QB1 – State Representation from Transport Closure in Scalar–Conformal NUVO Systems

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Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.
- The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-, +, +, +)$.

Abstract

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

We establish the existence of a minimal local state description for exchange-sector transport in scalar–conformal NUVO systems and show that this state admits a lossless complex representation. Starting from the closure density and transport-derived phase introduced in the preceding Q-series, we show that these quantities form a complete local description of admissible transport in the integrable regime. We then construct a unified complex encoding of this state and demonstrate that it preserves the closure and coherence structure established previously, without introducing new ontology.

The resulting representation provides a natural bridge to later quantum-mechanical state formalism while remaining entirely geometric and deterministic. We further clarify the scope of the representation, including its local validity and its relation to the underlying path-dependent phase structure. Finally, we record the compatibility of this state encoding with the Schrödinger-type representation obtained in the preceding work.

No probabilistic interpretation, wave ontology, or operator formalism is assumed. The purpose of the present paper is solely to establish the state-representational bridge between the transport closure system of the Q-series and the later formal developments of the QB and QM series.

This manuscript is mathematical in scope. It establishes definitions, structural identities, and variational consequences within a scalar–conformal setting. Sector reductions and correspondence limits are recorded only when explicitly stated as additional assumptions and are not used as premises in derivations.

No claim of full dynamical equivalence to general relativity, quantum mechanics, or classical field theories is made at the level of the present foundational development. Where later papers compare limiting behavior, those comparisons are presented as correspondence targets rather than as identity statements.

The NUVO program is organized as a sequence of internally consistent mathematical papers. Foundational papers (M-series) fix the scalar–conformal geometry, variational structure, and notation. Subsequent papers treat sectoral reductions (gravity, exchange, quantization, and bound-state structure) as controlled specializations of the foundational framework.

Throughout the series we distinguish between (i) definitions and theorems proved in the present manuscript, and (ii) external results used only for context. References are cited for orientation and comparison and are not treated as axioms unless explicitly declared.

All notation intended to be program-wide is centralized in the shared NUVO macro package and notation layer. This is done to maintain consistency across the series and to support future consolidation into a cohesive monograph-style presentation.

Scalar ontology. The scalar field Λ represents the *locally available structural capacity* of an underlying delivery field permeating spacetime. The baseline level Λ_0 denotes the availability supported by this intrinsic delivery structure in the absence of structural occupation. Localized structures or transport processes may reduce the available capacity relative to this baseline, but the intrinsic delivery baseline itself is not altered. Consequently the scalar field measures the *available portion* of structural capacity rather than the intrinsic production of the underlying field.

1 Introduction

1.1 Position Within the Program

The M-series [1, 2] established the scalar–conformal geometric framework of the NUVO program together with its support-sector and exchange-sector structure. The Q-series then developed the exchange sector through closure, coherence, transport, hydrogenic correspondence, and the emergence of a Schrödinger-type representation from the transport system [3–6].

Despite these developments, an important bridge remains to be stated explicitly. The preceding Q-series papers provide closure density, transport-derived phase, and a deterministic transport law, but they do not yet isolate the precise notion of *state representation* appropriate to that structure. In particular, the Q-series does not yet identify, in theorem form, the minimal local state variables, the exact sense in which they may be encoded by a single complex quantity, or the scope and limitations of such an encoding.

The purpose of the present paper is to establish this bridge. We show that the exchange-sector transport system already developed in the Q-series admits a minimal local state description by a pair of real quantities and that this pair can be encoded losslessly by a single complex-valued state representation. This construction is representational only: it does not alter the underlying ontology, does not introduce a wave substance, and does not assign probabilistic meaning to the resulting complex state.

1.2 Objective of the Present Work

The primary objective of the present paper is to prove four related claims.

1. In the local integrable transport regime, the exchange-sector state is fully specified by closure density and transport-derived phase.
2. This local state admits a lossless complex encoding.
3. The complex encoding preserves the closure and coherence structure already established in the Q-series.
4. The resulting encoded state is exactly the bridge object needed for later formal transition to quantum-mechanical representation, while remaining non-ontological and non-probabilistic at the present stage.

1.3 What the Present Paper Does Not Assume

The present manuscript does not introduce a probabilistic interpretation of closure density, does not formulate a measurement postulate, and does not assume operator formalism. Likewise, it does not treat the complex state as a primitive wave entity. The role of the present paper is strictly narrower: to establish the state representation naturally associated with the previously derived transport closure system.

1.4 Structure of the Paper

Section 2 recalls the transport closure structure from the Q-series needed for the present development. Section 3 identifies the minimal local state variables. Section 4 constructs the complex encoding of that state. Section 5 proves that the encoding preserves prior closure and coherence structure. Section 6 clarifies the local scope of the construction and its relation to the underlying path-dependent phase. Section 7 explains the naturalness of the complex form as a single-object state encoding. Section 8 records the compatibility of this representation with the Schrödinger-type evolution obtained previously. Section 9 concludes with interpretive remarks and the transition to the next QB paper.

2 Transport Closure Structure from the Q-Series

2.1 Closure Density and Phase

The Q-series established that exchange-sector transport admits a local description in terms of two scalar quantities: a closure density and a transport-derived phase.

The closure density

$$\rho : \mathcal{M} \times \mathbb{R} \to \mathbb{R}_{\geq 0}$$

measures the local distribution of admissible closure configurations. It is defined as a geometric quantity representing closure content and is not interpreted as a probability density.

The transport-derived phase

$$\phi : \mathcal{M} \times \mathbb{R} \to \mathbb{R}$$

arises from the cumulative exchange interaction experienced along admissible transport paths [7]. It is introduced as a derived scalar quantity encoding transport consistency and is not assumed as a wave property.

These quantities are not postulated in the present work but are recalled from the transport and closure structure developed in the preceding Q-series.

2.2 Local Transport Law

The evolution of closure structures is governed by a deterministic transport system coupling closure density, phase, and transport velocity.

The closure density satisfies the continuity relation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0, \tag{1}$$

where $v : \mathcal{M} \times \mathbb{R} \to \mathbb{R}^3$ is the transport velocity field.

The phase satisfies a transport-consistent evolution equation of the form

$$\partial_t \phi + v \cdot \nabla \phi = \mathcal{E}(x, t), \tag{2}$$

where $\mathcal{E}(x, t)$ is a scalar function determined by the exchange interaction along the transport trajectory.

The velocity field v is not an independent degree of freedom, but is determined through the transport structure, depending on the local phase gradient and scalar geometry. Accordingly, the system closes as a coupled evolution for ρ and ϕ.

These relations are recalled from the unified transport law established in the Q-series and are not derived in the present paper.

2.3 Integrable Local Regime

The phase $\phi(x, t)$ is derived from accumulated transport along admissible paths and is, in general, a path-dependent quantity. Consequently, a globally defined scalar phase need not exist in arbitrary configurations.

However, in a simply connected region $U \subset \mathcal{M}$ where the local phase increment is path-independent, there exists a scalar field ϕ whose differential reproduces the local phase accumulation. In such regions the phase admits a consistent local scalar representation.

We refer to such regions as *integrable local transport regimes*. Within these regions, the pair (ρ, ϕ) provides a well-defined local description of the transport state.

Outside this regime, the phase remains well-defined as a path-dependent quantity, but the scalar field $\phi(x, t)$ should be interpreted as a local encoding rather than a globally trivial phase function.

2.4 Interpretive Discipline Inherited from the Q-Series

The present work inherits the interpretive constraints established in the Q-series, which are maintained without modification.

- The quantities ρ and ϕ are geometric and transport-derived. They are not interpreted as probability density or wave amplitude.
- No wave ontology is introduced. The phase is not associated with a physical oscillatory medium.
- No probabilistic interpretation is assumed. The framework remains deterministic at the level of transport closure.
- No operator formalism is introduced. All quantities are defined directly in terms of transport and scalar geometry.

These constraints will remain in force throughout the QB-series unless explicitly relaxed in later work.

3 Minimal Local State Description

3.1 Statement of the Local State Problem

The transport closure structure recalled in the preceding section provides two scalar quantities, closure density $\rho(x,t)$ and transport-derived phase $\phi(x,t)$, together with a deterministic transport law governing their evolution. The present section addresses the following question:

> *What is the minimal set of local quantities required to specify the exchange-sector transport state in the integrable regime?*

While the Q-series introduces additional quantities such as transport velocity and exchange interaction rate, these appear only as auxiliary fields in the transport law. It is therefore necessary to determine whether such quantities represent independent state variables or are fully determined by the pair (ρ, ϕ).

3.2 The Local Transport State Theorem

Theorem 3.1 (Local transport state)**.** *Let $U \subset \mathcal{M} \times \mathbb{R}$ be a region in which:*

1. *the transport closure system holds,*
2. *the transport-derived phase admits a consistent local scalar representation,*
3. *the scalar field $\rho(x,t)$ is strictly positive.*

Then the local exchange-sector transport state on U is fully specified by the pair

$$(\rho(x,t), \phi(x,t)).$$

Equivalently, all auxiliary transport quantities appearing in the transport law are determined by (ρ, ϕ), and no additional independent local state variable is required.

3.3 Proof of the Local Transport State Theorem

The transport law recalled in Section 2 consists of two coupled relations:

1. the continuity equation
$$\partial_t \rho + \nabla \cdot (\rho v) = 0,$$

2. the phase evolution equation
$$\partial_t \phi + v \cdot \nabla \phi = \mathcal{E}(x, t),$$

together with a constitutive relation expressing the transport velocity v as a function of local transport quantities, typically depending on the phase gradient and scalar geometry.

By construction, the velocity field $v(x, t)$ does not appear as an independent dynamical variable, but is determined through the transport structure. In particular, the direction and rate of transport are encoded by the phase gradient $\nabla \phi$ together with the scalar field, so that v is functionally dependent on ϕ and λ.

Similarly, the exchange interaction term $\mathcal{E}(x, t)$ is not an independent state variable, but is determined by the local scalar field and its variation, and therefore depends on (ρ, ϕ) through the underlying transport configuration.

It follows that the evolution of the system is completely determined once ρ and ϕ are specified on an initial hypersurface. No additional independent local degree of freedom remains.

Therefore the pair (ρ, ϕ) constitutes a complete local specification of the exchange-sector transport state in the integrable regime. □

3.4 Minimality and Uniqueness

The preceding theorem establishes that (ρ, ϕ) is sufficient to determine the local transport state. It remains to clarify the sense in which this description is minimal.

Proposition 3.2 (Minimality of the local state)**.** *Within the integrable local transport regime, any local state description sufficient to determine the transport evolution must encode both:*

1. *the local distribution of closure content, and*

2. *the local phase structure governing transport consistency.*

Consequently, any equivalent state representation must contain the same information as the pair (ρ, ϕ)*.*

3.5 Proof of Minimality

The continuity equation shows that the evolution of closure content depends explicitly on ρ and the transport velocity v. Since v is determined through the phase structure, any description that omits ρ fails to capture closure distribution, and any description that omits ϕ fails to determine transport direction and phase compatibility.

Therefore both ρ and ϕ are necessary for a complete local description, and no strictly smaller set of scalar variables suffices.

□

3.6 Consequences for State Representation

The results of this section establish that the exchange-sector transport state admits a complete local description in terms of two real scalar fields.

This observation motivates the search for a unified representation in which these two quantities are encoded as a single object. Any such representation must satisfy two requirements:

1. it must be lossless, allowing recovery of both ρ and ϕ,
2. it must preserve the structural relations governing transport and closure.

The construction of such a representation is carried out in the following section.

4 Complex Encoding of the Local State

4.1 Motivation for a Unified Representation

The preceding section established that the local exchange-sector transport state is completely specified by the pair of real scalar fields $(\rho(x,t), \phi(x,t))$. While this description is sufficient, it is not minimal in the sense of object count, since it requires two independent fields.

It is therefore natural to ask whether these quantities may be encoded as a single object without loss of information. Such an encoding must satisfy two requirements:

1. it must be *lossless*, allowing recovery of both ρ and ϕ,
2. it must preserve the structural role of phase as an additive transport quantity.

The present section constructs such an encoding and establishes its basic properties.

4.2 Definition of the Complex State

Definition 4.1 (Complex state representation)**.** *Let $(\rho(x,t), \phi(x,t))$ be a local transport state with $\rho(x,t) > 0$. Fix a constant $\Phi_0 > 0$, referred to as the* representation scale*. The associated complex state is defined by [5, 8]*

$$\Psi(x,t) := \sqrt{\rho(x,t)}\, e^{i\phi(x,t)/\Phi_0}. \tag{3}$$

The representation scale Φ_0 is introduced solely for normalization of the phase and does not alter the underlying transport structure.

4.3 The Complex Encoding Theorem

Theorem 4.2 (Lossless complex encoding)**.** *Let $(\rho(x,t), \phi(x,t))$ be a local transport state with $\rho(x,t) > 0$, and let Ψ be defined as above. Then the map*

$$(\rho, \phi) \longmapsto \Psi$$

is locally invertible up to phase branch choice. In particular, the original state variables are recovered by

$$\rho(x,t) = |\Psi(x,t)|^2, \qquad \phi(x,t) = \Phi_0 \arg\big(\Psi(x,t)\big), \tag{4}$$

where arg *denotes any continuous branch of the complex argument.*

4.4 Proof of the Complex Encoding Theorem

The modulus of Ψ satisfies

$$|\Psi| = \sqrt{\rho},$$

so that $|\Psi|^2 = \rho$.

The phase of Ψ is given by

$$\arg(\Psi) = \frac{\phi}{\Phi_0}$$

modulo integer multiples of 2π. Thus

$$\phi = \Phi_0 \arg(\Psi)$$

up to branch choice, which is a representational ambiguity and does not affect local phase differences.

Therefore both ρ and ϕ are recoverable from Ψ, and the mapping is lossless in the region where $\rho > 0$.

□

4.5 Immediate Consequences

Corollary 4.3. *The pair* (ρ, ϕ) *and the complex state* Ψ *contain identical local state information in the integrable transport regime.*

Corollary 4.4. *The introduction of* Ψ *does not enlarge or reduce the physical content of the transport closure system.*

4.6 Role of the Representation Scale

The constant Φ_0 sets the normalization of the phase in the complex representation. Its choice determines the numerical scale at which phase periodicity is expressed in the encoded form, but does not alter the underlying coherence structure, which remains determined by the transport system.

In particular, any apparent periodicity of the phase in the complex representation arises from the choice of normalization and should not be interpreted as a fundamental periodicity of the underlying transport phase.

4.7 Interpretation of the Complex State

The complex state $\Psi(x, t)$ is a *representation* of the transport state and is not introduced as a new physical object.

- The modulus $|\Psi|^2$ represents closure density, not probability.
- The phase of Ψ represents transport-derived phase, not a physical oscillation.
- No wave ontology is assumed.

Accordingly, Ψ should be understood as a compact encoding of the pair (ρ, ϕ) rather than as a primitive dynamical entity.

4.8 Motivation for the Complex Form

The use of a complex exponential to encode phase is motivated by the additive structure of the transport-derived phase. For two phase contributions ϕ_1 and ϕ_2, one has

$$e^{i(\phi_1+\phi_2)/\Phi_0} = e^{i\phi_1/\Phi_0}\, e^{i\phi_2/\Phi_0},$$

so that additive phase accumulation is represented multiplicatively.

This property allows phase composition to be encoded algebraically within a single object and is essential for the later reformulation of the transport law in a linear representation.

4.9 Transition to Structural Preservation

Having established the existence and losslessness of the complex state representation, it remains to verify that this encoding preserves the closure and coherence structure derived in the Q-series. This is addressed in the following section.

5 Preservation of Closure and Coherence Structure

5.1 Statement of the Preservation Problem

The complex state Ψ introduced in the preceding section provides a lossless encoding of the pair (ρ, ϕ). However, admissibility of a representation requires more than losslessness: it must also preserve the structural relations governing the system.

In the present context, these relations consist of:

1. closure density evolution,
2. transport consistency encoded through phase,
3. coherence and closure conditions expressed through phase accumulation.

The purpose of this section is to show that the passage from (ρ, ϕ) to Ψ preserves these structures without modification.

5.2 The Structural Preservation Theorem

Theorem 5.1 (Structural preservation)**.** *Let $(\rho(x,t), \phi(x,t))$ be a local transport state in the integrable regime, and let*

$$\Psi(x,t) = \sqrt{\rho(x,t)}\, e^{\,i\phi(x,t)/\Phi_0}$$

be the associated complex state representation.

Then the encoding Ψ preserves the exchange-sector structure in the following sense:

1. ***Density preservation:***
$$\rho(x,t) = |\Psi(x,t)|^2.$$
2. ***Phase preservation:***
$$\phi(x,t) = \Phi_0 \arg\big(\Psi(x,t)\big)$$
up to branch choice.

3. ***Transport preservation:*** *The continuity relation and phase evolution equation are satisfied if and only if the corresponding relations expressed in terms of* Ψ *hold.*

4. ***Closure and coherence preservation:*** *All closure and coherence conditions previously expressed in terms of phase accumulation are preserved under the encoding.*

5.3 Proof of the Structural Preservation Theorem

The first two statements follow directly from the definition of Ψ and the recovery relations established in the Complex Encoding Theorem.

To establish transport preservation, observe that the transport system is formulated entirely in terms of ρ and ϕ. Since both quantities are recoverable from Ψ, any relation involving ρ and ϕ can be expressed equivalently in terms of Ψ.

In particular, the continuity equation

$$\partial_t \rho + \nabla \cdot (\rho v) = 0$$

and the phase evolution equation

$$\partial_t \phi + v \cdot \nabla \phi = \mathcal{E}(x, t)$$

are satisfied if and only if the corresponding relations obtained by substitution of

$$\rho = |\Psi|^2, \qquad \phi = \Phi_0 \arg(\Psi)$$

are satisfied.

Closure and coherence conditions in the Q-series are expressed in terms of accumulated phase along transport paths. Since the phase is preserved under the encoding, these conditions remain unchanged.

Finally, no new variables or relations are introduced in the construction of Ψ, and no existing variables are removed. The encoding is therefore conservative with respect to the exchange-sector structure.

□

5.4 Representation Faithfulness

The preceding theorem shows that the complex state representation is not merely a convenient relabeling, but a faithful encoding of the underlying transport structure.

Proposition 5.2 (Faithfulness of the representation)**.** *The mapping* $(\rho, \phi) \mapsto \Psi$ *is structure-preserving in the sense that all dynamical, closure, and coherence relations of the exchange sector are invariant under the encoding.*

5.5 Proof of Faithfulness

Since all such relations are formulated in terms of ρ and ϕ, and these quantities are recoverable from Ψ, the relations are invariant under substitution. No structural information is altered by the encoding.

□

5.6 Discussion

The structural preservation result establishes that the complex state Ψ carries exactly the same physical content as the pair (ρ, ϕ).

In particular:

- The representation does not introduce wave behavior; it encodes existing phase structure.
- The representation does not introduce probability; it encodes closure density.
- The representation does not alter transport dynamics; it reorganizes their expression.

Accordingly, the complex state should be interpreted as a *faithful representation* of the transport closure system rather than as a new physical entity.

5.7 Transition to Scope Considerations

While the encoding preserves local structure, the phase itself originates from path-dependent transport accumulation. It is therefore necessary to clarify the precise domain over which the complex state representation is valid.

This is addressed in the following section.

6 Local Scope and Path-Dependent Phase Structure

6.1 Need for a Scope Theorem

The transport-derived phase $\phi(x,t)$ introduced in the Q-series arises from the accumulation of exchange interaction along admissible transport paths. As such, it is fundamentally defined as a path-dependent quantity.

In the preceding sections, $\phi(x,t)$ has been treated as a scalar field, enabling the construction of a local state representation (ρ, ϕ) and its complex encoding Ψ. However, the validity of this scalar representation depends on the integrability of the underlying phase accumulation.

It is therefore necessary to make precise the conditions under which $\phi(x,t)$ exists as a well-defined scalar field and to clarify the scope of the resulting state representation.

6.2 Local Representability of Transport Phase

Lemma 6.1 (Local representability of transport phase)**.** *Let $U \subset \mathcal{M}$ be a simply connected region in which the transport-derived phase increment is locally path-independent. Then there exists a scalar field*

$$\phi : U \times \mathbb{R} \to \mathbb{R}$$

such that, for any admissible transport path γ contained in U,

$$\phi\big(x_2, t_2\big) - \phi\big(x_1, t_1\big)$$

reproduces the accumulated phase along γ between (x_1, t_1) and (x_2, t_2).

Consequently, within U, the transport-derived phase admits a consistent local scalar representation.

6.3 Proof of the Local Representability Lemma

The transport-derived phase is defined through accumulation of a locally defined phase increment along admissible transport paths. By assumption, this increment is locally path-independent in U, so that the accumulated phase depends only on the endpoints of the path.

It follows that the phase increment defines an exact differential in U, and therefore there exists a scalar function ϕ whose differential reproduces the local phase increment.

The simply connectedness of U ensures that this scalar function is globally well-defined within the region.

□

6.4 Local State Representation

The preceding lemma justifies the use of $\phi(x, t)$ as a scalar field within integrable regions. Accordingly, in such regions, the pair (ρ, ϕ) defines a well-posed local state description, and the complex state

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}$$

is well-defined.

We therefore interpret $\Psi(x, t)$ as a *local state representation*, valid within regions where the phase admits a consistent scalar encoding.

6.5 The Local-to-Global Scope Theorem

Theorem 6.2 (Scope of the state representation)**.** *The complex state representation $\Psi(x, t)$ is an exact encoding of the exchange-sector transport state on every region in which the transport-derived phase admits a consistent local scalar representation.*

In regions where the phase accumulation is globally path-dependent, the scalar field $\phi(x, t)$ serves as a local chart of the phase structure, and the corresponding complex state Ψ should be interpreted as a local state representation rather than a single globally trivial phase field.

6.6 Proof of the Scope Theorem

The first statement follows directly from the Local Representability Lemma, which ensures the existence of a scalar phase field in integrable regions, and from the Complex Encoding Theorem, which guarantees that Ψ provides a lossless encoding of (ρ, ϕ).

In regions where the phase accumulation depends on path, a single globally defined scalar field cannot represent the phase consistently. However, the local construction remains valid on sufficiently small or appropriately restricted regions, where the phase increment is locally exact.

Thus the complex state Ψ is well-defined as a local encoding of the transport state, even in the presence of nontrivial global phase structure.

□

6.7 Interpretive Consequences

The scope theorem clarifies that the state representation introduced in this paper is inherently local in character.

- The complex state $\Psi(x, t)$ is a local encoding of transport structure, not a globally defined wave field in the presence of nontrivial phase holonomy.

- Global phase structure may require patching of local representations or the inclusion of additional compatibility data associated with transport cycles.
- The local nature of the state representation is consistent with the underlying transport framework, in which phase is fundamentally defined through path accumulation.

This clarification does not weaken the state representation, but rather specifies its precise domain of validity and prepares the framework for later developments in which global structure may be treated explicitly.

6.8 Transition to Representation Choice

Having established the domain of validity of the state representation, it remains to justify the specific form of the complex encoding introduced in Section 4. In particular, one must show that this encoding is not merely admissible, but natural for the transport structure under consideration.

This is addressed in the following section.

7 Naturalness of the Complex State Form

7.1 Statement of the Representation Problem

The preceding sections established that the local transport state is fully specified by the pair (ρ, ϕ) and that this pair admits a lossless complex encoding

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}.$$

While this encoding is admissible, it is not yet clear whether it is preferred among possible representations. The present section addresses the following question:

Why is the complex exponential form a natural representation of the transport state?

The goal is not to assert uniqueness of complex numbers as a physical structure, but to identify the minimal representational properties required by the transport closure system and to show that the complex form satisfies them.

7.2 Requirements for a Single-Object State Encoding

Any representation of the local transport state by a single object must encode both:

1. a nonnegative magnitude field $\rho(x, t)$, and
2. an additive phase field $\phi(x, t)$.

In addition, the representation should respect the structural role of phase in the transport system. In particular, phase accumulation along transport paths is additive, so the representation should preserve this additive structure in a simple algebraic form.

We therefore require that a candidate representation satisfy:

1. **Recoverability:** both ρ and ϕ can be recovered from the representation,
2. **Additive compatibility:** phase addition is represented by a simple composition law,
3. **Locality:** the representation is defined pointwise in spacetime,
4. **Compatibility with transport dynamics:** the representation admits a reformulation of the transport law without introducing additional structure.

7.3 Exponential Encoding of Additive Phase

The additive structure of phase strongly constrains admissible encodings. Let ϕ_1 and ϕ_2 be two phase contributions. Then

$$\phi_{\text{total}} = \phi_1 + \phi_2.$$

A representation that maps additive phase into multiplicative composition is therefore natural. The exponential map provides precisely this property:

$$e^{i(\phi_1+\phi_2)/\Phi_0} = e^{i\phi_1/\Phi_0}\, e^{i\phi_2/\Phi_0}.$$

This converts additive phase accumulation into multiplicative structure within a single object.

7.4 Construction of the Minimal Encoding

To incorporate magnitude, we associate the nonnegative scalar field ρ with an amplitude $\sqrt{\rho}$. Combining amplitude and phase yields the complex-valued representation

$$\Psi = \sqrt{\rho}\, e^{\,i\phi/\Phi_0}.$$

This construction satisfies the required properties:

1. $\rho = |\Psi|^2$ ensures recoverability of the magnitude,
2. $\phi = \Phi_0 \arg(\Psi)$ ensures recoverability of the phase,
3. phase addition corresponds to multiplication of complex factors,
4. the representation is defined locally and algebraically.

7.5 Natural Minimal Encoding Lemma

Lemma 7.1 (Natural minimal complex encoding)**.** *Among single-object encodings of a nonnegative magnitude field ρ and an additive phase field ϕ, the representation*

$$\Psi = \sqrt{\rho}\, e^{\,i\phi/\Phi_0}$$

is a natural minimal encoding in the sense that it satisfies recoverability, preserves additive phase structure through multiplicative composition, and introduces no additional degrees of freedom beyond those already present in (ρ, ϕ).

7.6 Proof of the Natural Minimal Encoding Lemma

The recoverability conditions follow directly from the modulus and argument of Ψ.

The exponential map provides the simplest algebraic realization of additive phase through multiplication, and any alternative encoding that preserves additive structure must reproduce this behavior up to isomorphism.

The representation introduces no additional independent variables, since Ψ is fully determined by (ρ, ϕ) and vice versa.

Therefore the complex exponential form provides a minimal encoding satisfying the required properties.

□

7.7 Remarks on Non-Uniqueness

The preceding lemma establishes naturalness and minimality, but not strict uniqueness.

Alternative representations related by invertible transformations may exist. The present claim is only that the complex exponential form is the simplest encoding that simultaneously:

- captures magnitude and phase,
- preserves additive phase structure,
- supports algebraic manipulation of transport relations.

This level of justification is sufficient for its role as a bridge to later representation structures.

7.8 Relation to Subsequent Developments

The algebraic properties of the complex representation are essential for the reformulation of the transport closure system into a linear evolution equation, as established in the preceding Q-series. The use of a complex state is therefore not an arbitrary choice, but one aligned with the structure of the transport law itself.

7.9 Transition to Dynamical Compatibility

Having established the naturalness of the complex state representation, it remains to connect this representation explicitly to the dynamical form obtained in the Q-series.

This is addressed in the following section.

8 Compatibility with Schrödinger-Type Dynamics

8.1 Recall of the Preceding Result

The Q-series established that the transport closure system admits, under appropriate structural conditions, a representation in which the coupled evolution of closure density and phase may be expressed as a single complex-valued evolution equation [5, 6].

In that formulation, the transport system derived from closure, coherence, and exchange interaction was shown to admit a Schrödinger-type representation [9], without the introduction of wave ontology, probabilistic interpretation, or operator formalism.

The purpose of the present section is not to rederive this result, but to record its significance for the state representation constructed in the preceding sections.

8.2 Dynamic Compatibility Theorem

Theorem 8.1 (Dynamic compatibility). *Let* $\Psi(x,t) = \sqrt{\rho(x,t)}\, e^{i\phi(x,t)/\Phi_0}$ *be the complex state representation constructed from the local transport state* (ρ, ϕ).

Then, under the structural conditions identified in the Q-series, the evolution of Ψ *is governed by a Schrödinger-type equation of the form*

$$i\Phi_0 \partial_t \Psi = \mathcal{H}[\Psi],$$

where $\mathcal{H}$ *is a differential operator determined by the transport law and scalar geometry.*

Accordingly, the state representation Ψ *is dynamically compatible with the Schrödinger-type formulation of the transport closure system.*

8.3 Proof Sketch

The Q-series established that the coupled evolution equations for ρ and ϕ may be combined into a single complex-valued evolution equation through the substitution

$$\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}.$$

In that construction, the continuity equation and phase evolution equation are rewritten in terms of Ψ, yielding a second-order differential equation whose structure matches that of the Schrödinger equation.

Since the present paper has shown that Ψ is a lossless encoding of (ρ, ϕ) and preserves all transport structure, the same reformulation applies here without modification.

The resulting equation is therefore a representation of the underlying transport closure system, not an independently postulated dynamical law.

□

8.4 Interpretive Significance

The dynamic compatibility result establishes that the complex state representation introduced in this paper is precisely the object required to express the transport closure system in Schrödinger-type form.

This has several important consequences:

- The appearance of a Schrödinger-type equation is not an additional assumption, but a representation of the underlying transport dynamics.
- The complex state Ψ is not introduced to reproduce quantum mechanics, but arises naturally from the structure of transport closure.
- The standard wave-mechanical formalism is therefore interpreted as an encoding of a deeper geometric transport system.

8.5 Limitations of the Present Result

While the present construction establishes compatibility with Schrödinger-type dynamics, it does not yet provide:

- a probabilistic interpretation of $|\Psi|^2$,
- a measurement postulate,
- an operator formalism for observables.

These elements are not required for the present result and will be addressed in subsequent papers in the QB-series.

8.6 Transition to QB2

The present paper establishes the appropriate state representation for the transport closure system. The next step is to determine how observable quantities arise from this representation.

In particular, the subsequent paper [10] will examine the emergence of operator structure from transport generators and phase gradients, thereby connecting the present state representation to the standard formalism of quantum mechanics.

9 Interpretive Clarifications and Exclusions

9.1 No Wave Ontology

The complex state Ψ is not introduced as a physical wave or oscillatory medium. It is a representation of closure density and transport-derived phase and carries no independent ontological status.

9.2 No Probabilistic Interpretation

The quantity $|\Psi|^2$ represents closure density and is not interpreted as a probability density in the present work. No statistical interpretation or measurement postulate is assumed.

9.3 No Operator Formalism

No operator formalism is introduced in this paper. All quantities are defined directly in terms of transport and scalar geometry. Operator structure, if present, must emerge from this framework and is not assumed a priori.

9.4 Role of the Present Paper

The present paper establishes the state-representational bridge between the transport closure system of the Q-series and the later formal developments of quantum mechanics. It does not claim equivalence with quantum theory, but provides the minimal structure required for such a connection to be developed.

10 Conclusion

10.1 Summary of Results

We have established that the exchange-sector transport system admits a minimal local state description in terms of closure density and transport-derived phase, and that this state may be encoded losslessly as a single complex-valued function.

We have shown that this encoding preserves all closure, coherence, and transport structure derived in the Q-series, and have clarified the local scope of the representation in the presence of path-dependent phase accumulation.

We have further demonstrated that the complex state representation is naturally suited to the transport system and is dynamically compatible with the Schrödinger-type formulation obtained previously.

10.2 Programmatic Significance

The results of this paper provide the foundational bridge between geometric transport closure and quantum-mechanical state representation. The complex state Ψ emerges not as a primitive object, but as a compact encoding of transport structure.

This establishes the appropriate starting point for the introduction of observable structure and formal quantum-mechanical tools in subsequent work.

10.3 Next Steps

The next paper in the QB-series will develop the emergence of observable structure from the transport system. In particular, it will show how momentum, energy, and other operators arise from transport generators and phase gradients, thereby completing the transition from geometric transport closure to quantum-mechanical formalism.

A Dependence on Prior Q-Series Results

The present paper depends on the following results from the Q-series:

- the definition of closure density and its transport law,
- the introduction of transport-derived phase,
- the formulation of a unified transport law,
- the demonstration that the transport system admits a Schrödinger-type representation.

No additional dynamical assumptions are introduced in the present work.

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