Q13 – Closure-Density Curvature and the Madelung Form of Transport Closure in Scalar–Conformal NUVO Systems

Preprint, Version 1.0[*]

Rickey W. Austin
St Claire Scientific Research, Development, and Publishing

Abstract

The preceding Q-series developed a deterministic exchange-sector transport law for closure density and transport-derived phase, and showed that these variables admit a complex representation of Schrödinger type. However, full equivalence with the Madelung form of nonrelativistic quantum mechanics requires an additional amplitude-curvature contribution to the phase evolution equation. The purpose of the present manuscript is to identify this missing contribution within the NUVO framework.

We introduce a closure-density curvature functional measuring the structural cost of spatially inhomogeneous closure density. Under locality, positivity, rotational invariance, normalization compatibility, and lowest-order derivative assumptions, the minimal admissible curvature cost is proportional to

$$\mathcal{C}[\rho] = \int |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x = \frac{1}{4}\int \frac{|\nabla\rho|^2}{\rho} \, \mathrm{d}^3x.$$

When this cost is included in the transport action for closure density and phase, variation yields the continuity equation together with the Hamilton–Jacobi–Madelung phase equation containing the density-curvature term

$$Q_\rho = -\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

The resulting pair of real transport equations is then shown to be equivalent, under the representation

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0},$$

to the Schrödinger equation with scalar-geometric potential U_λ. In this interpretation, the quantum potential is not introduced as an external quantum postulate; it is the representation-level expression of closure-density curvature required for coherent transport across spatially inhomogeneous closure distributions.

1 Introduction

The preceding papers of the Q-series developed the exchange-sector transport structure of scalar–conformal NUVO systems [1, 2]. In that development, admissible bound configurations arise from closure conditions on exchange cycles rather than from externally imposed quantization postulates.

---

[*] Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

The early Q-series established holonomic return structure, closure-compatible excitation states, and the hydrogenic spectral ladder. The later Q-series then introduced transport-derived phase, closure density, and a deterministic transport law governing the evolution of these quantities.

In Q11 [3], the exchange-sector transport law was expressed in terms of a closure density $\rho(x,t)$, a transport-derived phase $\phi(x,t)$, and a velocity field determined by the admissible transport structure. In schematic form, the transport system contains a continuity relation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0, \tag{1}$$

together with a phase-evolution relation driven by exchange interaction and scalar geometry. This formulation is deterministic and geometric: ρ is not introduced as a probability density, ϕ is not introduced as the phase of a primitive wavefunction, and no operator formalism is assumed at the foundational level.

In Q12 [4], the pair (ρ, ϕ) was combined into the complex representation

$$\Psi = \sqrt{\rho}\, \exp\left(\frac{i\phi}{\Phi_0}\right), \tag{2}$$

where Φ_0 denotes the representation scale. This construction showed that the transport system admits a compact complex encoding and that, under appropriate normalization and structural conditions, the resulting representation takes a Schrödinger-type form. The purpose of that result was not to introduce a wave ontology, but to show that a familiar complex evolution equation can arise as a representation of underlying closure transport.

However, the passage from a Schrödinger-type representation to full nonrelativistic quantum correspondence requires a further step. In the standard Madelung decomposition of the Schrödinger equation [5, 6], writing

$$\Psi = \sqrt{\rho}\, e^{iS/\hbar} \tag{3}$$

yields two coupled real equations: a continuity equation and a Hamilton–Jacobi-type phase equation modified by the amplitude-curvature term

$$Q_\rho = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}. \tag{4}$$

This term is often called the quantum potential in the Madelung–Bohm formulation [5, 7]. Its presence is not optional: it is the term that makes the real phase equation equivalent to the full Schrödinger equation rather than to a purely classical Hamilton–Jacobi transport system.

The existing Q11–Q12 bridge establishes the closure-density and phase representation needed for such a structure, but it does not yet isolate the intrinsic origin of the amplitude-curvature contribution. If the phase evolution is driven only by an external scalar or exchange potential depending on the background scalar field $\lambda(x)$, then the resulting dynamics remain incomplete from the standpoint of full Madelung equivalence. The missing ingredient is a structural term that depends not on the external scalar geometry alone, but on the spatial inhomogeneity of the closure density itself.

The purpose of the present paper is to supply that missing ingredient. We show that an inhomogeneous closure-density distribution carries a minimal local deformation cost. This cost is determined by the spatial curvature of the amplitude $\sqrt{\rho}$ and is represented, at lowest derivative order, by the functional

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{2m} \int_\Omega |\nabla \sqrt{\rho}|^2 \, d^3x. \tag{5}$$

When the representation scale is identified with the empirical exchange action scale, $\Phi_0 = \hbar$, this becomes

$$\mathcal{C}[\rho] = \frac{\hbar^2}{2m} \int_\Omega |\nabla\sqrt{\rho}|^2 \, d^3x. \tag{6}$$

The central claim of this paper is that this term should not be understood as an imported quantum postulate. Within the NUVO exchange sector, it is interpreted as the lowest-order structural cost required to maintain coherent transport across a spatially nonuniform closure density. Uniform closure density carries no such cost. Spatially varying closure density, by contrast, requires additional structural compatibility between neighboring closure elements, and this compatibility is measured by the curvature of the represented amplitude.

We therefore introduce a closure-density transport action of the form

$$\mathcal{A}[\rho, \phi] = \int dt \, d^3x \left[\rho \left(\partial_t \phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x) \right) + \frac{\Phi_0^2}{2m} |\nabla\sqrt{\rho}|^2 \right], \tag{7}$$

where $U_\lambda(x)$ denotes the scalar/exchange potential associated with the ambient NUVO geometry. Variation with respect to ϕ yields the continuity equation, while variation with respect to ρ yields the modified phase equation

$$\partial_t \phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x) - \frac{\Phi_0^2}{2m} \frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{8}$$

Thus the amplitude-curvature term appears as a consequence of the closure-density deformation functional.

Together with the continuity equation

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla\phi}{m} \right) = 0, \tag{9}$$

this phase equation is precisely the Madelung system associated with the Schrödinger equation

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m} \nabla^2 \Psi + U_\lambda(x)\Psi. \tag{10}$$

Accordingly, the present paper upgrades the Q12 result from a Schrödinger-type representational correspondence to a full Madelung-equivalent transport representation, subject to the structural assumptions made explicit below.

The scope of the result is deliberately limited. We do not claim here to derive all of quantum mechanics, nor do we introduce measurement, probability, spin, relativistic dynamics, or many-body entanglement. Those topics belong to later representational and correspondence developments. The aim of the present paper is narrower and more foundational: to identify the missing closure-density curvature term required for exact equivalence between NUVO closure transport and the Madelung form of nonrelativistic Schrödinger dynamics.

The paper proceeds as follows. Section 2 reviews the transport-density and phase structure inherited from Q11 and Q12. Section 3 motivates the need for an intrinsic closure-density curvature cost. Section 4 derives the minimal local functional associated with closure-density inhomogeneity. Section 5 introduces the closure-density transport action. Sections 6 and 7 derive the continuity and modified phase equations by variation. Section 8 states the resulting Madelung transport theorem. Section 9 proves equivalence with the complex Schrödinger representation. Sections 10 and 11 interpret the amplitude-curvature term within the NUVO framework and clarify the relationship between the present result and Q12. The final sections record correspondence checks, limitations, and directions for subsequent development.

2 Review of Inputs from Q11 and Q12

The present paper builds on two structural inputs established in the preceding exchange-sector development. First, Q11 formulated a local transport law for closure density and transport-derived phase. Second, Q12 showed that this pair admits a lossless complex representation which, under suitable reduction conditions, takes a Schrödinger-type form. We recall only the elements needed for the present derivation.

2.1 Closure Density and Transport-Derived Phase

Let

$$\rho(x,t) \geq 0 \tag{11}$$

denote the local closure density of an admissible exchange-sector configuration. This quantity measures the local density of closure-compatible exchange structure. It is not introduced as a probability density at the foundational level, although probabilistic interpretations may arise later at the representational or measurement level.

Let

$$\phi(x,t) \tag{12}$$

denote the transport-derived phase associated with cumulative exchange transport. In the Q-series [8], this phase is not postulated as the phase of a primitive wavefunction. It is a geometric bookkeeping quantity measuring accumulated transport compatibility along admissible exchange paths.

In the local integrable regime, the pair

$$(\rho, \phi) \tag{13}$$

provides the minimal real state description of closure transport. The density ρ records how closure-compatible exchange structure is distributed, while the phase ϕ records the local transport orientation and accumulated coherence state. The present paper assumes this local integrable regime throughout, so that ρ and ϕ may be treated as smooth fields on a spatial domain $\Omega \subset \mathbb{R}^3$ over a time interval of interest.

2.2 Continuity Structure

The local conservation of closure transport gives the continuity equation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0, \tag{14}$$

where $v(x,t)$ is the admissible transport velocity field. This equation expresses conservation of closure density under exchange-sector transport. It does not require a probabilistic interpretation of ρ; it is a transport-conservation law for closure-compatible structure.

For the nonrelativistic Madelung correspondence, the admissible velocity field is specialized to the phase-gradient form

$$v = \frac{1}{m} \nabla \phi. \tag{15}$$

This relation is the local nonrelativistic reduction of the exchange-sector phase-gradient law: phase gradients determine the local direction and rate of admissible closure transport, and the parameter m supplies the inertial scale of the represented closure structure.

Substituting (15) into (14) gives

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0. \tag{16}$$

This is the continuity equation that appears in the Madelung decomposition of the nonrelativistic Schrödinger equation. In the present framework, however, it is interpreted as closure-density transport rather than as probability-current conservation at the foundational level.

2.3 Phase Evolution Structure

The transport-derived phase evolves under exchange interaction and scalar geometry. Prior to the refinement introduced in the present paper, the phase evolution may be represented schematically as

$$\partial_t \phi + v \cdot \nabla \phi = \kappa E(x), \tag{17}$$

where $E(x)$ denotes the exchange-sector driving contribution and κ is the corresponding conversion factor between exchange transport and phase accumulation.

For comparison with nonrelativistic Hamilton–Jacobi structure, the same phase evolution may be written in reduced form as

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda(x) = 0, \tag{18}$$

where $U_\lambda(x)$ denotes the scalar-geometric or exchange-sector potential contribution associated with the ambient NUVO geometry. The notation U_λ is used to emphasize that this term is determined by the scalar/exchange background and not by the internal shape of the closure-density distribution itself.

Equation (18) has the form of a classical Hamilton–Jacobi phase equation. By itself it is not yet the full Madelung phase equation. The missing contribution is an internal closure-density curvature term depending on the spatial inhomogeneity of ρ. The purpose of the present paper is to derive this term and to show that the refined phase equation becomes

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda(x) - \frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{19}$$

The final term in (19) is the closure-density curvature contribution. In conventional Madelung language it corresponds to the quantum potential. In the present framework it is interpreted instead as the representation-level image of the structural cost required to maintain coherent transport across a spatially inhomogeneous closure-density distribution.

2.4 Complex Representation

Q12 introduced the complex representation

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}, \tag{20}$$

where Φ_0 denotes the representation action scale. In the empirically calibrated nonrelativistic correspondence this scale is identified with the usual Planck-reduced action scale $\hbar$, but the notation Φ_0 is retained to emphasize its role as the NUVO representation scale.

The representation (20) is lossless wherever $\rho > 0$. Indeed,

$$\rho = |\Psi|^2, \qquad \phi = \Phi_0 \arg(\Psi), \tag{21}$$

up to the usual branch structure of phase. Thus Ψ does not add new ontology to the transport system. It repackages the two real fields (ρ, ϕ) into a single complex object.

This distinction is important for the present paper. The complex state Ψ is not assumed as fundamental, and the Schrödinger equation is not taken as a primitive dynamical law. Instead, the goal is to show that once the closure-density curvature contribution is included, the real transport system for (ρ, ϕ) is exactly equivalent to the standard complex Schrödinger representation

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m}\nabla^2\Psi + U_\lambda(x)\Psi. \tag{22}$$

The remaining sections derive the missing curvature term from a closure-density deformation functional and then prove the equivalence between the resulting real transport system and (22).

3 Closure-Density Curvature

The preceding section reviewed the closure-density and phase variables inherited from Q11 and Q12. Those variables are sufficient to describe local transport in the integrable regime, but the unrefined phase equation remains incomplete if it contains only a scalar-geometric potential contribution. Full Madelung equivalence requires an additional term depending on the spatial structure of the closure density itself.

The purpose of the present section is to identify the corresponding density-dependent contribution. We do not introduce it as a quantum postulate. Instead, we characterize it as the minimal local cost associated with maintaining coherent exchange transport across a spatially inhomogeneous closure-density distribution.

3.1 Motivation

A spatially uniform closure density carries no internal spatial deformation. If

$$\rho(x,t) = \rho_0(t) \tag{23}$$

on a spatial region, then neighboring points in that region carry the same closure-density weight. In such a region, coherent transport requires compatibility of phase accumulation, but there is no additional density-gradient structure to reconcile.

An inhomogeneous closure density is different. If

$$\nabla\rho \neq 0, \tag{24}$$

then the local amount of closure-compatible exchange structure varies from point to point. Maintaining coherent transport across such a distribution requires compatibility not only of phase gradients, but also of density gradients. Adjacent regions of different closure density must remain mutually compatible under transport, otherwise the represented exchange structure would fail to define a coherent local state.

Thus, in addition to the scalar-geometric or exchange-sector potential $U_\lambda(x)$, there must be an internal contribution measuring the structural cost of spatial variation in ρ. This cost should vanish when ρ is uniform, should be nonnegative, and should depend locally on the spatial variation of closure density in the lowest-order nonrelativistic theory.

It is useful to emphasize the limited role of this term. It is not a new force field, not a separate scalar source, and not an externally imposed quantum potential. It is a deformation cost internal to the closure-density representation. Its role is to account for the extra compatibility requirement introduced when closure support is spatially curved.

3.2 Admissibility Requirements

We now record the structural requirements imposed on such a cost functional. Let $\mathcal{C}[\rho]$ denote a scalar functional measuring the internal deformation cost of a closure-density distribution on a spatial domain $\Omega \subset \mathbb{R}^3$.

Principle 3.1 (Admissible closure-density curvature cost)**.** *A local closure-density curvature cost functional $\mathcal{C}[\rho]$ in the nonrelativistic integrable regime should satisfy the following conditions:*

(i) Locality*: the cost density depends locally on ρ and finitely many of its spatial derivatives;*

(ii) Rotational invariance*: the cost is invariant under spatial rotations of the coordinate frame;*

(iii) Nonnegativity*: the cost satisfies $\mathcal{C}[\rho] \geq 0$ for admissible closure-density configurations;*

(iv) Uniform-density normalization*: the cost vanishes for spatially uniform closure density;*

(v) Compatibility with total closure content*: the cost is compatible with fixing the total closure content*

$$\int_\Omega \rho \, \mathrm{d}^3x;$$

(vi) Lowest-order derivative dependence*: in the leading nonrelativistic approximation, the cost contains the lowest nontrivial spatial derivative contribution;*

(vii) Compatibility with the complex representation*: the cost is compatible with the representation*

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}.$$

These requirements do not uniquely determine all possible higher-order corrections. They determine the leading local curvature cost in the same sense that a lowest-order field theory is determined by symmetry, positivity, and derivative order. Additional terms involving higher derivatives, nonlinear density powers, or couplings to scalar geometry may be considered in later refinements, but they are not part of the minimal Madelung correspondence.

3.3 Minimal Curvature Functional

Because the cost must vanish for uniform density, the leading term must depend on spatial derivatives of ρ. Rotational invariance requires that the first-derivative contribution enter through the scalar combination

$$|\nabla f(\rho)|^2 \tag{25}$$

for some local function f. Compatibility with the complex representation singles out the amplitude

$$R := \sqrt{\rho}. \tag{26}$$

Indeed, in the representation

$$\Psi = Re^{i\phi/\Phi_0}, \tag{27}$$

the modulus R is the object whose spatial curvature appears in the Laplacian of the complex state. Therefore the leading local density-curvature cost should be quadratic in ∇R.

The minimal admissible functional is therefore proportional to

$$\int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x. \tag{28}$$

Equivalently, wherever $\rho > 0$,

$$|\nabla\sqrt{\rho}|^2 = \frac{1}{4}\frac{|\nabla\rho|^2}{\rho}, \tag{29}$$

and hence

$$\int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x = \frac{1}{4}\int_\Omega \frac{|\nabla\rho|^2}{\rho} \, \mathrm{d}^3x. \tag{30}$$

This expression is the Fisher-information-type density-gradient functional, but no probabilistic interpretation is being imposed here. In the present setting it is interpreted as the leading local measure of closure-density deformation.

The coefficient is fixed by dimensional correspondence with the transport mass m and the representation action scale Φ_0. The combination

$$\frac{\Phi_0^2}{2m} \tag{31}$$

has the required dimensions to convert the amplitude-gradient measure into an energy-density contribution after integration over space. This is also the coefficient required for equivalence with the standard nonrelativistic Schrödinger representation.

Definition 3.1 (Closure-density curvature functional)**.** *The closure-density curvature functional is*

$$\mathcal{C}[\rho] := \frac{\Phi_0^2}{2m}\int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x. \tag{32}$$

Equivalently, on regions where $\rho > 0$,

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{8m}\int_\Omega \frac{|\nabla\rho|^2}{\rho} \, \mathrm{d}^3x. \tag{33}$$

3.4 Basic Properties

The functional $\mathcal{C}[\rho]$ has the required structural properties. It is local, since its density depends on ρ and its first spatial derivatives. It is rotationally invariant, since it depends on the Euclidean scalar $|\nabla\sqrt{\rho}|^2$. It is nonnegative, because

$$|\nabla\sqrt{\rho}|^2 \geq 0 \tag{34}$$

pointwise. It vanishes for spatially uniform closure density, since

$$\nabla\rho = 0 \quad \Longrightarrow \quad \nabla\sqrt{\rho} = 0. \tag{35}$$

The functional is also compatible with fixed total closure content. If the admissible state space is restricted by

$$\int_\Omega \rho \, \mathrm{d}^3x = N_C, \tag{36}$$

where N_C denotes the total closure content, then $\mathcal{C}[\rho]$ measures only the spatial deformation of the density profile and not the total amount of closure content itself. Uniform redistributions carry zero curvature cost, while spatially structured distributions carry positive cost according to their amplitude gradients.

Finally, the use of $\sqrt{\rho}$, rather than ρ itself, is not arbitrary. It is required by compatibility with the complex representation. Since $\sqrt{\rho}$ is the amplitude of Ψ, the functional $\mathcal{C}[\rho]$ is precisely the amplitude-gradient contribution that appears when the Schrödinger kinetic term is written in polar variables. The next sections show that its variational derivative produces the closure-density curvature potential required for full Madelung form.

3.5 Interpretation

The functional $\mathcal{C}[\rho]$ measures the structural cost of maintaining coherent closure transport across a spatially curved density profile. It is not a probability postulate and does not presuppose wave ontology. It is a local deformation cost for closure density.

In NUVO language, a nonuniform closure-density profile represents a spatially varying distribution of exchange-sector closure support. The phase field ϕ governs the accumulated transport orientation of this support, while the amplitude $\sqrt{\rho}$ records how strongly closure support is represented at each point. When the amplitude varies spatially, neighboring regions of the exchange-sector representation must remain compatible under coherent transport. The curvature functional $\mathcal{C}[\rho]$ measures the leading cost of maintaining that compatibility.

This interpretation distinguishes the present construction from both a classical Hamilton–Jacobi theory and a postulated quantum theory. In a purely classical Hamilton–Jacobi system, the phase evolves under an external potential and no intrinsic amplitude-curvature contribution is present. In the present framework, by contrast, the closure-density distribution itself contributes to the phase evolution through its spatial curvature. At the same time, this contribution is not inserted as an external quantum rule. It follows from the admissible local curvature cost associated with inhomogeneous closure density.

Thus the conventional quantum potential is reinterpreted here as the Madelung representation of closure-density curvature. In later sections we will show explicitly that variation of $\mathcal{C}[\rho]$ contributes the term

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{37}$$

to the phase equation, thereby completing the real transport system needed for equivalence with the nonrelativistic Schrödinger equation.

4 Transport Action for Closure Density and Phase

The previous section identified the leading local curvature cost associated with spatially inhomogeneous closure density. We now combine that cost with the phase-gradient transport structure inherited from Q11 and Q12. The result is a variational principle for the pair (ρ, ϕ). Its Euler–Lagrange equations will yield the continuity equation and the modified phase equation containing the closure-density curvature term.

4.1 Definition of the Action

Let $\Omega \subset \mathbb{R}^3$ be a spatial domain and let $I \subset \mathbb{R}$ be a time interval. We assume that $\rho(x,t) > 0$ and $\phi(x,t)$ are sufficiently smooth on $\Omega \times I$, with boundary behavior specified below.

Define the closure-density transport action by

$$\mathcal{A}[\rho,\phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\rho\left(\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x)\right) + \frac{\Phi_0^2}{2m}|\nabla\sqrt{\rho}|^2\right]. \tag{38}$$

The first term records the coupling between closure density and phase evolution. The second term,

$$\rho\frac{|\nabla\phi|^2}{2m}, \tag{39}$$

is the nonrelativistic phase-gradient transport contribution. It is the local kinetic transport term obtained when the admissible velocity field is reduced to

$$v = \frac{\nabla\phi}{m}. \tag{40}$$

The third term,

$$\rho U_\lambda(x), \tag{41}$$

records the scalar-geometric or exchange-sector potential contribution. The final term,

$$\frac{\Phi_0^2}{2m}|\nabla\sqrt{\rho}|^2, \tag{42}$$

is the closure-density curvature cost introduced in the previous section.

The overall sign convention in (38) is not essential. One may multiply the entire action by -1 without changing the variational content, provided the sign convention is handled consistently. The convention chosen here is arranged so that variation with respect to ρ yields the Hamilton–Jacobi–Madelung phase equation in the form

$$\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x) - \frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{43}$$

It is sometimes useful to rewrite the curvature term directly in ρ. On regions where $\rho > 0$,

$$|\nabla\sqrt{\rho}|^2 = \frac{1}{4}\frac{|\nabla\rho|^2}{\rho}, \tag{44}$$

so that the action may equivalently be written as

$$\mathcal{A}[\rho,\phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\rho\left(\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x)\right) + \frac{\Phi_0^2}{8m}\frac{|\nabla\rho|^2}{\rho}\right]. \tag{45}$$

The amplitude form (38) will be used in the main derivation because it makes the origin of the curvature term more transparent.

4.2 Role of the Scalar-Geometric Potential

The term $U_\lambda(x)$ represents the scalar-geometric or exchange-sector contribution inherited from the NUVO scalar modulation. In applications to bound systems, U_λ encodes the effective exchange or scalar-geometric potential experienced by the closure structure. Its precise form depends on the sectoral reduction under consideration.

For the purposes of the present paper, U_λ is treated as a given external potential on the spatial domain. This does not mean that it is external to the NUVO framework. Rather, it means that U_λ is external to the local variation of ρ and ϕ performed in this paper. It is inherited from the ambient scalar/exchange configuration and is not produced by varying the closure-density curvature functional.

This distinction is essential. The scalar-geometric potential and the closure-density curvature contribution have different origins:

$$\text{external/scalar geometry: } U_\lambda(x), \qquad \text{internal density curvature: } Q_\rho[\rho]. \tag{46}$$

The first depends on the scalar or exchange background in which the closure structure is embedded. The second depends on the spatial shape of the closure density itself.

With the notation used here, the internal density-curvature contribution is

$$Q_\rho[\rho] := -\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{47}$$

Thus the refined phase equation takes the form

$$\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda(x) + Q_\rho[\rho] = 0. \tag{48}$$

The point of the present construction is precisely that $Q_\rho[\rho]$ is not absorbed into $U_\lambda(x)$. If the only potential contribution were $U_\lambda(x)$, the phase equation would have the structure of a classical Hamilton–Jacobi equation. Full Madelung equivalence requires the additional internal term $Q_\rho[\rho]$, which arises from the curvature of the closure-density amplitude.

4.3 Boundary Conditions

The variational calculation assumes boundary behavior sufficient to remove surface terms generated by integration by parts. We impose one of the following standard conditions:

(i) ρ and ϕ have compact spatial support in Ω;

(ii) ρ, ϕ, and their relevant derivatives decay sufficiently rapidly at spatial infinity;

(iii) the variations vanish on the spatial boundary,

$$\delta\rho\big|_{\partial\Omega} = 0, \qquad \delta\phi\big|_{\partial\Omega} = 0; \tag{49}$$

(iv) periodic boundary conditions are imposed, so that boundary contributions cancel pairwise.

For the time boundary, the variations are taken to vanish at the endpoints of the interval $I = [t_1, t_2]$:

$$\delta\rho(x,t_1) = \delta\rho(x,t_2) = 0, \qquad \delta\phi(x,t_1) = \delta\phi(x,t_2) = 0. \tag{50}$$

Only the vanishing of $\delta\phi$ at the time endpoints is needed for the phase variation, but it is convenient to impose endpoint conditions on both variables.

There is one further technical restriction. The expression

$$\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{51}$$

is singular at nodes where $\rho = 0$. The first theorem of the present paper is therefore stated on nodal-free regions where

$$\rho(x,t) > 0. \tag{52}$$

This restriction is standard in polar decompositions of complex fields. The treatment of nodes, branch changes of phase, and distributional contributions at zero-density sets is a separate technical problem and is deferred to later work. For the purpose of establishing the local Madelung correspondence, it is sufficient to work on connected regions where the closure density is strictly positive and the phase can be chosen smoothly.

5 Variation with Respect to Phase

We now derive the first Euler–Lagrange equation associated with the transport action. The phase field ϕ enters the action only through its time derivative and spatial gradient. Therefore variation with respect to ϕ yields a conservation law. In the present framework, this conservation law is the continuity equation for closure-density transport.

Lemma 5.1 (Phase variation yields continuity)**.** *Let $\rho > 0$ and ϕ be smooth on $\Omega \times I$, and assume boundary conditions such that all surface terms vanish. Variation of the action*

$$\mathcal{A}[\rho, \phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\rho \left(\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda(x) \right) + \frac{\Phi_0^2}{2m} |\nabla \sqrt{\rho}|^2 \right]$$

with respect to ϕ yields

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0. \tag{53}$$

Proof. Let $\phi_\epsilon = \phi + \epsilon\eta$, where η is a smooth test variation satisfying the boundary conditions specified in the previous section. The closure density ρ is held fixed during this variation. The terms in the action depending on ϕ are

$$\rho \partial_t \phi + \rho \frac{|\nabla \phi|^2}{2m}. \tag{54}$$

Therefore the first variation with respect to ϕ is

$$\delta_\phi \mathcal{A} = \frac{d}{d\epsilon} \mathcal{A}[\rho, \phi + \epsilon\eta] \bigg|_{\epsilon=0} \tag{55}$$

$$= \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\rho \, \partial_t \eta + \rho \frac{\nabla \phi}{m} \cdot \nabla \eta \right]. \tag{56}$$

We integrate the first term by parts in time:

$$\int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \, \rho \, \partial_t \eta = \left[\int_\Omega \rho\eta \, \mathrm{d}^3x \right]_{t_1}^{t_2} - \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \, (\partial_t \rho)\eta. \tag{57}$$

The endpoint term vanishes because the variation η is assumed to vanish at t_1 and t_2.

Similarly, integrating the spatial-gradient term by parts gives

$$\int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \, \rho \frac{\nabla \phi}{m} \cdot \nabla \eta = \int_I \mathrm{d}t \int_{\partial\Omega} \rho \frac{\nabla \phi}{m} \cdot n \, \eta \, \mathrm{d}S - \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \, \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) \eta. \tag{58}$$

The boundary term vanishes under the assumed spatial boundary conditions.

Combining the two integrations by parts, we obtain

$$\delta_\phi \mathcal{A} = -\int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) \right] \eta. \tag{59}$$

Since the variation $\eta = \delta\phi$ is arbitrary in the interior of $\Omega \times I$, stationarity of the action implies

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0. \tag{60}$$

This proves the lemma. □

The resulting equation has exactly the continuity form required for Madelung correspondence. In NUVO terms, however, it is interpreted as the conservation of closure density under admissible exchange-sector transport. The associated transport current is

$$J_\rho := \rho \frac{\nabla \phi}{m}. \tag{61}$$

Thus the continuity law may also be written as

$$\partial_t \rho + \nabla \cdot J_\rho = 0. \tag{62}$$

The current J_ρ is the local flux of closure-compatible exchange structure in the nonrelativistic integrable regime.

6 Variation with Respect to Closure Density

We next derive the second Euler–Lagrange equation associated with the transport action. This variation is the crucial step in the present paper. Variation with respect to the phase yielded closure-density continuity. Variation with respect to the closure-density amplitude yields the modified phase equation containing the density-curvature term.

6.1 Amplitude Variable

It is convenient to work with the amplitude variable

$$R := \sqrt{\rho}, \qquad \rho = R^2. \tag{63}$$

On nodal-free regions, $R > 0$, and the transformation between R and ρ is smooth and invertible.

In terms of R, the transport action becomes

$$\mathcal{A}[R, \phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3 x \left[R^2 \left(\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda \right) + \frac{\Phi_0^2}{2m} |\nabla R|^2 \right]. \tag{64}$$

Define the unrefined Hamilton–Jacobi expression

$$H_\phi := \partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda. \tag{65}$$

Then the action may be written compactly as

$$\mathcal{A}[R, \phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3 x \left[R^2 H_\phi + \frac{\Phi_0^2}{2m} |\nabla R|^2 \right]. \tag{66}$$

Lemma 6.1 (Amplitude variation yields the density-curvature phase equation)**.** *Let $R > 0$ and ϕ be smooth on $\Omega \times I$, and assume boundary conditions such that all surface terms vanish. Variation of $\mathcal{A}[R, \phi]$ with respect to R yields*

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0. \tag{67}$$

Equivalently, since $R = \sqrt{\rho}$,

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda + Q_\rho = 0, \tag{68}$$

where

$$Q_\rho := -\frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}. \tag{69}$$

Proof. Let $R_\epsilon = R + \epsilon\chi$, where χ is a smooth test variation satisfying the boundary conditions specified above. The phase ϕ is held fixed during this variation. Using (66), the first variation is

$$\delta_R \mathcal{A} = \frac{d}{d\epsilon} \mathcal{A}[R + \epsilon\chi, \phi]\bigg|_{\epsilon=0} \tag{70}$$

$$= \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[2RH_\phi \chi + \frac{\Phi_0^2}{m} \nabla R \cdot \nabla \chi\right]. \tag{71}$$

The second term is integrated by parts in space:

$$\int_\Omega \nabla R \cdot \nabla \chi \, \mathrm{d}^3x = \int_{\partial\Omega} \chi \, \nabla R \cdot n \, \mathrm{d}S - \int_\Omega (\nabla^2 R)\chi \, \mathrm{d}^3x. \tag{72}$$

The boundary term vanishes under the assumed spatial boundary conditions. Therefore

$$\delta_R \mathcal{A} = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[2RH_\phi - \frac{\Phi_0^2}{m} \nabla^2 R\right] \chi. \tag{73}$$

Since $\chi = \delta R$ is arbitrary in the interior of $\Omega \times I$, stationarity of the action implies

$$2RH_\phi - \frac{\Phi_0^2}{m} \nabla^2 R = 0. \tag{74}$$

On nodal-free regions where $R > 0$, division by $2R$ gives

$$H_\phi - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0. \tag{75}$$

Substituting the definition of H_ϕ from (65) yields

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0. \tag{76}$$

Finally, using $R = \sqrt{\rho}$ gives

$$Q_\rho = -\frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}}, \tag{77}$$

and hence the equivalent form

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda + Q_\rho = 0. \tag{78}$$

This proves the lemma. □

The density-curvature term has therefore appeared as the variational derivative of the closure-density curvature functional. It has not been introduced as an independent force, an external potential, or a probabilistic postulate. It is the phase-level contribution required by stationarity of the closure-density transport action when spatial inhomogeneity of the closure density is included.

In terms of the closure-density amplitude, the internal contribution is

$$Q_\rho = -\frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R}. \tag{79}$$

This is formally identical to the quantum potential of the standard Madelung decomposition. Within the present framework, however, it is interpreted as the closure-density curvature potential: the representation-level expression of the cost required to maintain coherent exchange transport across a nonuniform closure-density profile.

7 The Madelung Transport System

We now combine the two variational results. Variation with respect to ϕ produced the closure-density continuity equation, while variation with respect to the amplitude $R = \sqrt{\rho}$ produced the phase equation corrected by the density-curvature term. Together, these two equations form the Madelung transport system associated with closure-density dynamics in the nonrelativistic integrable regime.

Theorem 7.1 (NUVO Madelung transport system)**.** *Let* $\Omega \subset \mathbb{R}^3$ *be a spatial domain and let* $I \subset \mathbb{R}$ *be a time interval. Suppose*

$$\rho(x,t) > 0, \qquad \phi(x,t)$$

are smooth on $\Omega \times I$*, and assume boundary conditions such that all surface terms vanish in the variational calculation. In the local integrable nonrelativistic closure-transport regime, stationarity of the action*

$$\mathcal{A}[\rho,\phi] = \int_I \mathrm{d}t \int_\Omega \mathrm{d}^3x \left[\rho\left(\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda\right) + \frac{\Phi_0^2}{2m}|\nabla\sqrt{\rho}|^2\right]$$

with respect to independent variations of ρ *and* ϕ *yields the coupled system*

$$\partial_t\rho + \nabla\cdot\left(\rho\frac{\nabla\phi}{m}\right) = 0, \tag{80}$$

and

$$\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{81}$$

Proof. The first equation follows from variation of the action with respect to ϕ, as shown in the phase-variation lemma. The second equation follows from variation with respect to the amplitude $R = \sqrt{\rho}$, as shown in the amplitude-variation lemma. Since the variations are independent and arbitrary in the interior of $\Omega \times I$, stationarity requires both Euler–Lagrange equations to hold simultaneously. Substituting $R = \sqrt{\rho}$ into the amplitude equation gives exactly (81). ☐

7.1 Closure-Density Current

The first equation may be written in conservative form

$$\partial_t\rho + \nabla\cdot J_\rho = 0, \tag{82}$$

where the closure-density current is

$$J_\rho := \rho\frac{\nabla\phi}{m}. \tag{83}$$

This current is the nonrelativistic transport flux of closure-compatible exchange structure. In standard quantum notation, the analogous expression is interpreted as probability current. In the present framework, the more primitive interpretation is transport of closure density. Any probabilistic interpretation is therefore secondary to the closure-transport structure.

7.2 Closure-Density Curvature Potential

The second equation can be written more compactly by defining

$$Q_\rho[\rho] := -\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{84}$$

Then (81) becomes

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda + Q_\rho[\rho] = 0. \tag{85}$$

This is a Hamilton–Jacobi transport equation corrected by the closure-density curvature potential. The correction is not an external force and is not part of the ambient scalar-geometric potential U_λ. It is the phase-level expression of the structural cost of maintaining coherent transport across a spatially inhomogeneous closure-density profile.

The distinction between U_λ and Q_ρ is central:

$$U_\lambda \quad \text{depends on the scalar/exchange background,} \tag{86}$$

whereas

$$Q_\rho[\rho] \quad \text{depends on the shape of the closure density itself.} \tag{87}$$

Thus the refined phase equation contains both an external geometric contribution and an internal density-curvature contribution.

7.3 Interpretive Statement

The system

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0, \tag{88}$$

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda + Q_\rho[\rho] = 0 \tag{89}$$

is the NUVO Madelung transport system in the local nonrelativistic integrable regime.

Equation (88) expresses conservation of closure density under phase-gradient transport. Equation (89) expresses phase evolution under three contributions:

(i) the phase-gradient transport term

$$\frac{|\nabla \phi|^2}{2m};$$

(ii) the scalar-geometric or exchange-sector potential

$$U_\lambda;$$

(iii) the internal closure-density curvature potential

$$Q_\rho[\rho].$$

The third contribution is the term missing from the unrefined Hamilton–Jacobi transport equation. Its inclusion is precisely what distinguishes the full Madelung transport system from a classical Hamilton–Jacobi system with a transported density.

In conventional terminology, Q_ρ is the quantum potential. In the NUVO interpretation developed here, it is not treated as mysterious or as separately postulated. It is the representation-level image of the minimal local deformation cost associated with nonuniform closure-density support. When the closure density is spatially uniform, Q_ρ vanishes. When the closure density is spatially curved, Q_ρ contributes to phase evolution because coherent transport must remain compatible across the inhomogeneous density profile.

Thus the Madelung correction is reinterpreted as closure-density curvature. This closes the gap between the deterministic closure-transport law and the full real form of nonrelativistic Schrödinger dynamics.

8 Equivalence with the Schrödinger Representation

The preceding section derived the real transport system for closure density and transport-derived phase. We now show that this system is equivalent to a complex Schrödinger representation. This establishes the precise sense in which the Schrödinger equation arises in the present framework: it is the compact complex encoding of the closure-density continuity equation together with the density-curvature-corrected phase equation.

8.1 Complex Encoding

Let

$$R := \sqrt{\rho}, \qquad \Psi := Re^{i\phi/\Phi_0}. \tag{90}$$

On any connected nodal-free region where $R > 0$, this encoding is locally invertible up to the usual branch structure of phase:

$$\rho = |\Psi|^2, \qquad \phi = \Phi_0 \arg(\Psi). \tag{91}$$

Thus Ψ contains no additional local information beyond the pair (ρ, ϕ). It is a representation of the real closure-transport state, not an additional primitive field.

Theorem 8.1 (Equivalence with Schrödinger form)**.** *Assume $R > 0$, and suppose R, ϕ, and U_λ are sufficiently smooth on $\Omega \times I$. Then the Madelung transport system*

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0, \tag{92}$$

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0 \tag{93}$$

is equivalent, under the encoding

$$\Psi = Re^{i\phi/\Phi_0},$$

to the complex Schrödinger-type equation

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m} \nabla^2 \Psi + U_\lambda \Psi. \tag{94}$$

Proof. Let

$$\theta := \frac{\phi}{\Phi_0}, \qquad \Psi = Re^{i\theta}. \tag{95}$$

We compute the time derivative and Laplacian of Ψ:

$$\partial_t \Psi = e^{i\theta} \left(\partial_t R + iR\, \partial_t \theta \right), \tag{96}$$

and

$$\nabla^2 \Psi = e^{i\theta} \left[\nabla^2 R - R|\nabla \theta|^2 + i \left(2\nabla R \cdot \nabla \theta + R\nabla^2 \theta \right) \right]. \tag{97}$$

Substituting (96) and (97) into (94), and dividing by the common factor $e^{i\theta}$, gives

$$\begin{aligned} i\Phi_0 \partial_t R - \Phi_0 R \partial_t \theta = &-\frac{\Phi_0^2}{2m} \left[\nabla^2 R - R|\nabla \theta|^2 \right] + U_\lambda R \\ &- i\frac{\Phi_0^2}{2m} \left[2\nabla R \cdot \nabla \theta + R\nabla^2 \theta \right]. \end{aligned} \tag{98}$$

Equating imaginary parts yields

$$\Phi_0 \partial_t R = -\frac{\Phi_0^2}{2m} \left[2\nabla R \cdot \nabla \theta + R \nabla^2 \theta \right]. \tag{99}$$

Using $\theta = \phi/\Phi_0$, this becomes

$$\partial_t R = -\frac{1}{2m} \left[2\nabla R \cdot \nabla \phi + R \nabla^2 \phi \right]. \tag{100}$$

Multiplying by $2R$, and using $\rho = R^2$, gives

$$\begin{aligned} \partial_t (R^2) &= -\frac{1}{m} \left[2R\nabla R \cdot \nabla \phi + R^2 \nabla^2 \phi \right] && (101) \\ &= -\frac{1}{m} \nabla \cdot \left(R^2 \nabla \phi \right). && (102) \end{aligned}$$

Therefore

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0, \tag{103}$$

which is (92).

Equating real parts of (98) gives

$$-\Phi_0 R \partial_t \theta = -\frac{\Phi_0^2}{2m} \left[\nabla^2 R - R|\nabla \theta|^2 \right] + U_\lambda R. \tag{104}$$

Using again $\theta = \phi/\Phi_0$, we obtain

$$-R\partial_t \phi = -\frac{\Phi_0^2}{2m} \nabla^2 R + \frac{R}{2m} |\nabla \phi|^2 + U_\lambda R. \tag{105}$$

Dividing by $R > 0$ and rearranging gives

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0, \tag{106}$$

which is (93).

Thus the complex Schrödinger equation implies the two real Madelung transport equations.

Conversely, suppose that (92) and (93) hold. Reversing the preceding calculation shows that the imaginary part and the real part of (94) are both satisfied under the encoding $\Psi = Re^{i\phi/\Phi_0}$. Therefore the two real equations recombine into the single complex equation

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m} \nabla^2 \Psi + U_\lambda \Psi. \tag{107}$$

This proves the equivalence. □

8.2 Meaning of the Equivalence

The theorem establishes an exact local equivalence between two descriptions:

(i) a real closure-transport description in terms of (ρ, ϕ), consisting of a continuity equation and a density-curvature-corrected phase equation;

(ii) a complex representation in terms of $\Psi = \sqrt{\rho} e^{i\phi/\Phi_0}$, satisfying a Schrödinger-type equation.

The equivalence is mathematical, but its interpretation within NUVO is specific. The complex equation is not taken as the primitive starting point. Rather, it is the compact representation of the real closure-density transport system derived from the variational principle.

This is the point at which the gap in the earlier Q11–Q12 bridge is closed. Without the closure-density curvature term, the real phase equation would reduce to a classical Hamilton–Jacobi form and would not be equivalent to the full Schrödinger equation. With the curvature term included, the real transport system has exactly the Madelung structure required for recombination into the complex equation.

8.3 Representation Scale

The theorem has been stated using the NUVO representation scale Φ_0. In the empirical nonrelativistic correspondence, this scale is identified with the reduced Planck action $\hbar$. Under that identification,

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + U_\lambda\Psi, \tag{108}$$

which is the standard Schrödinger equation with potential U_λ.

Within the NUVO program, the role of Φ_0 is not merely to serve as a formal constant in the complex representation. It is the action scale inherited from exchange-cycle closure [9] and calibrated in the hydrogenic sector [10]. Thus the same scale that governs closure action also controls the curvature cost of the closure-density amplitude. This identification is what connects the local Madelung bridge developed here with the earlier Q-series derivation of the action scale.

8.4 Restriction to Nodal-Free Regions

The equivalence theorem has been stated on regions where $R > 0$. This restriction is not a physical claim that nodes cannot occur. It is a technical restriction required by the polar representation and by the term

$$\frac{\nabla^2 R}{R}. \tag{109}$$

At nodes, the phase may become singular or multi-valued, and additional topological or distributional structure may be required. Such behavior is familiar from the standard Madelung decomposition and is not unique to the present framework.

For the purpose of establishing the local equivalence between closure-density transport and Schrödinger representation, it is sufficient to work on connected nodal-free regions. A complete treatment of nodes, phase defects, vortices, and global branch structure belongs to a later technical development.

9 Relation to Q12

The present paper is best understood as a refinement and completion of the bridge begun in Q12. Q12 introduced the complex representation of closure density and transport-derived phase and showed that the transport system can be expressed in a Schrödinger-type form under appropriate structural conditions. The present work identifies one of those structural conditions explicitly and derives its consequence: the closure-density curvature term required for full Madelung equivalence.

9.1 What Q12 Established

Q12 established that the real exchange-sector variables

$$(\rho, \phi) \tag{110}$$

admit the complex encoding

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}. \tag{111}$$

This representation is lossless on regions where $\rho > 0$. The closure density is recovered from the modulus,

$$\rho = |\Psi|^2, \tag{112}$$

and the transport-derived phase is recovered from the argument,

$$\phi = \Phi_0 \arg(\Psi), \tag{113}$$

up to the ordinary branch structure of phase.

The significance of Q12 was not merely notational. It showed that the transport variables of the Q-series can be combined into a single complex representational object without introducing a primitive wave ontology, a probability postulate, or an operator formalism. The complex state Ψ was therefore interpreted as an encoding of closure transport rather than as a fundamental physical substance.

Q12 also showed that the complex representation naturally produces second-order spatial structure. In particular, the Laplacian of Ψ contains contributions from both the phase and the amplitude. When

$$\Psi = Re^{i\phi/\Phi_0}, \qquad R = \sqrt{\rho}, \tag{114}$$

the expression $\nabla^2\Psi$ contains terms involving

$$\nabla^2 R, \qquad |\nabla\phi|^2, \qquad \nabla R \cdot \nabla\phi, \qquad \nabla^2\phi. \tag{115}$$

Thus the mathematical structure needed for a Schrödinger-type representation was already present in the complex encoding.

However, Q12 left open the precise origin of the amplitude-curvature term in the real phase equation. It showed that the transport system can be represented in a Schrödinger-type form under structural conditions, but it did not fully isolate the closure-density admissibility condition responsible for the term

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{116}$$

As a result, Q12 established a strong representational bridge, but not yet the complete Madelung transport derivation.

9.2 What Q13 Adds

The present paper supplies the missing ingredient by identifying closure-density curvature admissibility as a structural condition of local coherent transport.

The key observation is that an inhomogeneous closure-density profile requires more than phase-gradient compatibility. If the closure density varies across space, then coherent transport must also preserve compatibility across density gradients. This introduces a local deformation cost depending on the spatial curvature of the amplitude

$$R = \sqrt{\rho}. \tag{117}$$

Under locality, rotational invariance, nonnegativity, vanishing for uniform density, lowest-order derivative dependence, and compatibility with the complex representation, the minimal such cost is

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{2m} \int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x. \tag{118}$$

Including this term in the closure-density transport action produces, by variation, the density-curvature potential

$$Q_\rho[\rho] = -\frac{\Phi_0^2}{2m} \frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{119}$$

This is the precise term that appears in the Madelung decomposition of the Schrödinger equation. In the present framework, however, it is not introduced as an external quantum potential. It is derived as the phase-level expression of the structural cost of maintaining coherent transport across a spatially inhomogeneous closure-density profile.

Thus Q13 upgrades the Q12 bridge in two ways:

(i) it identifies the missing structural condition needed for full Madelung equivalence;

(ii) it derives the amplitude-curvature term from a closure-density curvature functional rather than inserting it as an external quantum postulate.

The result is that the real transport system no longer has the structure of a classical Hamilton–Jacobi equation with a transported density. Instead, it becomes the full Madelung system:

$$\partial_t\rho + \nabla \cdot \left(\rho \frac{\nabla\phi}{m} \right) = 0, \tag{120}$$

$$\partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{121}$$

9.3 Revised Claim

The combined Q11–Q13 result should therefore be stated in a more precise form than the preliminary Q12 language. Rather than saying only that a Schrödinger-type equation emerges, the revised claim is:

> The exchange-sector transport system of scalar–conformal NUVO theory admits a local integrable regime in which closure density and transport-derived phase satisfy the Madelung equations. Under the complex representation
>
> $$\Psi = \sqrt{\rho} e^{i\phi/\Phi_0},$$
>
> this system is equivalent to the nonrelativistic Schrödinger equation with scalar-geometric potential U_λ.

This formulation is stronger than the original Q12 claim because it specifies the real transport equations and includes the density-curvature term required for exact Schrödinger equivalence. It is also more precise because it states the assumptions under which the claim holds:

(i) the local integrable transport regime is assumed;

(ii) the closure density is positive on the region considered;

(iii) the transport velocity is given by the nonrelativistic phase-gradient reduction

$$v = \frac{\nabla \phi}{m};$$

(iv) the scalar-geometric contribution enters through a potential U_λ;

(v) the closure-density curvature functional is included as the minimal local cost of density inhomogeneity;

(vi) the representation scale Φ_0 is identified with the exchange action scale in the nonrelativistic correspondence.

These qualifications are important. They prevent the result from being overstated while preserving its significance. Q13 does not claim to derive every aspect of quantum theory. It establishes a specific and load-bearing bridge: the deterministic closure-transport variables of the Q-series can satisfy the full Madelung form, and therefore can be represented exactly by the nonrelativistic Schrödinger equation in the regime described above.

9.4 Implication for the Q-Series

With this refinement, the logical sequence of the Q-series becomes more stable. The early papers establish closure, holonomic return, action scale, dynamic-loop coupling, and hydrogenic spectral structure. Q10 and Q11 then introduce phase and closure-density transport. Q12 introduces the complex representation. Q13 supplies the missing closure-density curvature condition that converts the representation from Schrödinger-type to Madelung-equivalent.

The resulting chain may be summarized as

$$\text{closure transport} \longrightarrow (\rho, \phi) \longrightarrow \text{closure-density curvature} \longrightarrow \text{Madelung system} \longrightarrow \text{Schrödinger representat} \tag{122}$$

This sequence preserves the NUVO interpretive order. The complex Schrödinger equation is not the starting point. It is the terminal representation of a real closure-density and phase transport system once the minimal curvature cost of density inhomogeneity is included.

10 Physical and Structural Interpretation

The preceding sections established the variational origin of the density-curvature term and proved equivalence between the resulting Madelung transport system and the complex Schrödinger representation. We now clarify the physical and structural meaning of this term within the NUVO framework.

The interpretive point is important. The term

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}$$

has the same mathematical form as the quantum potential in the standard Madelung decomposition. In the present framework, however, it is not introduced through Bohmian ontology, probabilistic assumptions, or an external quantum rule. It is derived as the local curvature contribution associated with spatially inhomogeneous closure density.

10.1 Closure-Density Curvature Versus Quantum Potential

In the standard Madelung rewriting of the Schrödinger equation, the term

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} \tag{123}$$

is usually called the quantum potential. It appears when a complex wavefunction is decomposed into amplitude and phase,

$$\Psi = \sqrt{\rho}\, e^{iS/\hbar}.$$

The real part of the Schrödinger equation then becomes a Hamilton–Jacobi-type equation corrected by Q.

In the present paper the same mathematical structure appears, but its interpretation is different. The corresponding NUVO term is

$$Q_\rho[\rho] = -\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}, \tag{124}$$

and is interpreted as the *closure-density curvature potential*. It measures how the spatial curvature of the closure-density amplitude contributes to the phase evolution of an admissible exchange-sector transport state.

The terminology reflects the origin of the term. In the NUVO derivation, ρ is not introduced as a probability density. It is the local density of closure-compatible exchange structure. Its square root,

$$R = \sqrt{\rho}, \tag{125}$$

is the amplitude in the complex representation, and spatial variation of this amplitude carries a deformation cost. The variational derivative of that cost produces $Q_\rho[\rho]$.

Thus the conventional phrase "quantum potential" is mathematically accurate in the sense of Madelung correspondence, but it is not the most structurally transparent term within NUVO. The term is better understood as the potential-like expression generated by closure-density curvature.

10.2 Why This Is Not a Force

The closure-density curvature potential should not be interpreted as a primitive force field. It is not an independently existing field on space, and it is not sourced in the same manner as the scalar-geometric potential U_λ. It is a functional of the closure density itself:

$$Q_\rho = Q_\rho[\rho]. \tag{126}$$

This distinguishes it from the scalar-geometric contribution $U_\lambda(x)$. The term $U_\lambda(x)$ represents the ambient scalar/exchange background experienced by the closure structure. Its origin lies in the scalar-conformal and exchange-sector environment. By contrast, $Q_\rho[\rho]$ depends on the internal spatial shape of the represented closure-density distribution.

The distinction may be summarized as

$$U_\lambda(x) \quad \text{is an external scalar/exchange contribution,} \tag{127}$$

while

$$Q_\rho[\rho] \quad \text{is an internal density-curvature contribution.} \tag{128}$$

For this reason, the closure-density curvature potential does not represent a force acting on the closure density from outside. Rather, it is the phase-level consequence of requiring coherent transport to be maintained across an inhomogeneous density profile. If the density is spatially uniform, there is no amplitude curvature and the contribution vanishes. If the density varies spatially, the transport phase must adjust to preserve coherence across the curved density profile.

In this sense, Q_ρ is closer to an internal compatibility pressure or deformation response than to a primitive force. It records the fact that nonuniform closure support cannot be transported coherently as if each point were independent of its neighbors. Spatially adjacent regions of different closure density are linked by the requirement of coherent exchange transport, and the curvature potential is the representation-level expression of that linkage.

10.3 Why This Is Not a Probability Postulate

The curvature functional introduced above may be written as

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{8m} \int_\Omega \frac{|\nabla \rho|^2}{\rho} \, \mathrm{d}^3 x, \tag{129}$$

on regions where $\rho > 0$. This expression resembles the Fisher information functional when ρ is interpreted as a probability density. However, no such interpretation is assumed in the present derivation.

In this paper, ρ remains closure density. The functional $\mathcal{C}[\rho]$ measures the spatial deformation of closure support, not the information content of a probability distribution. The similarity to Fisher information arises because both constructions seek a positive, local, lowest-order measure of density variation. The mathematical form is shared, but the interpretation is different.

This distinction is necessary for the logical order of the NUVO program. The present paper operates before the introduction of measurement weights, event frequencies, or probabilistic interpretation. Its only inputs are closure density, transport-derived phase, the scalar-geometric potential, and the local curvature cost of inhomogeneous closure density.

Probabilistic interpretation, if introduced later, belongs to the QB/QM measurement correspondence. In that later context, one may interpret $|\Psi|^2$ as a weight or probability density under additional structural assumptions involving event projectors, coherence-gated interactions, and frequency realization. None of those assumptions are required for the derivation of the Madelung transport system itself.

Thus the derivation proceeds in the following order:

$$\text{closure density} \longrightarrow \text{curvature cost} \longrightarrow \text{Madelung transport} \longrightarrow \text{Schrödinger representation.} \tag{130}$$

Only after this chain has been established may one ask whether the resulting representation also supports a probability interpretation.

10.4 Structural Meaning of Amplitude Curvature

The use of the amplitude $R = \sqrt{\rho}$ is not merely a formal device. It reflects the fact that the complex representation stores closure density through the modulus:

$$|\Psi| = R, \qquad |\Psi|^2 = \rho. \tag{131}$$

The amplitude is therefore the field whose spatial curvature enters the second-order representation.

A nonzero value of

$$\nabla^2 R \tag{132}$$

indicates that the closure-density amplitude is spatially curved. The ratio

$$\frac{\nabla^2 R}{R} \tag{133}$$

then measures this curvature relative to the local amplitude itself. The corresponding contribution to phase evolution is

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2 R}{R}. \tag{134}$$

This form has a natural structural interpretation. Regions where the amplitude bends sharply impose a larger compatibility burden on coherent transport than regions where the amplitude is nearly flat. The curvature potential measures this burden locally. Its dependence on the ratio $\nabla^2 R/R$ expresses the fact that the relevant quantity is not absolute curvature alone, but curvature relative to the local amount of closure support.

10.5 Relation to Classical Hamilton–Jacobi Theory

If the density-curvature contribution is omitted, the phase equation reduces to

$$\partial_t \phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda = 0, \tag{135}$$

which is the classical Hamilton–Jacobi form for a phase or action function evolving under the potential U_λ. Coupled to a continuity equation, this gives a classical ensemble-like transport system.

The Madelung system differs precisely by the addition of $Q_\rho[\rho]$:

$$\partial_t \phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda + Q_\rho[\rho] = 0. \tag{136}$$

This term couples the phase evolution to the shape of the density profile. The phase is no longer determined only by the external potential and its own gradient. It also responds to the curvature of the closure-density amplitude.

This is the structural feature that makes the system equivalent to the Schrödinger equation. In NUVO terms, it says that coherent exchange-sector transport is not merely classical transport of a density along phase-gradient trajectories. It is transport constrained by the curvature of the closure-density support itself.

10.6 Summary of the Interpretation

The term conventionally called the quantum potential is reinterpreted in this paper as the closure-density curvature potential:

$$Q_\rho[\rho] = -\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}. \tag{137}$$

Its role is threefold:

(i) it measures the local phase-level effect of spatial curvature in the closure-density amplitude;

(ii) it arises from the variational derivative of the minimal closure-density deformation functional;

(iii) it supplies the missing term needed to make the real closure-transport system exactly equivalent to the nonrelativistic Schrödinger representation.

It is not a primitive force, not an external scalar source, and not a probability postulate. It is the internal structural contribution required when coherent transport is maintained across a spatially inhomogeneous closure-density distribution.

11 Hydrogenic and Free-Particle Correspondence

The preceding section established the mathematical equivalence between the NUVO Madelung transport system and the Schrödinger representation. We now record three immediate correspondence consequences: free-particle spreading, interference, and hydrogenic stationary states. These examples are not introduced as new assumptions. They follow from the Schrödinger representation once the closure-density curvature term has been included.

The purpose of this section is therefore limited. We do not rederive the full phenomenology of nonrelativistic quantum mechanics. Rather, we identify how familiar quantum behavior is interpreted in the NUVO closure-transport language.

11.1 Free-Particle Spreading

Consider the case

$$U_\lambda = 0. \tag{138}$$

The Schrödinger representation then becomes

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m}\nabla^2\Psi. \tag{139}$$

This is the free nonrelativistic Schrödinger equation with representation scale Φ_0. It supports the usual dispersive evolution of localized wave packets.

In the Madelung variables, the corresponding real system is

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla\phi}{m}\right) = 0, \tag{140}$$

$$\partial_t \phi + \frac{|\nabla\phi|^2}{2m} - \frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}} = 0. \tag{141}$$

The second equation shows that even in the absence of an external scalar-geometric potential, a nonuniform closure-density profile contributes to phase evolution through its curvature. This is the source of free-particle spreading in the real transport description.

In NUVO language, free-particle dispersion is therefore not interpreted as the spreading of a primitive wave substance. It is the evolution of a represented closure-density distribution under phase-gradient transport, corrected by the internal curvature of the closure-density amplitude. A sharply localized profile has significant amplitude curvature and therefore a nontrivial curvature contribution to phase evolution. A broad, slowly varying profile has a smaller curvature contribution and correspondingly weaker dispersive behavior.

Thus the free-particle case makes clear why the density-curvature term is essential. Without it, the system would reduce to classical Hamilton–Jacobi transport with no intrinsic mechanism for the standard Schrödinger dispersion of localized states.

11.2 Interference

Interference is also naturally represented once the complex encoding is available. Suppose two locally admissible represented states are written as

$$\Psi_1 = R_1 e^{i\phi_1/\Phi_0}, \qquad \Psi_2 = R_2 e^{i\phi_2/\Phi_0}. \tag{142}$$

Their representational superposition is

$$\Psi = \Psi_1 + \Psi_2. \tag{143}$$

The represented closure-density profile associated with Ψ is

$$\begin{aligned} |\Psi|^2 &= |\Psi_1 + \Psi_2|^2 \qquad (144) \\ &= R_1^2 + R_2^2 + 2R_1R_2\cos\left(\frac{\phi_1 - \phi_2}{\Phi_0}\right). \qquad (145) \end{aligned}$$

The cross term depends on the relative transport-derived phase. This is the usual interference structure of the Schrödinger representation.

In NUVO language, interference is not introduced as the collision or overlap of physical wave substances. It is the phase-sensitive recombination of represented closure-density amplitudes. The underlying real quantities remain closure density and transport-derived phase. The complex representation encodes how different admissible transport branches recombine when expressed in a common representational space.

The density-curvature term is again essential. Interference patterns are spatially structured density profiles. Once such profiles form, their subsequent evolution depends on the curvature of $\sqrt{\rho}$. Thus the same structural term responsible for Madelung equivalence also governs the evolution of the spatial density structure produced by phase-sensitive recombination.

This interpretation is compatible with the earlier Q-series account of matter-wave coherence. The observable interference pattern is not taken as evidence for a primitive matter wave. Rather, it is the representation-level density structure generated by coherent phase relations among admissible exchange-transport histories.

11.3 Hydrogenic Stationary States

Now consider a scalar-geometric or exchange-sector potential

$$U_\lambda(r) \tag{146}$$

corresponding to the hydrogenic proton–electron exchange sector. In the Schrödinger representation, stationary states have the form

$$\Psi(x,t) = \psi_n(x)e^{-iE_nt/\Phi_0}, \tag{147}$$

where ψ_n satisfies the time-independent equation

$$-\frac{\Phi_0^2}{2m}\nabla^2\psi_n + U_\lambda(r)\psi_n = E_n\psi_n. \tag{148}$$

For the Coulombic hydrogenic correspondence, this equation admits the standard discrete bound-state spectrum and stationary mode structure.

Within the NUVO program, this result should be compared with, but not used to replace, the independent closure-hierarchy derivation of the hydrogen spectral ladder developed earlier in the Q-series. In that earlier derivation, the spectral structure arises from exchange-cycle closure, holonomic admissibility, and the discrete hierarchy of closure-compatible configurations. The present Schrödinger representation supplies a complementary local transport representation of the same nonrelativistic sector.

The relationship should therefore be understood as follows. The Q7 closure-hierarchy derivation identifies the discrete hydrogenic ladder from global exchange-cycle closure. The present Madelung bridge shows that, once closure-density curvature is included, the local closure-transport representation is equivalent to the usual Schrödinger equation whose stationary solutions also exhibit the hydrogenic spectrum.

Thus the two descriptions support one another but play different roles:

(i) the closure-hierarchy analysis supplies the global holonomic origin of the discrete ladder;

(ii) the Madelung/Schrödinger representation supplies the local differential equation governing closure-density and phase distributions in the nonrelativistic regime.

This distinction prevents circularity. The hydrogen spectrum is not first assumed through the Schrödinger equation and then claimed as a closure result. Rather, the closure hierarchy and the Schrödinger representation are two corresponding descriptions of the same hydrogenic exchange sector, one global and holonomic, the other local and differential.

11.4 Summary of Correspondence

The inclusion of closure-density curvature provides the missing mechanism needed for the usual nonrelativistic quantum behaviors to appear in the local representation:

(i) free-particle spreading arises from density-curvature-driven phase evolution when $U_\lambda = 0$;

(ii) interference arises from phase-sensitive recombination of represented closure-density amplitudes;

(iii) hydrogenic stationary states arise from the Schrödinger representation with the scalar-geometric hydrogenic potential $U_\lambda(r)$, while remaining conceptually linked to the independent closure-hierarchy derivation of the spectral ladder.

These correspondences support the central claim of the present paper: the closure-density curvature term is the structural ingredient needed to convert the Q11–Q12 transport representation into a full Madelung-equivalent description of the nonrelativistic quantum regime.

12 Limitations and Open Problems

The present paper closes a specific gap in the Q-series: it identifies the closure-density curvature term required for full Madelung equivalence and shows that the resulting real transport system is equivalent to the nonrelativistic Schrödinger representation. The scope of this result is intentionally limited. Several technical and conceptual issues remain outside the present derivation and should be treated in later work.

12.1 Nodes and Singular Density Profiles

The derivation above assumes that the closure density is strictly positive on the region of interest:

$$\rho(x,t) > 0. \tag{149}$$

Equivalently, the amplitude

$$R = \sqrt{\rho} \tag{150}$$

is assumed to satisfy $R > 0$. This assumption allows the phase ϕ to be chosen smoothly and permits division by R in the density-curvature term

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2 R}{R}. \tag{151}$$

This restriction is standard in local Madelung-type decompositions, but it is not sufficient for a complete global theory. In ordinary Schrödinger systems, nodes occur naturally in excited states, interference patterns, angular momentum eigenstates, and scattering configurations. At such points the amplitude vanishes, the phase may become undefined or multi-valued, and the ratio $\nabla^2 R/R$ may become singular.

A full NUVO treatment of nodes must therefore address at least four issues:

(i) how closure-density transport is represented at points or surfaces where $\rho = 0$;

(ii) how phase branch structure is handled around nodal sets;

(iii) whether vortex-like or circulation defects correspond to admissible exchange-sector holonomies;

(iv) how distributional or weak formulations of the closure-density curvature term should be defined.

The present paper avoids these complications by proving the local result on nodal-free regions. This is sufficient for establishing the Madelung bridge, but not for a complete global account of all Schrödinger states. A later technical development should treat nodes, phase defects, and singular closure-density profiles explicitly.

12.2 Relativistic Extension

The present derivation is nonrelativistic. This restriction enters in several places. First, the admissible transport velocity is reduced to the phase-gradient form

$$v = \frac{\nabla\phi}{m}. \tag{152}$$

Second, the kinetic transport contribution is taken to be

$$\frac{|\nabla\phi|^2}{2m}. \tag{153}$$

Third, the resulting complex representation is the nonrelativistic Schrödinger equation

$$i\Phi_0\partial_t\Psi = -\frac{\Phi_0^2}{2m}\nabla^2\Psi + U_\lambda\Psi. \tag{154}$$

These are appropriate assumptions for the local nonrelativistic closure-transport regime, but they are not expected to remain exact in relativistic settings. A relativistic extension must account for:

(i) Lorentz-covariant transport structure;

(ii) relativistic phase and action relations;

(iii) coupling to scalar–conformal spacetime geometry rather than only a spatial potential $U_\lambda(x)$;

(iv) spinorial or multi-component representations where appropriate;

(v) compatibility with the RQM series and the eventual relativistic quantum correspondence.

Accordingly, the present result should not be interpreted as a derivation of relativistic quantum mechanics. It establishes the nonrelativistic Madelung bridge. The relativistic extension belongs to the RQM development [11], where the density-curvature idea may require a covariant reformulation.

12.3 Many-Body Configuration Space

The present derivation is written over physical three-space for a single closure-density field:

$$\rho = \rho(x,t), \qquad x \in \mathbb{R}^3. \tag{155}$$

This is sufficient for the single-body or effective one-body nonrelativistic correspondence. However, standard many-body quantum mechanics is formulated on configuration space, with wavefunctions of the form

$$\Psi(x_1,\dots,x_N,t). \tag{156}$$

The associated density lives on a $3N$-dimensional configuration space rather than ordinary physical space.

A full many-body NUVO extension must therefore clarify whether the appropriate object is:

(i) a configuration-space closure density

$$\rho(x_1,\dots,x_N,t);$$

(ii) a collection of coupled physical-space closure densities;

(iii) a multi-bundle closure-density structure defined over admissible bundle configurations;

(iv) or a deeper exchange-sector object whose configuration-space representation emerges only after reduction.

This issue is not merely technical. Entanglement, nonseparability, and multi-particle interference depend on the structure of the state space. If NUVO is to reproduce the many-body quantum formalism, it must explain why and when a configuration-space representation is required, and how that representation relates to physical exchange-sector closure among bundled structures.

The present paper does not solve that problem. It establishes the single-field Madelung bridge. The many-body extension should be treated as a separate development, likely connecting the Q-series transport framework to the QB/QM treatment of representational spaces, projectors, and entanglement [12, 13].

12.4 Derivation of the Coefficient

The coefficient of the closure-density curvature functional was chosen as

$$\frac{\Phi_0^2}{2m}. \tag{157}$$

This coefficient is required for dimensional compatibility and for exact correspondence with the Schrödinger kinetic operator. With this choice, the variational derivative produces

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}, \tag{158}$$

and the resulting Madelung system recombines into

$$i\Phi_0\partial_t\Psi = -\frac{\Phi_0^2}{2m}\nabla^2\Psi + U_\lambda\Psi. \tag{159}$$

This is sufficient for the present correspondence theorem, but it is not yet the deepest possible NUVO derivation of the coefficient. A stronger future derivation should connect both factors in the coefficient to prior NUVO structures.

First, the action scale Φ_0 should be tied explicitly to the closure-action scale developed in the Q-series. Earlier Q-series work identified an invariant exchange-cycle action scale through closure transport and hydrogenic calibration. The present paper uses Φ_0 as the representation scale controlling both phase and closure-density curvature. A deeper derivation should show directly why the same closure-action scale controls the amplitude-curvature cost.

Second, the mass parameter m should be connected to the support-sector invariant intake structure. In the support sector, persistent anchored structures are characterized by invariant structural intake. The nonrelativistic transport mass appearing in the Madelung system should therefore be interpreted not as an imported inertial parameter, but as the reduction of the support-sector persistence scale into the exchange-sector transport representation.

The deeper coefficient problem can therefore be stated as:

$$\frac{\Phi_0^2}{2m} \quad \text{should arise from} \quad \frac{(\text{closure-action scale})^2}{2(\text{support-sector persistence scale})}. \tag{160}$$

The present paper assumes this coefficient at the level required for Madelung correspondence. A later foundational refinement should derive it from the already developed NUVO action and support-sector structures.

12.5 Status of the Curvature Functional

The closure-density curvature functional

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{2m}\int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x \tag{161}$$

was motivated as the minimal local functional satisfying the admissibility requirements listed above. This is a strong structural argument, but it is not the same as deriving the functional uniquely from a more primitive substrate dynamics.

The present paper therefore establishes a conditional result:

> If the local nonrelativistic closure-density deformation cost is given by the minimal admissible curvature functional, then the resulting transport system is exactly Madelung-equivalent and hence Schrödinger-equivalent.

A stronger future result would derive this functional directly from a deeper NUVO substrate model. Possible routes include:

(i) deriving the amplitude-gradient term from finite-capacity smoothing of closure-density gradients;

(ii) deriving it from a discrete closure-network continuum limit;

(iii) deriving it from an exchange-cycle stability condition;

(iv) deriving it from an information-geometric or coherence-metric structure intrinsic to the closure state space.

For the purposes of closing the Q11–Q12 Madelung gap, the minimal functional is adequate. But for a fully foundational derivation, the status of $\mathcal{C}[\rho]$ should remain marked as a structural principle awaiting deeper substrate-level justification.

12.6 Scope of the Result

The result of the present paper should be stated carefully. It does not claim to derive the entirety of quantum mechanics. It does not derive measurement, Born weights, spin, relativistic wave equations, field quantization, or many-body entanglement. Those topics require additional structures developed elsewhere in the NUVO program.

What the paper does establish is narrower and more precise:

(i) closure density and transport-derived phase admit a variational transport action;

(ii) the minimal closure-density curvature functional supplies the amplitude-curvature term missing from the unrefined phase equation;

(iii) the resulting real transport system is exactly the Madelung system;

(iv) the Madelung system is locally equivalent to the nonrelativistic Schrödinger equation under the complex representation

$$\Psi = \sqrt{\rho}e^{i\phi/\Phi_0}.$$

This closes a specific structural gap in the Q-series. It does not complete the entire quantum reconstruction program. Rather, it secures one of its central bridges: the passage from deterministic closure-density transport to the full nonrelativistic Madelung form.

13 Conclusion

The present manuscript addressed a specific structural gap in the exchange-sector development of scalar–conformal NUVO theory. Previous work established closure density, transport-derived phase, and a lossless complex representation of the pair

$$(\rho, \phi)$$

through

$$\Psi = \sqrt{\rho}e^{i\phi/\Phi_0}.$$

That construction provided a Schrödinger-type representation of closure transport, but it did not yet isolate the internal density-dependent contribution required for full Madelung equivalence.

The missing contribution is the amplitude-curvature term

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}.$$

In standard Madelung language this term is called the quantum potential. In the present framework it has been reinterpreted as the closure-density curvature potential. It is not introduced as an external force, an independent scalar source, a probability postulate, or a primitive quantum assumption. It arises as the phase-level expression of the minimal local deformation cost associated with a spatially inhomogeneous closure-density profile.

The central structural input was the closure-density curvature functional

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{2m} \int_\Omega |\nabla \sqrt{\rho}|^2 \ \mathrm{d}^3x.$$

This functional is local, nonnegative, rotationally invariant, vanishes for uniform closure density, and is compatible with the complex representation. Including it in the closure transport action gives

$$\mathcal{A}[\rho, \phi] = \int \mathrm{d}t \ \mathrm{d}^3x \left[\rho \left(\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda \right) + \frac{\Phi_0^2}{2m} |\nabla \sqrt{\rho}|^2 \right].$$

Variation with respect to the phase yields the continuity equation

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0,$$

while variation with respect to the amplitude $R = \sqrt{\rho}$ yields the corrected phase equation

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = 0.$$

Together these equations form the NUVO Madelung transport system in the local nonrelativistic integrable regime.

Under the complex representation

$$\Psi = \sqrt{\rho} e^{i\phi/\Phi_0},$$

the two real transport equations are equivalent to

$$i\Phi_0 \partial_t \Psi = -\frac{\Phi_0^2}{2m} \nabla^2 \Psi + U_\lambda \Psi.$$

Thus the Schrödinger equation appears as the compact complex representation of the real closure-density transport system once the density-curvature cost is included.

This result sharpens the claim of the preceding Q-series papers. The appropriate statement is not merely that a Schrödinger-type equation can be written for closure transport. Rather, in the local integrable nonrelativistic regime, closure density and transport-derived phase satisfy the Madelung equations, and their complex encoding is equivalent to the Schrödinger equation with scalar-geometric potential U_λ.

The conclusion should be read with its limitations intact. The present paper does not derive measurement theory, Born weights, spin, relativistic quantum dynamics, field quantization, or many-body entanglement. It also assumes a nodal-free local region and takes the coefficient $\Phi_0^2/2m$ at the correspondence level, with deeper derivation from closure action and support-sector persistence deferred to future work. What it does establish is narrower but essential: the specific amplitude-curvature term needed to close the gap between deterministic closure transport and full Madelung form arises naturally as closure-density curvature.

Accordingly, the conventional quantum potential is reinterpreted within NUVO as the representation-level expression of the structural cost of maintaining coherent transport across a spatially inhomogeneous closure-density distribution. This closes the transport-to-Madelung bridge and provides a more precise foundation for the subsequent Schrödinger, Hilbert-space, and quantum-measurement correspondences in the broader NUVO program.

A Appendix A: Variation of the Closure-Density Curvature Functional

This appendix records the detailed variation of the closure-density curvature functional used in the main text. Let

$$R = \sqrt{\rho}$$

and consider the spatial functional

$$\mathcal{K}[R] := \int_\Omega |\nabla R|^2 \, \mathrm{d}^3x.$$

We assume that R is smooth on the spatial domain $\Omega \subset \mathbb{R}^3$, and that variations satisfy boundary conditions sufficient to remove surface terms. For example, one may assume compact support, sufficiently rapid decay at infinity, periodic boundary conditions, or

$$\delta R\big|_{\partial\Omega} = 0.$$

Let

$$R_\epsilon = R + \epsilon\chi,$$

where $\chi = \delta R$ is a smooth test variation. Then

$$\mathcal{K}[R_\epsilon] = \int_\Omega |\nabla R_\epsilon|^2 \, \mathrm{d}^3x = \int_\Omega |\nabla R + \epsilon\nabla\chi|^2 \, \mathrm{d}^3x.$$

Expanding to first order in ϵ, we obtain

$$\mathcal{K}[R_\epsilon] = \int_\Omega \left(|\nabla R|^2 + 2\epsilon \, \nabla R \cdot \nabla\chi + \epsilon^2 |\nabla\chi|^2\right) \mathrm{d}^3x.$$

Therefore,

$$\delta\mathcal{K} = \frac{d}{d\epsilon}\mathcal{K}[R_\epsilon]\bigg|_{\epsilon=0} = 2\int_\Omega \nabla R \cdot \nabla\chi \, \mathrm{d}^3x.$$

Using integration by parts,

$$\int_\Omega \nabla R \cdot \nabla\chi \, \mathrm{d}^3x = \int_{\partial\Omega} \chi \, \nabla R \cdot n \, \mathrm{d}S - \int_\Omega (\nabla^2 R)\chi \, \mathrm{d}^3x,$$

where n denotes the outward unit normal on $\partial\Omega$.

Under the assumed boundary conditions, the boundary term vanishes:

$$\int_{\partial\Omega} \chi \, \nabla R \cdot n \, \mathrm{d}S = 0.$$

Hence

$$\delta\mathcal{K} = -2\int_\Omega (\nabla^2 R)\chi \, \mathrm{d}^3x.$$

Since $\chi = \delta R$, this gives

$$\boxed{\delta\int_\Omega |\nabla R|^2 \, \mathrm{d}^3x = -2\int_\Omega (\nabla^2 R)\delta R \, \mathrm{d}^3x}.$$

Multiplying by the coefficient appearing in the closure-density curvature functional,

$$\frac{\Phi_0^2}{2m},$$

we obtain

$$\delta\left[\frac{\Phi_0^2}{2m}\int_\Omega |\nabla R|^2 \, \mathrm{d}^3x\right] = -\frac{\Phi_0^2}{m}\int_\Omega (\nabla^2 R)\delta R \, \mathrm{d}^3x.$$

This is the term used in the amplitude variation in Section 6.

When combined with the variation of

$$\int_\Omega R^2 H_\phi \, \mathrm{d}^3x,$$

where

$$H_\phi = \partial_t\phi + \frac{|\nabla\phi|^2}{2m} + U_\lambda,$$

one obtains

$$\delta_R\mathcal{A} = \int \mathrm{d}t \, \mathrm{d}^3x \left[2RH_\phi - \frac{\Phi_0^2}{m}\nabla^2 R\right]\delta R.$$

Stationarity for arbitrary δR gives

$$2RH_\phi - \frac{\Phi_0^2}{m}\nabla^2 R = 0.$$

On nodal-free regions where $R > 0$, division by $2R$ yields

$$H_\phi - \frac{\Phi_0^2}{2m}\frac{\nabla^2 R}{R} = 0,$$

which is the density-curvature-corrected phase equation.

B Appendix B: Direct Polar Decomposition of the Schrödinger Equation

This appendix records the standard polar decomposition of the Schrödinger representation used in the equivalence theorem. Consider

$$i\Phi_0\partial_t\Psi = -\frac{\Phi_0^2}{2m}\nabla^2\Psi + U_\lambda\Psi,$$

and write

$$\Psi = Re^{i\phi/\Phi_0}, \qquad R = \sqrt{\rho}.$$

For compactness, define

$$\theta := \frac{\phi}{\Phi_0}.$$

Then

$$\Psi = Re^{i\theta}.$$

First compute the time derivative:

$$\partial_t\Psi = \partial_t(Re^{i\theta}) = e^{i\theta}\left(\partial_t R + iR\partial_t\theta\right).$$

Therefore,

$$i\Phi_0\partial_t\Psi = e^{i\theta}\left(i\Phi_0\partial_t R - \Phi_0 R\partial_t\theta\right).$$

Since $\theta = \phi/\Phi_0$,

$$\Phi_0\partial_t\theta = \partial_t\phi,$$

so

$$i\Phi_0\partial_t\Psi = e^{i\theta}\left(i\Phi_0\partial_t R - R\partial_t\phi\right).$$

Next compute the spatial derivatives. The gradient is

$$\nabla\Psi = e^{i\theta}\left(\nabla R + iR\nabla\theta\right).$$

Taking another divergence gives

$$\nabla^2\Psi = \nabla\cdot\Big[e^{i\theta}\left(\nabla R + iR\nabla\theta\right)\Big].$$

Using $\nabla e^{i\theta} = ie^{i\theta}\nabla\theta$, we get

$$\nabla^2\Psi = e^{i\theta}\left[\nabla^2 R - R|\nabla\theta|^2 + i\left(2\nabla R\cdot\nabla\theta + R\nabla^2\theta\right)\right].$$

Since

$$\nabla\theta = \frac{\nabla\phi}{\Phi_0}, \qquad \nabla^2\theta = \frac{\nabla^2\phi}{\Phi_0},$$

this may also be written as

$$\nabla^2\Psi = e^{i\phi/\Phi_0}\left[\nabla^2 R - \frac{R}{\Phi_0^2}|\nabla\phi|^2 + \frac{i}{\Phi_0}\left(2\nabla R\cdot\nabla\phi + R\nabla^2\phi\right)\right].$$

Substituting into the Schrödinger equation and dividing by $e^{i\theta}$, we obtain

$$i\Phi_0\partial_t R - R\partial_t\phi = -\frac{\Phi_0^2}{2m}\left[\nabla^2 R - \frac{R}{\Phi_0^2}|\nabla\phi|^2 + \frac{i}{\Phi_0}\left(2\nabla R\cdot\nabla\phi + R\nabla^2\phi\right)\right] + U_\lambda R.$$

Separating real and imaginary parts gives two equations.

The imaginary part is

$$\Phi_0\partial_t R = -\frac{\Phi_0}{2m}\left(2\nabla R\cdot\nabla\phi + R\nabla^2\phi\right).$$

Canceling Φ_0,

$$\partial_t R = -\frac{1}{2m}\left(2\nabla R\cdot\nabla\phi + R\nabla^2\phi\right).$$

Multiplying by $2R$, we find

$$2R\partial_t R = -\frac{1}{m}\left(2R\nabla R\cdot\nabla\phi + R^2\nabla^2\phi\right).$$

Since $\rho = R^2$,

$$2R\partial_t R = \partial_t\rho,$$

and

$$2R\nabla R = \nabla(R^2) = \nabla\rho.$$

Thus

$$\partial_t\rho = -\frac{1}{m}\left(\nabla\rho\cdot\nabla\phi + \rho\nabla^2\phi\right).$$

Using

$$\nabla \cdot (\rho \nabla \phi) = \nabla \rho \cdot \nabla \phi + \rho \nabla^2 \phi,$$

we obtain

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0.$$

This is the Madelung continuity equation.

The real part is

$$-R\partial_t \phi = -\frac{\Phi_0^2}{2m} \nabla^2 R + \frac{R}{2m} |\nabla \phi|^2 + U_\lambda R.$$

Dividing by $R > 0$ gives

$$-\partial_t \phi = -\frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} + \frac{|\nabla \phi|^2}{2m} + U_\lambda.$$

Rearranging,

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 R}{R} = 0.$$

Since $R = \sqrt{\rho}$, this becomes

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = 0.$$

This is the Madelung phase equation.

Thus the polar decomposition of the Schrödinger representation yields the two real equations

$$\partial_t \rho + \nabla \cdot \left(\rho \frac{\nabla \phi}{m} \right) = 0,$$

and

$$\partial_t \phi + \frac{|\nabla \phi|^2}{2m} + U_\lambda - \frac{\Phi_0^2}{2m} \frac{\nabla^2 \sqrt{\rho}}{\sqrt{\rho}} = 0.$$

Conversely, if these two real equations hold, reversing the calculation recombines them into the complex Schrödinger representation.

C Appendix C: Uniqueness of the Minimal Curvature Functional

This appendix gives a controlled structural argument for the use of

$$\int_\Omega |\nabla \sqrt{\rho}|^2 \, \mathrm{d}^3 x$$

as the minimal local curvature cost for closure density. The result is not presented as a final global uniqueness theorem. Rather, it is a minimality lemma: under the stated local assumptions and to lowest nontrivial derivative order, this is the natural scalar functional compatible with the NUVO closure-density representation and with Madelung correspondence.

Lemma C.1 (Minimal local closure-density curvature cost)**.** *Let $\rho > 0$ be a smooth closure-density field on a spatial domain $\Omega \subset \mathbb{R}^3$. Suppose a local curvature cost functional $\mathcal{C}[\rho]$ satisfies the following leading-order requirements:*

(i) it is local in ρ and its spatial derivatives;

(ii) it is rotationally invariant;

(iii) it is nonnegative;

(iv) it vanishes for spatially uniform ρ;

(v) it has lowest nontrivial dependence on spatial derivatives;

(vi) it is compatible with the amplitude representation

$$\Psi = \sqrt{\rho} e^{i\phi/\Phi_0};$$

(vii) it produces a second-order local contribution to the phase equation under variation.

Then, to lowest derivative order and up to an overall positive coefficient, the natural admissible curvature cost is

$$\mathcal{C}[\rho] \propto \int_\Omega |\nabla\sqrt{\rho}|^2 \, \mathrm{d}^3x.$$

Structural argument. Because the cost must vanish for spatially uniform closure density, its leading nontrivial contribution must depend on spatial derivatives of ρ. A zeroth-order term of the form

$$\int_\Omega F(\rho) \, \mathrm{d}^3x$$

would generally not vanish for uniform density unless F were trivial or specially constrained. Such a term would represent a local density potential rather than a curvature or deformation cost. It is therefore excluded from the leading curvature functional.

The lowest nontrivial derivative order is first order in spatial gradients. Rotational invariance requires that first derivatives enter through scalar contractions. Thus the leading local density-gradient cost must have the schematic form

$$\int_\Omega A(\rho)|\nabla\rho|^2 \, \mathrm{d}^3x,$$

or equivalently,

$$\int_\Omega |\nabla f(\rho)|^2 \, \mathrm{d}^3x$$

for some monotone local amplitude variable $f(\rho)$.

Nonnegativity requires the coefficient of the quadratic gradient term to be nonnegative. The cost then vanishes for uniform ρ, since

$$\nabla\rho = 0 \quad \Longrightarrow \quad |\nabla f(\rho)|^2 = 0.$$

The remaining question is which local amplitude variable $f(\rho)$ is selected. Compatibility with the complex representation selects

$$R = \sqrt{\rho}.$$

This is because the complex encoding is

$$\Psi = Re^{i\phi/\Phi_0},$$

with

$$R = |\Psi|, \qquad \rho = |\Psi|^2.$$

Thus R, rather than ρ itself, is the amplitude field whose spatial curvature appears in the Laplacian of the complex representation.

The Schrödinger kinetic term is

$$-\frac{\Phi_0^2}{2m}\nabla^2\Psi.$$

When $\Psi = Re^{i\phi/\Phi_0}$, the real part of this second-order term contains

$$-\frac{\Phi_0^2}{2m}\frac{\nabla^2 R}{R}.$$

Therefore a closure-density curvature functional compatible with the Schrödinger/Madelung representation must vary to produce a term proportional to

$$\frac{\nabla^2 R}{R}$$

in the phase equation.

The functional

$$\int_\Omega |\nabla R|^2 \ \mathrm{d}^3x$$

has exactly this property. Its variation is

$$\delta \int_\Omega |\nabla R|^2 \ \mathrm{d}^3x = -2\int_\Omega (\nabla^2 R)\delta R \ \mathrm{d}^3x,$$

and, when combined with the variation of the term $R^2 H_\phi$, it produces

$$H_\phi - \frac{\Phi_0^2}{2m}\frac{\nabla^2 R}{R} = 0.$$

Thus the amplitude-gradient functional is the minimal first-derivative functional whose Euler–Lagrange contribution has the required second-order curvature form.

Since $R = \sqrt{\rho}$,

$$\int_\Omega |\nabla R|^2 \ \mathrm{d}^3x = \int_\Omega |\nabla\sqrt{\rho}|^2 \ \mathrm{d}^3x.$$

Equivalently,

$$|\nabla\sqrt{\rho}|^2 = \frac{1}{4}\frac{|\nabla\rho|^2}{\rho},$$

so

$$\int_\Omega |\nabla\sqrt{\rho}|^2 \ \mathrm{d}^3x = \frac{1}{4}\int_\Omega \frac{|\nabla\rho|^2}{\rho} \ \mathrm{d}^3x.$$

This is the leading Fisher-information-type density-gradient functional, although no probabilistic interpretation is assumed in the present context.

Other local functionals are possible if one allows higher derivatives, higher powers, nonminimal density weights, or additional coupling to the scalar background. For example, one could consider terms of the form

$$\int_\Omega (\nabla^2 R)^2 \ \mathrm{d}^3x,$$

$$\int_\Omega |\nabla R|^4 \ \mathrm{d}^3x,$$

or

$$\int_\Omega B(\lambda)|\nabla R|^2 \, \mathrm{d}^3x.$$

Such terms may be relevant as higher-order corrections or sectoral refinements, but they are not part of the minimal lowest-order curvature cost.

Therefore, under the stated leading-order assumptions, the minimal local scalar cost compatible with the closure-density representation is

$$\mathcal{C}[\rho] = C \int_\Omega |\nabla \sqrt{\rho}|^2 \, \mathrm{d}^3x,$$

with $C > 0$. Madelung–Schrödinger correspondence fixes

$$C = \frac{\Phi_0^2}{2m}.$$

This yields

$$\mathcal{C}[\rho] = \frac{\Phi_0^2}{2m} \int_\Omega |\nabla \sqrt{\rho}|^2 \, \mathrm{d}^3x.$$

□

Interpretive Note

The preceding lemma should be read as a controlled structural minimality argument. It does not claim that no other density-dependent terms can ever appear in a more complete NUVO theory. Rather, it shows that if one seeks the lowest-order local, positive, rotationally invariant curvature cost compatible with the amplitude representation

$$\Psi = \sqrt{\rho} e^{i\phi/\Phi_0},$$

then the amplitude-gradient functional

$$\int_\Omega |\nabla \sqrt{\rho}|^2 \, \mathrm{d}^3x$$

is the natural minimal choice.

This is sufficient for the purpose of the present manuscript: to supply the missing density-curvature contribution required for local Madelung equivalence. A deeper future derivation may attempt to obtain the same functional from an underlying discrete closure network, a finite-capacity substrate model, or a more primitive exchange-sector coherence metric.

D Appendix D: Notation Ledger

Symbol	Meaning
ρ	closure density
ϕ	transport-derived phase
$R = \sqrt{\rho}$	closure amplitude
$\Phi_0 = \Phi_0$	representation/action scale
m	transported closure mass parameter
U_λ	scalar-geometric or exchange-sector potential
Q_ρ	closure-density curvature potential
Ψ	complex representation of (ρ, ϕ)

References

[1] Rickey W. Austin. M1: Scalar–conformal geometry and the variational structure of the scalar capacity field. NUVO M-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[2] Rickey W. Austin. M2: Structural capacity availability and the interpretation of the canonical nuvo equation. NUVO M-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[3] Rickey W. Austin. Q11: Unified transport law for closure and exchange in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[4] Rickey W. Austin. Q12: Emergent schrödinger representation from transport closure in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[5] Erwin Madelung. Quantentheorie in hydrodynamischer form. *Zeitschrift für Physik*, 40(3-4):322–326, 1927.

[6] Erwin Schrödinger. Quantisierung als eigenwertproblem. *Annalen der Physik*, 384(4):361–376, 1926.

[7] David Bohm. A suggested interpretation of the quantum theory in terms of “hidden” variables. *Physical Review*, 85(2):166–193, 1952.

[8] Rickey W. Austin. Q10: Scalar transport and coherent phase evolution in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[9] Rickey W. Austin. Q5: Closure action and the energy–frequency law in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[10] Rickey W. Austin. Q7: The hydrogen spectrum from exchange-cycle closure in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[11] Rickey W. Austin. RQM1: Nuvo rqm-series paper 1 – relativistic quantum foundation. NUVO RQM-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[12] Rickey W. Austin. QB1: Nuvo qb-series paper 1 – quantum building blocks. NUVO QB-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[13] Rickey W. Austin. QM1: Nuvo qm-series paper 1 – quantum measurement. NUVO QM-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.