $\Psi(x,t)$ does not imply the existence of a wavefunction in the conventional sense. The complex representation is an encoding of closure density and phase, and its use is optional.

No probabilistic interpretation is introduced by this construction. Any such interpretation must be regarded as an emergent description of the transport system under specific conditions.

10.7 Transition to Subsequent Work

The representation introduced here provides a bridge between the transport-based formulation developed in the Q-series and more familiar dynamical equations.

In the subsequent paper [15], this representation will be developed in detail, and the conditions under which Schrödinger-type equations emerge will be examined explicitly. The goal of that development is not to assume such equations, but to derive them as representations of the underlying transport law.

11 Summary

In this paper, we have derived a unified transport law governing the evolution of closure structures in scalar–conformal NUVO space. This development extends the kinematic framework established in Q8–Q10 to a fully consistent dynamical description of transport, exchange, and scalar geometry.

The key results of this work are as follows:

- A closure density field $\rho(x,t)$ was introduced to describe the spatial distribution of admissible closure structures, together with a transport velocity field $v(x,t)$ representing the flow of closure content;
- A continuity equation
$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$
was derived from the requirement of closure persistence under admissible transport;
- A phase evolution equation
$$\partial_t \phi + v \cdot \nabla \phi = \kappa\, \mathcal{E}(x)$$
was obtained from the transport-based definition of phase, linking phase dynamics to exchange interaction;
- The coupled system was reduced to a closed formulation in terms of $\rho(x,t)$ and $\phi(x,t)$ by expressing transport in terms of phase structure, leading to an emergent second-order description of the system;
- The scalar field $\lambda(x)$ was shown to enter the transport law through both exchange and transport structure, establishing geometry as an active determinant of dynamical behavior.
- The normalization of phase evolution was shown to be fixed by the closure-action scale established in Q10, ensuring that the transport law introduces no additional free parameters;

These results demonstrate that a complete dynamical framework for the exchange sector can be derived directly from transport consistency and scalar geometry, without the introduction of external dynamical assumptions, wave ontology, or probabilistic interpretation.

The transport law obtained here is fully deterministic and geometric in character. It provides a closed system governing the evolution of closure density and phase, and reproduces the coherence and closure conditions established in earlier work.

Furthermore, it has been shown that this system admits alternative representations in which density and phase may be combined into a single object. In such representations, familiar dynamical equations may emerge under appropriate conditions, but these are not fundamental to the construction.

The present work therefore establishes the first complete dynamical description of the exchange sector within the NUVO framework. It serves as the foundation for subsequent developments in which the transport law will be expressed in alternative representations and connected to established dynamical formalisms.

In particular, the next paper will develop the representation introduced in Section 9 and examine explicitly the conditions under which Schrödinger-type equations arise as encodings of the transport structure derived here.

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