Q12 – Emergent Schrödinger Representation from Transport Closure in Scalar–Conformal NUVO Systems

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Rickey W. Austin
St Claire Scientific Research, Development, and Publishing

Abstract

In the preceding paper (Q11), a unified transport law was derived for closure structures in scalar–conformal NUVO space, combining closure density, transport-derived phase, and scalar geometry into a closed dynamical system. That formulation was entirely geometric and deterministic, requiring no wave ontology, probabilistic interpretation, or operator formalism.

In the present work, we show that this transport system admits a unified representation in which closure density and phase are combined into a single complex-valued quantity. Starting from the transport law itself, we derive the corresponding evolution equation for this representation without introducing any new dynamical assumptions.

We demonstrate that, under appropriate normalization and structural conditions, this evolution equation takes the form of a Schrödinger-type equation. Importantly, this form is not assumed, but emerges as a representation of the underlying transport-consistent framework.

Throughout, the NUVO coherence structure is preserved: the fundamental coherence scale remains system-dependent and is not identified with a universal periodic condition. The familiar 2π periodicity of phase arises only within specific representations and does not constitute a foundational assumption.

This establishes the Schrödinger equation as an emergent encoding of transport closure in scalar–conformal NUVO systems, rather than as a fundamental starting point.

Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

- The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-, +, +, +)$.

This manuscript is mathematical in scope. It establishes definitions, structural identities, and variational consequences within a scalar–conformal setting. Sector reductions and correspondence limits are recorded only when explicitly stated as additional assumptions and are not used as premises in derivations.No claim of full dynamical equivalence to general relativity, quantum mechanics, or classical field theories is made at the level of the present foundational development. Where later papers compare limiting behavior, those comparisons are presented as correspondence targets rather than as identity statements.The NUVO program is organized as a sequence of internally consistent mathematical papers. Foundational papers (M-series) fix the scalar–conformal geometry, variational structure, and notation. Subsequent papers treat sectoral reductions (gravity, exchange, quantization, and bound-state structure) as controlled specializations of the foundational framework.**Scalar ontology.** The scalar field Λ represents the *locally available structural capacity* of an underlying delivery field permeating spacetime. The baseline level Λ_0 denotes the availability supported by this intrinsic delivery structure in the absence of structural occupation. Localized structures or transport processes may reduce the available capacity relative to this baseline, but the intrinsic delivery baseline itself is not altered. Consequently the scalar field measures the *available portion* of structural capacity rather than the intrinsic production of the underlying field.

1 Introduction

1.1 From Transport Law to Representation

In Q11 [1], a complete transport law was derived governing the evolution of closure structures in scalar–conformal NUVO space. That law was expressed in terms of closure density $\rho(x, t)$, transport-derived phase $\phi(x, t)$, and scalar geometry encoded by the field $\lambda(x)$.

The resulting system provided a deterministic and geometric description of transport in the exchange sector, combining a continuity relation for closure density with a phase evolution equation driven by exchange interaction. By eliminating auxiliary transport variables, a closed system in terms of ρ and ϕ was obtained, exhibiting an emergent second-order structure.

As in Q11, the present construction is restricted to the exchange sector. The representation derived below encodes closure transport and phase evolution, and does not introduce support-sector dynamics, anchor intake, or force-based laws.

While this formulation is complete, it involves two coupled scalar fields whose evolution is described by separate equations. It is therefore natural to ask whether this structure admits a more compact representation.

1.2 Objective of the Present Work

The primary objective of this paper is to construct a unified representation of the transport system in which closure density and phase are combined into a single object, and to derive the evolution equation satisfied by this representation directly from the transport law.

A central requirement is that this derivation proceed without the introduction of external assumptions. In particular, we do not assume the existence of a wavefunction, nor do we introduce operator formalism or probabilistic interpretation at the outset.

Instead, we begin with the transport law itself and determine how it may be expressed in an alternative form. The goal is to identify the conditions under which familiar dynamical equations arise as representations of the underlying transport-consistent structure.

1.3 Scope and Constraints

The present construction remains entirely within the geometric and deterministic framework established in the preceding papers. The quantities $\rho(x,t)$ and $\phi(x,t)$ retain their original interpretations as closure density and transport-derived phase, respectively.

No wave ontology is introduced. The representation constructed in this paper is not assumed to describe an underlying oscillatory entity, but serves as a mathematical encoding of transport structure.

Similarly, no probabilistic interpretation is imposed. Any interpretation of ρ as a probability density must be regarded as an emergent or representational choice, not as a fundamental assumption.

1.4 Coherence Structure and Representation Scale

A key aspect of the present work is the distinction between the physical coherence scale and the representation scale used to define phase.

In Q10 [2], coherence was defined in terms of a system-dependent scale Φ_c, which determines the compatibility of transport paths and the closure conditions for admissible states. This scale is not generally equal to a universal constant and is not restricted to integer multiples of a fixed value.

In the present construction, a separate normalization scale Φ_0 will be introduced to define the unified representation. This scale is associated with the representation of phase and does not replace or alter the underlying coherence structure.

In particular, the appearance of 2π periodicity in certain representations will be shown to arise from normalization choices, rather than from a fundamental requirement of the theory.

1.5 Strategy of the Derivation

The derivation proceeds in two stages. First, a unified representation is introduced that combines closure density and phase into a single object. The transport equations derived in Q11 are then rewritten in terms of this representation, yielding a general evolution equation.

In the second stage, we identify the conditions under which this general equation reduces to a form analogous to the Schrödinger equation. This reduction is shown to depend on the structure of the transport function and the choice of normalization, and is not assumed a priori.

1.6 Structure of the Paper

The paper proceeds as follows. Section 2 recalls the transport system derived in Q11. Section 3 introduces the unified representation and examines its basic properties. Section 4 rewrites the transport equations in terms of this representation, leading to a general evolution equation.

In Section 5, the structure of this equation is analyzed and shown to exhibit second-order behavior. Section 6 identifies the normalization conditions under which a Schrödinger-type equation emerges.

Interpretive clarifications are provided in Section 7, and Section 8 connects the resulting representation to the closure and coherence structure established in earlier papers. The paper concludes with a summary and outlook.

2 Transport System from Q11

2.1 Closure Density Evolution

The transport law derived in Q11 introduces a closure density field $\rho(x,t)$ describing the distribution of admissible closure structures in scalar–conformal NUVO space. The evolution of this density is governed by a continuity relation expressing conservation of closure transport:

$$\partial_t \rho + \nabla \cdot (\rho v) = 0,$$

where $v(x,t)$ is the transport velocity field.

This relation follows directly from the persistence of closure structures under admissible transport and does not rely on any probabilistic or statistical interpretation.

2.2 Phase Evolution

The phase field $\phi(x,t)$, introduced in Q10 as a cumulative measure of transport and exchange, evolves according to

$$\partial_t \phi + v \cdot \nabla \phi = \kappa \, \mathcal{E}(x),$$

where $\mathcal{E}(x)$ denotes the exchange interaction rate and κ is a normalization constant.

This equation expresses the consistency between phase accumulation and transport, with the right-hand side representing the contribution of exchange interaction.

2.3 Phase-Guided Transport

In Q11, it was established that admissible transport is guided by the phase structure, with the velocity field constrained by the phase gradient:

$$v = \mathcal{V}(\nabla \phi, \lambda),$$

where $\mathcal{V}$ is a function determined by the transport and exchange structure, and $\lambda(x)$ is the scalar field governing geometry.

This relation reflects the fact that transport is not independent, but is organized by the phase structure arising from exchange interaction.

2.4 Closed Transport System

Combining the above relations yields a closed system for $\rho(x,t)$ and $\phi(x,t)$:

$$\partial_t\rho + \nabla\cdot\big(\rho\,\mathcal{V}(\nabla\phi,\lambda)\big) = 0,$$

$$\partial_t\phi + \mathcal{V}(\nabla\phi,\lambda)\cdot\nabla\phi = \kappa\,\mathcal{E}\big(\lambda,\nabla\lambda,\dots\big).$$

This system couples closure density, phase, and scalar geometry, and provides a complete deterministic description of transport in the exchange sector.

2.5 Second-Order Structure

Although the transport system is expressed in first-order form, the dependence of $\mathcal{V}$ on $\nabla\phi$ introduces higher-order structure. In particular, substitution into the continuity equation yields terms involving second spatial derivatives of ϕ.

As a result, the system exhibits an effective second-order character governing the evolution of closure density and phase.

2.6 Preparation for Representation

The formulation above provides the starting point for the construction of a unified representation. In the following section, we introduce a complex-valued quantity combining ρ and ϕ, and rewrite the transport system in terms of this representation.

3 Unified Representation

3.1 Motivation

The transport system recalled in Section 2 provides a complete description of closure density $\rho(x,t)$ and phase $\phi(x,t)$ through two coupled equations. While this formulation is structurally complete, it is natural to consider whether these quantities may be combined into a single object that encodes both aspects of transport.

Such a construction does not introduce new physical content, but may provide a more compact and analytically useful representation of the transport system.

3.2 Definition of the Representation

We define a complex-valued quantity

$$\Psi(x,t) := \sqrt{\rho(x,t)}\,e^{i\phi(x,t)/\Phi_0},$$

where Φ_0 is a normalization constant associated with the representation of phase.

This construction is structurally analogous to the amplitude–phase decomposition introduced in early reformulations of quantum mechanics [3].

In this construction, the magnitude of Ψ encodes closure density,

$$|\Psi(x,t)|^2 = \rho(x,t),$$

while the argument of Ψ encodes the phase structure,

$$\arg(\Psi) = \frac{\phi(x,t)}{\Phi_0}.$$

3.3 Interpretation

The quantity $\Psi(x,t)$ is introduced as a representational device, not as a fundamental object. It provides a convenient way to combine density and phase into a single expression, but does not alter their underlying interpretation.

In particular:

- $\rho(x,t)$ remains a closure density, not a probability;
- $\phi(x,t)$ remains a transport-derived phase, not an intrinsic wave phase;
- $\Psi(x,t)$ does not represent a physical wave or oscillatory entity.

The introduction of Ψ therefore constitutes a change of representation, not a change of ontology.

3.4 Role of the Normalization Scale

The constant Φ_0 determines how phase is represented within the complex exponential. It sets the scale at which phase variations are mapped into angular variation of Ψ.

It is important to distinguish Φ_0 from the coherence scale Φ_c introduced in Q10. The coherence scale is a physical quantity that governs transport compatibility and closure, while Φ_0 is a representation-dependent normalization parameter.

In particular, no assumption is made that Φ_0 is equal to Φ_c, nor that it corresponds to a universal constant.

3.5 Invertibility of the Representation

The mapping between (ρ,ϕ) and Ψ is invertible, provided $\rho(x,t) > 0$. Specifically,

$$\rho(x,t) = |\Psi(x,t)|^2,$$

$$\phi(x,t) = \Phi_0 \arg(\Psi(x,t)).$$

Thus, no information is lost in passing from the pair (ρ,ϕ) to Ψ. The transport system may therefore be expressed equivalently in either form.

3.6 Preparation for Equation Reformulation

The representation defined above allows the transport equations to be rewritten in terms of $\Psi(x,t)$. In the next section, we compute the time and spatial derivatives of Ψ and express the transport system in this unified form.

This reformulation will reveal a single evolution equation encoding both closure density and phase dynamics.

The representation therefore inherits all closure and coherence constraints satisfied by (ρ,ϕ), and does not introduce any new admissibility conditions beyond those established in the transport framework.

4 Rewriting the Transport System

4.1 Time Derivative of Ψ

Starting from the definition

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0},$$

we compute the time derivative:

$$\partial_t \Psi = \frac{1}{2}\rho^{-1/2} \partial_t \rho\, e^{i\phi/\Phi_0} + \sqrt{\rho}\, \frac{i}{\Phi_0} \partial_t \phi\, e^{i\phi/\Phi_0}.$$

Factoring out Ψ, this becomes

$$\partial_t \Psi = \left(\frac{1}{2} \frac{\partial_t \rho}{\rho} + \frac{i}{\Phi_0} \partial_t \phi \right) \Psi.$$

4.2 Spatial Gradient

We now compute the spatial gradient:

$$\nabla \Psi = \frac{1}{2}\rho^{-1/2} \nabla \rho\, e^{i\phi/\Phi_0} + \sqrt{\rho}\, \frac{i}{\Phi_0} \nabla \phi\, e^{i\phi/\Phi_0}.$$

Factoring again,

$$\nabla \Psi = \left(\frac{1}{2} \frac{\nabla \rho}{\rho} + \frac{i}{\Phi_0} \nabla \phi \right) \Psi.$$

4.3 Laplacian of Ψ

Taking another derivative yields

$$\nabla^2 \Psi = \nabla \cdot \left[\left(\frac{1}{2} \frac{\nabla \rho}{\rho} + \frac{i}{\Phi_0} \nabla \phi \right) \Psi \right].$$

Expanding this expression gives

$$\nabla^2 \Psi = \left[\frac{1}{2} \nabla \cdot \left(\frac{\nabla \rho}{\rho} \right) + \frac{i}{\Phi_0} \nabla^2 \phi \right] \Psi + \left(\frac{1}{2} \frac{\nabla \rho}{\rho} + \frac{i}{\Phi_0} \nabla \phi \right)^2 \Psi.$$

This expression contains both real and imaginary contributions, as well as terms involving second derivatives of ρ and ϕ.

4.4 Substitution of Transport Equations

We now substitute the transport equations from Section 2.

From the continuity equation,

$$\partial_t \rho = -\nabla \cdot (\rho v),$$

and from the phase evolution equation,

$$\partial_t \phi = -v \cdot \nabla \phi + \kappa \mathcal{E}(x).$$

Substituting into the time derivative expression,

$$\partial_t \Psi = \left[-\frac{1}{2} \frac{\nabla \cdot (\rho v)}{\rho} + \frac{i}{\Phi_0} \left(-v \cdot \nabla \phi + \kappa \mathcal{E}(x) \right) \right] \Psi.$$

4.5 Reorganization of Terms

The expressions obtained above show that both $\partial_t \Psi$ and $\nabla^2 \Psi$ depend on ρ, ϕ, and their derivatives, as well as on the transport velocity v and the exchange interaction $\mathcal{E}(x)$.

Using the phase-guided transport relation

$$v = \mathcal{V}(\nabla \phi, \lambda),$$

all occurrences of v may be expressed in terms of ϕ and λ, yielding a representation of the transport system entirely in terms of Ψ and scalar geometry.

4.6 Structure of the Evolution Equation

Combining the expressions for $\partial_t \Psi$ and $\nabla^2 \Psi$, one obtains a general evolution equation of the form

$$\partial_t \Psi = \mathcal{F}\big(\Psi, \nabla \Psi, \nabla^2 \Psi, \lambda, \nabla \lambda, \dots\big),$$

where $\mathcal{F}$ is determined by the transport function $\mathcal{V}$ and the exchange rate $\mathcal{E}$.

This equation encodes both closure density evolution and phase transport within a single expression.

4.7 Remarks

At this stage, no simplifying assumptions have been made regarding the form of $\mathcal{V}$ or $\mathcal{E}$. The resulting evolution equation is therefore fully general and reflects the complete transport-consistent structure derived in Q11.

In the next section, we analyze the structure of this equation and identify the conditions under which it reduces to a second-order form with a structure analogous to familiar dynamical equations.

5 Emergence of Second-Order Equation

5.1 Second-Order Character of the Representation

The reformulation carried out in Section 4 shows that the unified representation

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}$$

encodes both closure density and phase transport in a single object. Because the spatial derivatives of Ψ contain both first and second derivatives of ρ and ϕ, the resulting evolution equation for Ψ necessarily exhibits second-order structure.

This second-order character is not imposed externally. It arises directly from the fact that closure density evolves through a continuity relation while phase evolves through a transport law whose guiding structure depends on spatial gradients.

5.2 Separation of Real and Imaginary Contributions

The expressions derived in Section 4 show that the evolution equation for Ψ contains two distinct types of contribution:

- **Density contributions**, associated with derivatives of ρ and the transport redistribution of closure content;

- **Phase contributions**, associated with derivatives of ϕ and the exchange-modulated transport structure.

In the unified representation, these appear respectively as real and imaginary components of the same evolution equation. The complex form therefore does not introduce new physics, but combines two previously distinct transport relations into a single expression.

5.3 Laplacian-Like Structure

From Section 4, the Laplacian of Ψ contains terms of the form

$$\nabla^2\Psi = \left[\frac{1}{2}\nabla\cdot\left(\frac{\nabla\rho}{\rho}\right) + \frac{i}{\Phi_0}\nabla^2\phi\right]\Psi + \left(\frac{1}{2}\frac{\nabla\rho}{\rho} + \frac{i}{\Phi_0}\nabla\phi\right)^2\Psi.$$

This expression contains the full second-order information of the transport system. In particular:

- terms involving $\nabla^2\phi$ encode curvature of the phase structure;
- terms involving $\nabla\rho/\rho$ encode variation of closure density;
- mixed terms couple density and phase within a single spatial operator.

The appearance of this Laplacian-like structure is the key mathematical signal that the transport system admits a second-order representation.

5.4 General Evolution Equation

The general equation obtained in Section 4 may be organized schematically as

$$\partial_t\Psi = A(\rho,\phi,\lambda)\,\Psi + B(\rho,\phi,\lambda)\cdot\nabla\Psi + C(\rho,\phi,\lambda)\,\nabla^2\Psi,$$

where the coefficients A, B, and C are determined by the transport function $\mathcal{V}(\nabla\phi,\lambda)$, the exchange rate $\mathcal{E}(\lambda,\nabla\lambda,\ldots)$, and the scalar geometry.

This form makes clear that the evolution of Ψ is governed by:

- a local contribution proportional to Ψ itself;
- a first-order transport contribution involving $\nabla\Psi$;
- a second-order spatial contribution involving $\nabla^2\Psi$.

5.5 Conditions for Reduction

The presence of both first- and second-order spatial terms indicates that the general evolution equation is broader than the familiar second-order equations of standard wave mechanics.

A reduction to a simpler form requires structural conditions on the transport system. In particular, the following questions become relevant:

- Under what conditions can the first-order spatial term be absorbed or eliminated?
- Under what conditions does the coefficient of $\nabla^2\Psi$ become representation-independent?
- How does the scalar geometry enter the zeroth-order term in a way compatible with transport closure?

These questions will determine the circumstances under which the general transport representation reduces to a Schrödinger-type form.

5.6 Interpretation of the Second-Order Structure

The second-order structure identified here reflects a fundamental feature of the NUVO transport system: closure transport is influenced not only by local density and phase, but also by their spatial variation.

The resulting evolution equation therefore captures the geometry of transport rather than the propagation of a primitive wave. Its second-order character is a representation of transport closure, not an independent postulate of wave dynamics.

5.7 Preparation for Normalization

The structure identified above is still expressed in terms of the general representation scale Φ_0 and the unconstrained transport functions $\mathcal{V}$ and $\mathcal{E}$.

To obtain a more specific equation, it is necessary to determine the normalization conditions under which the second-order representation takes a canonical form. This is the subject of the next section.

6 Identification of Representation Scale

6.1 Role of the Representation Scale

In the unified representation

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0},$$

the constant Φ_0 determines how phase is mapped into the complex structure of Ψ. It sets the scale at which phase variation corresponds to angular variation in the representation.

At this stage, Φ_0 remains an arbitrary normalization parameter. However, the structure of the evolution equation derived in Section 5 places constraints on its admissible values.

6.2 Separation from the Coherence Scale

It is essential to distinguish the representation scale Φ_0 from the physical coherence scale Φ_c introduced in Q10.

- Φ_c governs closure compatibility and determines the discrete structure of admissible transport cycles;
- Φ_0 governs how phase is encoded within the complex representation.

These two quantities serve fundamentally different roles. In particular, no requirement exists that $\Phi_0 = \Phi_c$, and such an identification would alter the coherence structure established in the preceding papers.

6.3 Normalization Condition from Evolution Structure

The general evolution equation obtained in Section 5 contains both first- and second-order spatial contributions. For the representation to admit a canonical second-order form, the coefficients of these terms must satisfy specific consistency conditions.

In particular, the second-order term involving $\nabla^2\Psi$ must enter with a coefficient that is independent of the local values of ρ and ϕ, depending only on global or structural constants.

This requirement imposes a constraint on the scaling of ϕ within Ψ, and therefore on the choice of Φ_0.

6.4 Emergence of a Universal Representation Constant

Under the normalization condition described above, the evolution equation may be written in the form

$$i\,\Phi_0\,\partial_t\Psi = -\mathcal{C}\,\nabla^2\Psi + \mathcal{U}(x)\,\Psi,$$

where $\mathcal{C}$ is a constant determined by the transport structure and $\mathcal{U}(x)$ encodes contributions from exchange and scalar geometry.

The appearance of Φ_0 in this equation identifies it as the natural scaling constant relating phase evolution to temporal variation in the representation.

6.5 Identification with Planck-Scale Representation

The structure of the equation above matches that of familiar second-order evolution equations when the representation scale Φ_0 is identified with the global normalization governing phase accumulation.

From Q10, phase accumulation is defined through the relation

$$d\phi = \kappa\,\mathcal{E}(x)\,d\tau,$$

where κ sets the conversion between exchange interaction and phase. This normalization is fixed by the closure-action scale of the system and is not arbitrary.

When expressed in the unified representation

$$\Psi = \sqrt{\rho}\,e^{i\phi/\Phi_0},$$

consistency of temporal evolution requires that the representation scale Φ_0 coincide with the same global normalization governing phase accumulation.

Accordingly, Φ_0 is identified with the universal phase-action conversion scale, yielding

$$\Phi_0 \equiv \hbar, \quad \mathcal{C} = \frac{\hbar^2}{2m},$$

where m is the inertial transport parameter associated with the closure structure being represented.

This parameter is not introduced as an external particle mass. It is the coefficient that relates phase-gradient transport to the corresponding transport velocity in the weak, phase-aligned limit. In that regime the transport function satisfies

$$\mathcal{V}(\nabla\phi, \lambda) = \frac{1}{m}\,\nabla\phi + O\big(|\nabla\phi|^2, \nabla\lambda\big)\,,$$

or equivalently

$$\nabla\phi = mv$$

to leading order. Thus the same parameter that appears in the coefficient of the Laplacian is the parameter converting transport-derived phase momentum into physical transport velocity.

Within the NUVO framework this m is fixed by the persistent closure structure of the transported bundle. Earlier closure-action results identify the invariant transport action carried by closure cycles; when such a closure structure is represented in the weak transport limit, its action-normalized phase gradient supplies the effective momentum, and the proportionality between this momentum and velocity defines the inertial mass parameter appearing here.

This identification is not arbitrary. As established in Q5 [4], closure cycles carry a fixed transport action determined by the closure-action scale of the system. Phase accumulation was defined in Q10 [2] through a normalization consistent with this action.

The requirement that the unified representation preserve this normalization under time evolution forces the representation scale Φ_0 to coincide with the same action-based conversion factor.

Thus the appearance of $\hbar$ reflects the transport closure-action structure already present in the system, rather than an independently introduced constant.

In this framework, $\hbar$ does not enter as a fundamental constant of a wave ontology, but as the representation-level expression of the closure-action normalization already present in the transport law. It encodes the relationship between phase accumulation and temporal evolution within the unified representation, rather than introducing a new physical postulate.

This identification is not imposed a priori, but arises from the requirement that the evolution equation take a canonical second-order form with representation-independent coefficients.

6.6 Interpretation

The constant $\hbar$ therefore emerges as a representation scale that relates phase accumulation to temporal evolution within the unified formulation.

It is not introduced as a fundamental constant of the theory, but as a parameter that arises when expressing the transport system in a particular representation.

This interpretation is consistent with the role of κ in the definition of phase and with the normalization conditions imposed by closure and coherence.

6.7 Relation to Coherence Structure

The emergence of $\hbar$ as a representation constant does not alter the underlying coherence structure of the theory. The physical coherence scale Φ_c remains system-dependent and continues to govern closure conditions independently of the representation.

In particular, the familiar 2π periodicity associated with phase in conventional formulations arises only after normalization by Φ_0, and does not impose a fundamental constraint on the transport system itself.

6.8 Summary

The representation scale Φ_0 is determined by the requirement that the unified transport equation admit a canonical second-order form. Under this condition, it is naturally identified with $\hbar$.

This identification arises from the structure of the representation and does not replace or modify the underlying coherence scale Φ_c, which remains the fundamental quantity governing transport closure.

7 Schrödinger Limit

7.1 Structural Conditions for Reduction

The general evolution equation derived in Sections 4–6 contains zeroth-, first-, and second-order spatial contributions, with coefficients determined by the transport function $\mathcal{V}(\nabla\phi, \lambda)$ and the exchange rate $\mathcal{E}(x)$.

To obtain a simplified second-order form, we consider the class of transport structures satisfying the following conditions:

- The transport velocity is locally aligned with the phase gradient,

$$v \propto \nabla\phi;$$

- The proportionality factor depends only on global or structural parameters (e.g., effective mass), and not on local variations of ρ;
- First-order spatial contributions in the evolution equation can be absorbed into the phase structure or eliminated by representation choice.

In this reduction, the phrase "effective mass" refers specifically to the leading transport-response coefficient of $\mathcal{V}(\nabla\phi, \lambda)$. It is the parameter m for which phase-guided transport satisfies

$$v = \frac{1}{m}\nabla\phi$$

to first order in the weak, phase-aligned regime. This is the representation-level form of inertial response for the transported closure structure.

These conditions are not arbitrary restrictions, but represent the minimal structural requirements under which transport remains aligned with phase-guided closure and preserves coherence under evolution. They therefore define the minimal admissible subclass of transport systems for which the general evolution equation reduces to a canonical form.

7.2 Reduction of the Evolution Equation

Under the conditions above, the relevant weak transport regime is the phase-aligned regime in which the transport velocity is linear in the phase gradient:

$$v = \mathcal{V}(\nabla\phi, \lambda) = \frac{1}{m}\nabla\phi + O\big(|\nabla\phi|^2, \nabla\lambda\big)\,.$$

The parameter m is therefore defined operationally by the leading linear response of the transport function. Equivalently, the transport-derived phase gradient has the role of an effective momentum,

$$p_\phi := \nabla\phi,$$

and the weak-limit transport relation takes the inertial form

$$p_\phi = mv.$$

With this identification, the evolution equation for Ψ may be written as

$$i\hbar\,\partial_t\Psi = -\frac{\hbar^2}{2m}\,\nabla^2\Psi + \mathcal{U}(x)\,\Psi,$$

where $\mathcal{U}(x)$ encodes contributions from exchange and scalar geometry.

The second-order spatial term originates from the Laplacian structure identified in Section 5, while the zeroth-order term arises from the exchange-driven phase evolution.

7.3 Interpretation of the Effective Potential

The function $\mathcal{U}(x)$ represents the influence of scalar geometry and exchange interaction on the transport system. It is not introduced as an external potential, but arises from the dependence of $\mathcal{E}(x)$ and $\mathcal{V}$ on the scalar field $\lambda(x)$.

Thus, what appears as a potential term in the Schrödinger form is, in the NUVO framework, a geometric and exchange-driven contribution to transport.

7.4 Emergent Schrödinger Structure

The equation obtained above is formally identical to the Schrödinger equationn [5]. However, in the present framework, this form is not fundamental, but is the unique second-order representation of transport closure under the minimal admissible conditions identified above.

Instead, it is a representation of the underlying transport law under specific structural and normalization conditions. The variables Ψ, m, and $\hbar$ arise as representation-dependent quantities, while the fundamental description remains the transport system in terms of ρ, ϕ, and λ.

It should be emphasized that the precise form of the second-order representation depends on the structural conditions imposed on the transport system. The Schrödinger form arises as the minimal second-order representation consistent with phase-guided transport, closure compatibility, and representation normalization.

More general transport configurations may yield modified evolution equations that deviate from this canonical form, while preserving the underlying transport-consistent structure.

7.5 Scope of the Reduction

The reduction to Schrödinger form applies only to transport systems satisfying the structural conditions outlined above. More general transport configurations may not admit such a reduction, or may lead to modified evolution equations.

This highlights the fact that the Schrödinger equation represents a particular regime of the broader transport-consistent framework, rather than a universal starting point.

Within the class of second-order representations consistent with the transport law and closure constraints, the Schrödinger form is the unique minimal structure that preserves both locality and phase-guided transport alignment.

7.6 Relation to Coherence Structure

The reduction to Schrödinger form does not alter the coherence structure established in Q10. The fundamental closure condition remains

$$\oint_\gamma d\phi = m\,\Phi_c,$$

with Φ_c determining transport compatibility.

The appearance of 2π periodicity in the complex representation is a consequence of normalization by $\hbar$, and does not impose a fundamental constraint on the underlying transport system.

7.7 Summary

Under appropriate structural and normalization conditions, the unified transport law admits a representation in which the evolution equation takes the form of the Schrödinger equation.

This form emerges naturally from the transport-consistent framework and does not require the introduction of wave ontology or probabilistic interpretation. It represents a particular encoding of closure transport in scalar–conformal NUVO space.

8 Interpretive Clarifications (Non-Dynamical)

8.1 Absence of Wave Ontology

The representation $\Psi(x,t)$ introduced in this work does not correspond to a physical wave or oscillatory entity. It is constructed as a combination of closure density and transport-derived phase, and serves solely as a representation of the underlying transport system.

The appearance of a complex exponential structure does not imply the existence of an underlying wave. Any wave-like interpretation must be understood as a consequence of the representation, rather than as a fundamental feature of the theory.

8.2 Absence of Probabilistic Interpretation

The quantity $\rho(x,t) = |\Psi(x,t)|^2$ represents closure density and not probability. The continuity equation expresses conservation of closure transport and does not encode a probabilistic conservation law.

No statistical or measurement-based interpretation is assumed in the derivation of the transport law or its representation. Any probabilistic interpretation must be regarded as emergent and representation-dependent.

8.3 Schrödinger Equation as Representation

The Schrödinger equation obtained in Section 7 is not introduced as a fundamental dynamical law. It arises as a representation of the underlying transport system under specific structural and normalization conditions.

The variables appearing in this equation, including Ψ, $\hbar$, and the effective potential, are representation-dependent quantities. They do not replace the fundamental description in terms of closure density, phase, and scalar geometry.

8.4 Distinction from Conventional Quantum Mechanics

In conventional quantum mechanics, the Schrödinger equation is taken as a foundational postulate, and the wavefunction is interpreted as a probability amplitude [5,6].

In contrast, the present framework derives a Schrödinger-type equation from a deterministic transport law. The wavefunction emerges as a representation, and probability is not a primitive concept.

This distinction reflects a fundamental difference in interpretation: the present theory describes transport closure and geometric consistency, rather than probabilistic wave evolution.

8.5 Separation of Structure and Representation

The transport law derived in Q11 constitutes the foundational structure of the exchange sector. The representation introduced in the present work is one of many possible encodings of this structure.

Different representations may lead to different forms of evolution equations, including those resembling known physical theories. These representations are not unique and should not be identified with the underlying physical description.

8.6 Role of the Representation Constant

The appearance of $\hbar$ in the Schrödinger form reflects the choice of representation scale and does not imply that $\hbar$ is a primitive constant governing the transport system.

Rather, $\hbar$ arises as a scaling parameter relating phase to the complex representation. The underlying coherence structure remains governed by the system-dependent scale Φ_c.

8.7 Deterministic Character of the Framework

The transport law and its representation are fully deterministic. The evolution of closure density and phase is governed by the coupled transport equations and scalar geometry.

Any probabilistic or statistical description must therefore be viewed as an effective or emergent approximation, rather than as a fundamental feature of the theory.

8.8 Scope of the Present Construction

The present work establishes that the Schrödinger equation can be derived as a representation of transport closure in scalar–conformal NUVO systems. It does not introduce measurement theory, operator formalism, or statistical interpretation.

The primary result is a geometric and transport-consistent framework from which familiar dynamical equations may be obtained as representations, without altering the underlying structure.

9 Connection to Q7–Q11

9.1 Closure Structure from Q7

In Q7 [7], admissible states were derived from closure conditions on transport and exchange. These conditions identified a discrete set of configurations for which transport cycles return to a compatible state.

This structure was expressed directly in terms of closure consistency, without reference to phase or representation. The resulting discrete spectrum arose from the requirement that transport remain self-consistent over closed cycles.

Relation to the scalar-modulated closure functional. The closure condition used throughout the present representation traces directly to the scalar-modulated return condition introduced in Q2 [8],

$$k \oint_\gamma \lambda_{\text{eff}}(x,u)\, ds = L_\gamma.$$

The phase-based condition

$$\oint_\gamma d\phi = m\, \Phi_c$$

arises as a representation of this functional under the identification of phase with normalized transport action. Accordingly, the Schrödinger-type equation derived in this work ultimately encodes the same closure compatibility condition expressed in transport variables.

9.2 Phase Reformulation in Q10

In Q10, the closure condition was reformulated in terms of phase accumulation along admissible transport paths:

$$\oint_\gamma d\phi = m\,\Phi_c.$$

This provided a scalar measure of closure consistency and introduced phase as a transport-derived quantity. The coherence scale Φ_c was identified as the fundamental quantity governing compatibility of transport paths.

Importantly, this formulation did not assume periodicity of 2π, but allowed for system-dependent coherence scales.

9.3 Transport Law from Q11

In Q11, a complete transport law was derived governing the evolution of closure density $\rho(x,t)$ and phase $\phi(x,t)$ under exchange interaction and scalar geometry.

This law combined continuity of closure transport with phase evolution and phase-guided transport, yielding a closed system with emergent second-order structure.

The resulting framework provided a deterministic and geometric description of transport in the exchange sector.

9.4 Representation in Q12

In the present work, the variables $\rho(x,t)$ and $\phi(x,t)$ have been combined into a unified representation

$$\Psi = \sqrt{\rho}\,e^{i\phi/\Phi_0}.$$

This representation does not introduce new physical content, but provides an alternative encoding of the transport system derived in Q11.

The evolution equation for Ψ was obtained directly from the transport law, preserving all structure established in earlier papers.

9.5 Consistency of Closure Conditions

The closure condition derived in Q7 and reformulated in Q10 remains unchanged in the present representation. Specifically,

$$\oint_\gamma d\phi = m\,\Phi_c$$

continues to govern admissible transport cycles.

In the Ψ representation, this condition corresponds to a compatibility requirement on the phase of Ψ around closed paths. The appearance of periodicity in the complex representation reflects the normalization scale Φ_0, not a change in the underlying closure structure.

9.6 Relation to Schrödinger Form

The Schrödinger equation obtained in Section 7 is therefore a representation of the transport law derived in Q11, which itself is consistent with the closure and coherence structure established in Q7 and Q10.

All discrete structure, coherence conditions, and transport relations are preserved under this representation. No new assumptions are introduced, and no prior results are modified.

9.7 Continuity of the Q-Series Development

The sequence Q7–Q12 may therefore be understood as a continuous development:

- Q7 establishes closure structure and discrete admissibility;
- Q10 introduces phase as a scalar measure of transport consistency;
- Q11 derives a unified transport law governing closure density and phase;
- Q12 provides a representation of this transport law in which a Schrödinger-type equation emerges.

Each step builds on the previous ones without introducing external assumptions, forming a coherent and internally consistent framework.

9.8 Summary

The Schrödinger equation obtained in this work is fully consistent with the closure, coherence, and transport structures established in Q7–Q11. It represents an encoding of these structures within a particular representation and does not replace the underlying transport-consistent description.

10 Summary

In this paper, we have shown that the transport law governing closure structures in scalar–conformal NUVO space admits a unified representation in which closure density and transport-derived phase are combined into a single complex-valued quantity.

Starting from the transport system derived in Q11, we constructed the representation

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}$$

and derived its evolution equation directly from the underlying transport relations. No additional dynamical assumptions were introduced in this process.

The key results of this work are as follows:

- A unified representation was introduced that encodes closure density and phase within a single object without altering their underlying interpretation;
- The transport equations for $\rho(x, t)$ and $\phi(x, t)$ were rewritten in terms of this representation, yielding a general evolution equation with both first- and second-order spatial structure;
- A normalization condition was identified under which the evolution equation admits a canonical second-order form;
- Under this normalization, a representation constant emerges that is naturally identified with $\hbar$, relating phase evolution to temporal variation in the representation;
- Under appropriate structural conditions on the transport system, the evolution equation reduces to a form identical to the Schrödinger equation.
- The representation constant $\hbar$ was shown to arise from the same normalization governing phase accumulation in Q10, ensuring continuity between closure-action structure and the emergent Schrödinger representation;

These results demonstrate that the Schrödinger equation can be derived as a representation of transport closure and exchange in scalar–conformal NUVO systems. It is not introduced as a fundamental postulate, but arises from the structure of the transport law under specific representation and normalization conditions.

Throughout the derivation, the coherence structure established in Q10 is preserved. The fundamental closure condition

$$\oint_\gamma d\phi = m\,\Phi_c$$

remains unchanged, with Φ_c governing transport compatibility. The appearance of 2π periodicity in the complex representation is a consequence of normalization by $\hbar$ and does not impose a fundamental constraint on the transport system.

The framework developed here is fully deterministic and geometric. It does not rely on wave ontology, probabilistic interpretation, or operator formalism. Instead, it provides a transport-consistent description from which familiar dynamical equations may be obtained as representations.

Taken together with Q7–Q11, this work completes the derivation of a Schrödinger-type equation from closure structure, phase accumulation, and transport consistency within the NUVO framework. The resulting structure provides a coherent and internally consistent foundation for further developments.

Future work [9] will extend this framework to more general transport configurations, explore alternative representations, and examine the connection to relativistic and multi-component systems.

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