QB4 – Closure Partitions and Projector-Based Event Structure in Scalar–Conformal NUVO Systems

Preprint, Version 1.0[*]

Rickey W. Austin
St Claire Scientific Research, Development, and Publishing

Abstract

Building on the pre-Hilbert structure established in QB3, we derive a projector-based event framework for the hydrogenic sector of scalar–conformal NUVO systems. Interaction-induced constraints are modeled as selectors acting on the closure hierarchy, partitioning the set of closure classes into mutually exclusive outcome channels.

We show that these closure-class partitions induce an orthogonal decomposition of the representational space, giving rise to a family of projection operators. These projectors are demonstrated to be intrinsic to the closure structure and independent of the specific interaction context used to realize them.

Extending this construction, we show that the inner product structure supports the full algebra of orthogonal projectors, yielding a complete event space. Events are thus identified with subspaces of the representational space, and exclusivity is encoded through orthogonality.

No probabilistic interpretation is assumed. The resulting projector algebra provides the structural foundation required for the formulation of consistent weight assignments in subsequent work.

1 Introduction

In the preceding paper [1], we established that the hydrogenic stationary sector of scalar–conformal NUVO systems admits a natural complex pre-Hilbert structure derived from its holonomic coherence properties. In particular, stationary closure modes were shown to form an orthonormal family with respect to an inner product induced by an invariant coherence functional. This construction was obtained without invoking probabilistic or measurement-theoretic assumptions.

The existence of a comparison structure raises a natural next question: how are physically admissible outcomes represented within this framework? In standard formulations of quantum theory, measurement outcomes are associated with subspaces of a Hilbert space and are represented by orthogonal projectors. In the present work, we do not assume such a structure a priori. Instead, we seek to derive an analogous event structure directly from the closure and coherence properties of the hydrogenic sector.

The key idea is to consider interaction-induced constraints on the system. Physical interactions impose geometric and structural conditions on admissible configurations, restricting the set of closure-compatible outcomes. These constraints act to partition the hydrogenic closure-class

---

[*] Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

structure into mutually exclusive classes, each corresponding to a distinct class of admissible post-interaction configurations.

The central objective of this paper is to show that such closure-class partitions induce a natural family of subspaces of the representational space constructed in Paper 1. These subspaces, in turn, give rise to a collection of orthogonal projectors. In this way, we obtain a projector-based event structure derived from the intrinsic geometry of the system rather than introduced as a postulate.

It is important to emphasize that no probabilistic interpretation is introduced at this stage. The projectors constructed here represent admissible outcome classes, but no weights are assigned to them. The goal of this paper is purely structural: to establish the correspondence

$$\text{closure partitions} \longrightarrow \text{subspaces} \longrightarrow \text{projectors}.$$

A further essential requirement is that the identification of outcomes be independent of the specific interaction context. That is, if two different interaction configurations induce the same partition of the closure-class structure, they must correspond to the same subspace and hence the same projector. This context-independence property will be established within the hydrogenic spectral sector.

The results of this paper provide the event-level structure required for the subsequent development of a consistent weighting schemee [2]. In particular, they supply the projector algebra on which admissible weight assignments can be defined. In the next stage of the program, we will show that consistency conditions on such weight assignments lead, via a Gleason-type argument, to a unique form of weighting determined by the inner product structure.

The organization of the paper is as follows. In Section 2, we recall the representational structure of the hydrogenic sector established in Paper 1. Section 3 introduces interaction selectors as geometric constraints on admissible closure configurations. In Section 4, we show that such selectors induce partitions of the closure-class set. Section 5 constructs the corresponding subspaces of the representational space. In Section 6, we define the associated projectors and establish their basic properties. Section 7 proves the context-independence of the resulting projector structure. Section 8 extends the construction to the full projector algebra of the representational space. Sections 9 and 10 discuss interpretation and outline the transition to the development of probabilistic structure in subsequent work.

2 Hydrogenic Representational Structure

We briefly recall the structural results established in the preceding paper that will be used throughout the present work. The focus here is on the representational space associated with the hydrogenic stationary sector and its induced comparison structure.

2.1 Stationary Closure Modes

The hydrogenic system admits a discrete set of stationary closure classes indexed by

$$\mathcal{I}_H = \{n\},$$

where each n labels a closure-compatible exchange configuration.

To each closure class, there corresponds a represented stationary mode

$$\Psi_n = \sqrt{\rho_n}\, e^{i\phi_n/\Phi_0},$$

where ρ_n is the closure density and ϕ_n is the transport-derived phase associated with the stationary configuration.

Relation to the scalar-modulated closure functional. The discrete closure classes indexing the hydrogenic stationary sector originate from the scalar-modulated return condition established in Q2 [3],

$$k \oint_{\gamma} \lambda_{\text{eff}}(x,u)\, ds = L_{\gamma}.$$

In the stationary hydrogenic regime, this condition manifests as a discrete hierarchy of closure-compatible configurations, represented here by the index set $\mathcal{I}_H$. The partition structure developed in this work therefore acts on a hierarchy ultimately determined by the underlying scalar-modulated closure functional.

2.2 Orthonormal Structure

In Paper 1, it was shown that the stationary modes $\{\Psi_n\}$ form an orthonormal family with respect to the inner product

$$\langle \Psi_m, \Psi_n \rangle_H = \int_{\Gamma_H} \Psi_m^* \Psi_n \, d\mu_H,$$

where $d\mu_H$ is a measure invariant under the hydrogenic return map.

In particular,

$$\langle \Psi_m, \Psi_n \rangle_H = \delta_{mn}.$$

This orthogonality arises from the distinct eigencharacter structure of the modes under the return map and does not rely on any probabilistic interpretation.

2.3 Representational Space

We define the hydrogenic representational space as

$$\mathcal{V}_H^{\text{fin}} := \operatorname{span}_{\mathbb{C}}\{\Psi_n\}.$$

Elements of $\mathcal{V}_H^{\text{fin}}$ are finite linear combinations of stationary modes:

$$\Phi = \sum_n c_n \Psi_n, \qquad c_n \in \mathbb{C}.$$

The inner product extends to this space by sesquilinearity, yielding

$$\langle \Phi, \Xi \rangle_H = \sum_n \overline{c_n} d_n,$$

for $\Xi = \sum_n d_n \Psi_n$.

2.4 Pre-Hilbert Structure

The space $\mathcal{V}_H^{\text{fin}}$, equipped with the inner product $\langle \cdot, \cdot \rangle_H$, forms a complex pre-Hilbert space. The stationary modes $\{\Psi_n\}$ provide an orthonormal basis for this space.

We emphasize that this structure is derived from holonomic coherence and return-map symmetry, and is not assumed as an independent postulate.

2.5 Representational Scope

The space $\mathcal{V}_H^{\text{fin}}$ serves as a comparison space for analyzing coherence relations and structural properties of the hydrogenic sector. While it admits arbitrary linear combinations of stationary modes, we do not assume that every element of this space corresponds to a physically realized configuration.

Instead, physical admissibility remains governed by closure compatibility conditions. The representational space provides a framework in which structural relations, such as orthogonality and decomposition, can be expressed and analyzed.

2.6 Role in the Present Work

In the sections that follow, we will use the structure of $\mathcal{V}_H^{\text{fin}}$ to represent outcome classes induced by interaction constraints. Specifically, partitions of the closure-class set will be shown to define subspaces of $\mathcal{V}_H^{\text{fin}}$, which will in turn give rise to a family of orthogonal projectors.

This provides the bridge from closure structure to event structure that is the focus of the present paper.

3 Interaction Selectors

In order to relate the representational structure of the hydrogenic sector to physically admissible outcomes, we introduce the notion of interaction selectors. These objects model the effect of external constraints on the set of closure-compatible configurations, without invoking probabilistic or statistical assumptions.

3.1 Interaction-Induced Constraints

Physical interactions impose geometric and structural constraints on admissible configurations of the system. In the NUVO framework, such constraints act by restricting the set of closure-compatible exchange configurations that can be realized under the interaction.

Crucially, these constraints do not create new closure configurations. Rather, they restrict the admissible subset of the existing closure-class structure.

3.2 Definition of Interaction Selector

Definition 3.1 (Interaction selector)**.** *An interaction selector $\mathcal{S}$ is a configuration of external constraints acting on the hydrogenic system such that:*

- *it restricts the set of admissible closure-compatible configurations,*
- *it partitions the admissible configurations into mutually exclusive outcome classes,*
- *and each admissible interaction realization resolves into exactly one such outcome class.*

Thus, an interaction selector acts as a structural filter on the closure hierarchy, organizing admissible configurations into distinct classes.

This partition property reflects the requirement that a single interaction realization resolves the system into a uniquely determined closure-compatible configuration, ensuring that the outcome structure is well-defined at the level of closure compatibility.

3.3 Hydrogenic Spectral Selectors

In this work, we restrict attention to a particular class of selectors.

Definition 3.2 (Hydrogenic spectral selector). *A hydrogenic spectral selector is an interaction selector whose constraints depend only on closure compatibility within the hydrogenic sector. In particular, it does not resolve configurations beyond their classification by closure class.*

This restriction ensures that the outcome structure is determined solely by the intrinsic closure hierarchy, without dependence on additional degrees of freedom.

3.4 Outcome Channels

Definition 3.3 (Outcome channel). *An outcome channel of a selector $\mathcal{S}$ is an equivalence class of admissible configurations that are indistinguishable under the constraints imposed by $\mathcal{S}$.*

Each outcome channel represents a class of configurations that are compatible with the same interaction constraints and therefore correspond to the same structural outcome.

3.5 Closure-Class Compatibility

Given a selector $\mathcal{S}$ and an outcome channel C_α, we define the associated set of closure classes:

$$I_\alpha := \{n \in \mathcal{I}_H \mid \text{closure class } n \text{ is compatible with } C_\alpha\}.$$

Thus, each outcome channel corresponds to a subset of the hydrogenic closure-class index set.

3.6 Interpretation

The selector framework provides a structural description of how interaction constraints organize the hydrogenic closure hierarchy into distinct outcome classes.

It is important to emphasize that:

- selectors do not assign probabilities,
- outcome channels do not carry weights,
- and no statistical interpretation is introduced.

Instead, the role of selectors is purely to define a partition of admissible configurations. In the following section, we will show that this partition induces a corresponding partition of the closure-class index set, which forms the basis for the construction of subspaces and projectors.

4 Closure-Class Partitions

In this section, we show that hydrogenic spectral selectors induce a canonical partition of the closure-class index set. This result provides the bridge from interaction constraints to the representational structure developed in Section 2.

4.1 Closure-Class Index Set

We recall that the hydrogenic stationary sector is organized by a discrete set of closure classes

$$\mathcal{I}_H = \{n\},$$

where each n labels a closure-compatible exchange configuration.

4.2 Selector-Induced Subsets

Let $\mathcal{S}$ be a hydrogenic spectral selector, and let $\{C_\alpha\}_{\alpha \in A}$ denote its outcome channels.

For each outcome channel C_α, define the associated subset of closure classes

$$I_\alpha := \{n \in \mathcal{I}_H \mid \text{closure class } n \text{ is compatible with } C_\alpha\}.$$

Thus, each outcome channel determines a subset of the closure-class index set.

4.3 Selector–Closure Correspondence

We now establish the central structural result of this section.

Lemma 4.1 (Selector–closure correspondence)**.** *Let $\mathcal{S}$ be a hydrogenic spectral selector with outcome channels $\{C_\alpha\}$. Then the associated subsets $\{I_\alpha\}$ form a partition of the closure-class index set:*

$$\mathcal{I}_H = \bigsqcup_{\alpha \in A} I_\alpha.$$

Proof. We prove disjointness and completeness.

Disjointness. By definition of an interaction selector (Section 3), each admissible configuration resolves into exactly one outcome channel. Suppose, for contradiction, that there exists a closure class n such that

$$n \in I_\alpha \cap I_\beta \quad \text{with } \alpha \neq \beta.$$

Then closure class n would be compatible with two distinct outcome channels under the same selector, contradicting the uniqueness of outcome resolution. Therefore,

$$I_\alpha \cap I_\beta = \varnothing \quad \text{for } \alpha \neq \beta.$$

Completeness. By definition, a hydrogenic spectral selector acts on the entire hydrogenic stationary sector and partitions the admissible configurations into outcome channels. Since every closure class corresponds to an admissible configuration in the stationary sector, each $n \in \mathcal{I}_H$ must be compatible with at least one outcome channel. Hence,

$$\bigcup_{\alpha \in A} I_\alpha = \mathcal{I}_H.$$

Combining disjointness and completeness yields

$$\mathcal{I}_H = \bigsqcup_{\alpha \in A} I_\alpha.$$

□

4.4 Uniqueness of the Induced Partition

Lemma 4.2 (Uniqueness of partition)**.** *For a fixed hydrogenic spectral selector $\mathcal{S}$, the induced partition $\{I_\alpha\}$ is uniquely determined.*

Proof. The partition is determined entirely by the compatibility relation between closure classes and the outcome channels defined by the selector $\mathcal{S}$. Since the selector constraints are fixed, the compatibility relation is fixed, and therefore the induced subsets I_α are uniquely determined. $\square$

4.5 Interpretation

The selector–closure correspondence shows that interaction constraints act at the level of closure classes by partitioning the hydrogenic closure hierarchy into mutually exclusive subsets.

Each outcome channel corresponds not to a single configuration, but to a class of closure-compatible configurations grouped by the selector constraints. The resulting partition reflects a structural organization of the closure hierarchy induced by interaction.

4.6 Role in Subsequent Construction

The partition

$$\mathcal{I}_H = \bigsqcup_{\alpha} I_\alpha$$

provides the key input for the next stage of the construction. In the following section, we will show that each subset I_α defines a subspace of the representational space $\mathcal{V}_H^{\mathrm{fin}}$, and that these subspaces form an orthogonal decomposition.

This will allow the construction of projectors associated with outcome channels, completing the transition from closure structure to event structure.

5 Induced Subspaces

In the previous section, we showed that a hydrogenic spectral selector induces a partition of the closure-class index set

$$\mathcal{I}_H = \bigsqcup_{\alpha \in A} I_\alpha.$$

We now lift this partition to the representational space $\mathcal{V}_H^{\mathrm{fin}}$, thereby obtaining a corresponding family of subspaces.

5.1 Subspaces Associated with Closure Partitions

Definition 5.1 (Selector-induced subspace)**.** *For each subset $I_\alpha \subseteq \mathcal{I}_H$, define the associated subspace*

$$\mathcal{H}_\alpha := \mathrm{span}_{\mathbb{C}}\{\Psi_n \mid n \in I_\alpha\}.$$

Thus, each outcome channel of the selector corresponds to a subspace of $\mathcal{V}_H^{\mathrm{fin}}$ generated by the stationary modes compatible with that channel.

5.2 Orthogonality of Subspaces

Lemma 5.2. *If $\alpha \neq \beta$, then*

$$\mathcal{H}_\alpha \perp \mathcal{H}_\beta.$$

Proof. Let $\Phi \in \mathcal{H}_\alpha$ and $\Xi \in \mathcal{H}_\beta$. Then

$$\Phi = \sum_{n \in I_\alpha} a_n \Psi_n, \qquad \Xi = \sum_{m \in I_\beta} b_m \Psi_m.$$

Using the inner product,

$$\langle \Phi, \Xi \rangle_H = \sum_{n \in I_\alpha} \sum_{m \in I_\beta} \overline{a_n} b_m \langle \Psi_n, \Psi_m \rangle_H.$$

Since $\{\Psi_n\}$ is an orthonormal family (Section 2), we have

$$\langle \Psi_n, \Psi_m \rangle_H = \delta_{nm}.$$

But $I_\alpha \cap I_\beta = \varnothing$ for $\alpha \neq \beta$, so no term satisfies $n = m$. Therefore,

$$\langle \Phi, \Xi \rangle_H = 0.$$

□

5.3 Direct Sum Decomposition

Theorem 5.3 (Orthogonal decomposition)**.** *The representational space decomposes as an orthogonal direct sum:*

$$\mathcal{V}_H^{\mathrm{fin}} = \bigoplus_{\alpha \in A} \mathcal{H}_\alpha.$$

Proof. We prove that every element of $\mathcal{V}_H^{\mathrm{fin}}$ can be uniquely decomposed into components in the subspaces $\mathcal{H}_\alpha$.

Let $\Phi \in \mathcal{V}_H^{\mathrm{fin}}$, with

$$\Phi = \sum_{n \in \mathcal{I}_H} c_n \Psi_n.$$

Using the partition $\mathcal{I}_H = \bigsqcup_\alpha I_\alpha$, we may rewrite

$$\Phi = \sum_{\alpha \in A} \left(\sum_{n \in I_\alpha} c_n \Psi_n \right).$$

Define

$$\Phi_\alpha := \sum_{n \in I_\alpha} c_n \Psi_n \in \mathcal{H}_\alpha.$$

Thus,

$$\Phi = \sum_{\alpha \in A} \Phi_\alpha.$$

Orthogonality of the subspaces (previous lemma) ensures that this decomposition is orthogonal. Uniqueness follows from the orthonormality of the basis $\{\Psi_n\}$.

Therefore,

$$\mathcal{V}_H^{\mathrm{fin}} = \bigoplus_{\alpha \in A} \mathcal{H}_\alpha.$$

□

5.4 Interpretation

The decomposition above shows that the partition of closure classes induced by a selector translates directly into a decomposition of the representational space into orthogonal subspaces.

Each subspace $\mathcal{H}_\alpha$ corresponds to a distinct class of closure-compatible configurations determined by the interaction constraints. The orthogonality of these subspaces reflects the incompatibility of different closure-class subsets under the induced comparison structure.

5.5 Role in Projector Construction

The orthogonal decomposition

$$\mathcal{V}_H^{\text{fin}} = \bigoplus_\alpha \mathcal{H}_\alpha$$

provides the foundation for the construction of projection operators associated with outcome channels. In the next section, we will define these projectors explicitly and establish their algebraic properties.

This completes the transition from closure partitions to a subspace structure in the representational space.

6 Projector Realization

In the previous section, we showed that a hydrogenic spectral selector induces an orthogonal decomposition

$$\mathcal{V}_H^{\text{fin}} = \bigoplus_{\alpha \in A} \mathcal{H}_\alpha,$$

where each subspace $\mathcal{H}_\alpha$ corresponds to a subset of closure classes. We now construct projection operators associated with these subspaces and establish their algebraic properties.

6.1 Definition of Projectors

Definition 6.1 (Selector-induced projector)**.** *Let $\mathcal{H}_\alpha \subseteq \mathcal{V}_H^{\text{fin}}$ be a subspace associated with a closure-class subset I_α. The projector onto $\mathcal{H}_\alpha$ is the linear operator*

$$P_\alpha : \mathcal{V}_H^{\text{fin}} \to \mathcal{V}_H^{\text{fin}}$$

defined by

$$P_\alpha \left(\sum_{n \in \mathcal{I}_H} c_n \Psi_n \right) = \sum_{n \in I_\alpha} c_n \Psi_n.$$

Thus, P_α extracts the component of a vector lying in the subspace $\mathcal{H}_\alpha$.

6.2 Basic Properties

Theorem 6.2 (Projector properties)**.** *The operators $\{P_\alpha\}$ satisfy:*

1. *Idempotence:*

$$P_\alpha^2 = P_\alpha.$$

2. *Self-adjointness:*

$$\langle P_\alpha \Phi, \Xi \rangle_H = \langle \Phi, P_\alpha \Xi \rangle_H \quad \textit{for all } \Phi, \Xi \in \mathcal{V}_H^{\mathrm{fin}}.$$

3. *Orthogonality:*

$$P_\alpha P_\beta = 0 \quad \textit{for } \alpha \neq \beta.$$

4. *Completeness:*

$$\sum_{\alpha \in A} P_\alpha = I,$$

where I is the identity operator on $\mathcal{V}_H^{\mathrm{fin}}$.

(1) Idempotence. Applying P_α twice,

$$P_\alpha^2 \left(\sum_n c_n \Psi_n \right) = P_\alpha \left(\sum_{n \in I_\alpha} c_n \Psi_n \right) = \sum_{n \in I_\alpha} c_n \Psi_n = P_\alpha \left(\sum_n c_n \Psi_n \right).$$

(2) Self-adjointness. Let $\Phi = \sum_n a_n \Psi_n$ and $\Xi = \sum_n b_n \Psi_n$. Then

$$\langle P_\alpha \Phi, \Xi \rangle_H = \left\langle \sum_{n \in I_\alpha} a_n \Psi_n, \ \sum_m b_m \Psi_m \right\rangle_H = \sum_{n \in I_\alpha} \overline{a_n} b_n.$$

Similarly,

$$\langle \Phi, P_\alpha \Xi \rangle_H = \sum_{n \in I_\alpha} \overline{a_n} b_n.$$

Thus, the two expressions are equal.

(3) Orthogonality. For $\alpha \neq \beta$, we have $I_\alpha \cap I_\beta = \varnothing$. Hence,

$$P_\alpha P_\beta \left(\sum_n c_n \Psi_n \right) = P_\alpha \left(\sum_{n \in I_\beta} c_n \Psi_n \right) = 0.$$

(4) Completeness. For any $\Phi = \sum_n c_n \Psi_n$, we have

$$\sum_\alpha P_\alpha \Phi = \sum_\alpha \sum_{n \in I_\alpha} c_n \Psi_n.$$

Since the sets $\{I_\alpha\}$ form a partition of $\mathcal{I}_H$, every index n appears in exactly one subset. Therefore,

$$\sum_\alpha P_\alpha \Phi = \sum_n c_n \Psi_n = \Phi,$$

which implies

$$\sum_\alpha P_\alpha = I.$$

□

6.3 Projectors as Subspace Maps

Each projector P_α is uniquely associated with the subspace $\mathcal{H}_\alpha$. In particular,

$$\mathrm{Im}(P_\alpha) = \mathcal{H}_\alpha, \qquad \ker(P_\alpha) = \bigoplus_{\beta \neq \alpha} \mathcal{H}_\beta.$$

Thus, the family $\{P_\alpha\}$ provides a complete algebraic representation of the subspace decomposition induced by the selector.

6.4 Interpretation

The projectors constructed above represent the structural outcome classes induced by the interaction selector. Each P_α corresponds to a distinct subset of closure classes and acts by isolating the component of a state compatible with that subset.

It is important to emphasize that these projectors are derived from closure partitions and the induced inner product structure. They are not introduced as primitive objects or postulated independently.

6.5 Role in Event Structure

The family $\{P_\alpha\}$ provides a representation of mutually exclusive outcome classes in the hydrogenic sector. In the next section, we will show that this representation is independent of the particular selector used to induce the partition, provided the resulting closure-class subsets coincide.

This context-independence property will ensure that the projectors represent intrinsic structural features of the system rather than artifacts of specific interaction configurations.

7 Context Independence

In the previous sections, we constructed projectors associated with closure-class partitions induced by interaction selectors. We now establish that these projectors depend only on the induced partition of the closure-class index set and not on the specific form of the selector that generates it.

This property ensures that the resulting event structure is intrinsic to the hydrogenic sector and not tied to particular interaction realizations.

7.1 Equivalent Selectors

Definition 7.1 (Equivalent selectors). *Two hydrogenic spectral selectors $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ are said to be equivalent if they induce the same partition of the closure-class index set:*

$$\mathcal{I}_H = \bigsqcup_{\alpha \in A} I_\alpha.$$

Thus, equivalent selectors may differ in their physical realization or constraint mechanisms, but they organize the closure classes into the same subsets.

7.2 Equality of Induced Subspaces

Lemma 7.2. *If two selectors $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ are equivalent, then they induce the same family of subspaces $\{\mathcal{H}_\alpha\}$.*

Proof. Proof. By definition, the subspace associated with a subset I_α is

$$\mathcal{H}_\alpha = \operatorname{span}\{\Psi_n : n \in I_\alpha\}.$$

If the two selectors induce the same subsets $\{I_\alpha\}$, then the corresponding subspaces are identical by construction. □

7.3 Context-Independence of Projectors

We now establish the main result of this section.

Theorem 7.3 (Context independence of projectors)**.** *Let $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ be equivalent hydrogenic spectral selectors. Let $P_\alpha^{(1)}$ and $P_\alpha^{(2)}$ be the corresponding projectors onto the subspaces $\mathcal{H}_\alpha$. Then*

$$P_\alpha^{(1)} = P_\alpha^{(2)}.$$

Proof. From the previous lemma, the subspaces $\mathcal{H}_\alpha$ induced by $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ are identical.

The projector P_α is uniquely determined by its action as the orthogonal projection onto $\mathcal{H}_\alpha$ with respect to the inner product $\langle \cdot, \cdot \rangle_H$.

Since both $P_\alpha^{(1)}$ and $P_\alpha^{(2)}$ project onto the same subspace $\mathcal{H}_\alpha$, it follows that

$$P_\alpha^{(1)} = P_\alpha^{(2)}.$$

□

7.4 Intrinsic Nature of Projectors

The above result shows that the projectors $\{P_\alpha\}$ depend only on the partition of the closure-class index set and not on the specific selector used to realize that partition.

This establishes that the projectors are intrinsic to the closure structure of the hydrogenic sector.

7.5 Interpretation

Context independence expresses the principle that the identification of outcome classes is determined by structural compatibility rather than by the details of the interaction that probes the system.

In particular:

- different interaction configurations that impose the same structural constraints lead to the same projectors,
- the projectors therefore represent equivalence classes of outcome structure,
- and the resulting event structure is independent of the measurement context in which it is realized.

This property will be essential for the development of a consistent weighting scheme in subsequent work.

7.6 Role in Event Structure

The context-independence theorem ensures that the family of projectors constructed in Section 6 forms a well-defined event structure on $\mathcal{V}_H^{\mathrm{fin}}$.

In the next section, we will extend this structure beyond selector-induced partitions to the full projector algebra associated with the representational space. This extension is necessary for the formulation of general consistency conditions on weight assignments.

8 Extension to Full Projector Algebra

In the preceding sections, we constructed a family of projectors $\{P_\alpha\}$ associated with closure-class partitions induced by hydrogenic spectral selectors. These projectors correspond to subspaces spanned by subsets of the stationary basis $\{\Psi_n\}$ and therefore generate a class of diagonal projectors with respect to this basis.

In this section, we show that the representational structure established in Paper 1 naturally extends this class to the full projector algebra on $\mathcal{V}_H^{\mathrm{fin}}$. This extension is essential for the formulation of general consistency conditions on weight assignments in subsequent work.

8.1 Spectral Projectors and Basis Structure

The projectors constructed in Section 6 are of the form

$$P_I = \sum_{n \in I} |\Psi_n\rangle \langle \Psi_n| \, ,$$

for subsets $I \subseteq \mathcal{I}_H$. These projectors are diagonal with respect to the orthonormal basis $\{\Psi_n\}$ and generate all subspaces spanned by subsets of this basis.

While selector-induced projectors arise directly from closure-class partitions and therefore correspond to decompositions aligned with the stationary basis, the inner product structure of $\mathcal{V}_H^{\mathrm{fin}}$ admits a broader class of subspaces not aligned with this basis.

These additional subspaces do not arise from a single closure partition, but from coherent superpositions of closure classes within the representational structure. Their inclusion reflects the fact that the comparison structure derived from holonomic coherence supports all orthogonal decompositions compatible with the inner product.

Accordingly, the full projector algebra is not introduced as an independent physical postulate, but as the completion of the representational structure under admissible linear combinations.

However, these projectors do not exhaust the set of all subspaces of $\mathcal{V}_H^{\mathrm{fin}}$, as general subspaces may not be aligned with the stationary basis.

8.2 Representational Closure Under Linear Combinations

The representational space $\mathcal{V}_H^{\mathrm{fin}}$ is a complex inner product space admitting arbitrary finite linear combinations of the stationary modes:

$$\Phi = \sum_n c_n \Psi_n .$$

Given any nonzero vector $\Phi \in \mathcal{V}_H^{\mathrm{fin}}$, we may define the one-dimensional subspace

$$\mathrm{span}\{\Phi\}.$$

This subspace is not, in general, aligned with any subset of the stationary basis unless Φ is proportional to a single Ψ_n.

8.3 General Orthogonal Projections

Definition 8.1 (General projector)**.** *For any subspace* $\mathcal{H} \subseteq \mathcal{V}_H^{\text{fin}}$*, the orthogonal projector* $P_{\mathcal{H}}$ *is defined as the unique linear operator satisfying*

$$P_{\mathcal{H}}^2 = P_{\mathcal{H}}, \qquad P_{\mathcal{H}}^\dagger = P_{\mathcal{H}}, \qquad \mathrm{Im}(P_{\mathcal{H}}) = \mathcal{H}.$$

Such projectors exist and are uniquely determined by the inner product structure of $\mathcal{V}_H^{\text{fin}}$.

8.4 Generation of the Full Projector Algebra

Theorem 8.2. *The inner product structure on* $\mathcal{V}_H^{\text{fin}}$ *induces the full lattice of orthogonal projectors on the space. In particular, every subspace* $\mathcal{H} \subseteq \mathcal{V}_H^{\text{fin}}$ *is associated with a unique orthogonal projector.*

Proof. Since $\mathcal{V}_H^{\text{fin}}$ is a finite-dimensional inner product space (Section 9 of Paper 1), every subspace admits an orthonormal basis. Given such a basis $\{\Phi_i\}$ for $\mathcal{H}$, the orthogonal projector onto $\mathcal{H}$ is given by

$$P_{\mathcal{H}} = \sum_i |\Phi_i\rangle \langle\Phi_i| .$$

This operator is independent of the choice of orthonormal basis for $\mathcal{H}$ and satisfies the defining properties of an orthogonal projector. ☐

8.5 Relation to Selector-Induced Projectors

The selector-induced projectors $\{P_\alpha\}$ form a distinguished subset of the full projector algebra, corresponding to subspaces generated by closure-class partitions.

Thus:

- selector-induced projectors correspond to *spectral decompositions* aligned with the stationary basis,
- general projectors correspond to arbitrary subspaces of $\mathcal{V}_H^{\text{fin}}$.

The former arise directly from closure structure, while the latter are made available by the inner product structure derived from holonomic coherence.

8.6 Interpretation

The extension to the full projector algebra reflects the fact that once a consistent inner product structure has been established, the representational space supports all orthogonal decompositions compatible with that structure.

This extension is purely structural and does not introduce additional physical assumptions. It provides a complete event space in which different decompositions correspond to different admissible ways of resolving the system into mutually exclusive outcome classes.

Representational versus admissible event structure. While the inner product structure supports the full lattice of orthogonal projectors at the representational level, not all such projectors necessarily correspond to physically realizable outcome structures.

Selector-induced projectors arise directly from closure-class partitions and therefore correspond to admissible structural outcomes under interaction constraints. General projectors, by contrast, reflect the representational completeness of the inner product space and may correspond to coherent combinations of closure classes that are not directly realizable as a single closure partition.

Accordingly, the full projector algebra provides the space of admissible comparisons, while physically realizable events are those compatible with selector-induced structure or its consistent extensions.

General projectors therefore correspond to outcome structures that cannot be reduced to a single closure-class partition, but instead represent coherent combinations of closure-compatible configurations. These structures will play a central role in the formulation of consistent weight assignments, where different decompositions of the same state must yield compatible weights.

8.7 Role in Subsequent Development

The availability of the full projector algebra is essential for the formulation of general consistency conditions on weight assignments. In particular, it allows one to consider all orthogonal decompositions of the representational space, not only those aligned with a fixed closure-class partition.

This generality is required for the application of Gleason-type arguments, which rely on the existence of a sufficiently rich family of projectors.

The next section will formalize the event structure associated with this projector algebra and prepare the ground for the introduction of weight assignments in subsequent work.

9 Event Structure

In the preceding sections, we constructed a family of orthogonal projectors on the representational space $\mathcal{V}_H^{\mathrm{fin}}$, beginning from closure-class partitions and extending to the full projector algebra. We now interpret these projectors as events and formalize the associated event structure.

9.1 Events as Projectors

Definition 9.1 (Event). *An event in the hydrogenic sector is represented by an orthogonal projector*

$$P : \mathcal{V}_H^{\mathrm{fin}} \to \mathcal{V}_H^{\mathrm{fin}}.$$

Thus, each event corresponds to a subspace of the representational space, with the projector acting as the map that isolates the component of a state lying in that subspace.

9.2 Elementary and Composite Events

- An *elementary event* is a one-dimensional projector of the form

  $$P_\Phi = |\Phi\rangle \langle\Phi| ,$$

  for a normalized vector $\Phi \in \mathcal{V}_H^{\mathrm{fin}}$.

- A *composite event* is a projector onto a higher-dimensional subspace,

  $$P = \sum_i |\Phi_i\rangle \langle\Phi_i| ,$$

  where $\{\Phi_i\}$ is an orthonormal basis of the subspace.

Selector-induced projectors correspond to composite events aligned with closure-class partitions, while general projectors correspond to arbitrary event classes in the representational space.

9.3 Orthogonality and Exclusivity

Definition 9.2 (Exclusive events)**.** *Two events P and Q are said to be exclusive if*

$$PQ = 0.$$

This condition is equivalent to orthogonality of the corresponding subspaces. In this case, the events represent mutually incompatible outcome classes.

9.4 Event Decompositions

Definition 9.3 (Event decomposition)**.** *A collection of events $\{P_\alpha\}$ is said to form a decomposition of the identity if*

$$\sum_\alpha P_\alpha = I, \qquad P_\alpha P_\beta = 0 \quad (\alpha \neq \beta).$$

Such a decomposition represents a complete resolution of the system into mutually exclusive outcome classes.

Selector-induced partitions provide a distinguished class of such decompositions, but the full projector algebra allows for more general decompositions.

9.5 Lattice Structure

The set of all projectors on $\mathcal{V}_H^{\text{fin}}$ forms a lattice under the operations [4]:

- meet: $P \wedge Q =$ projection onto $\mathrm{Im}(P) \cap \mathrm{Im}(Q)$,
- join: $P \vee Q =$ projection onto $\mathrm{span}(\mathrm{Im}(P) \cup \mathrm{Im}(Q))$.

This lattice encodes the logical structure of events in the representational space.

9.6 Interpretation

The identification of events with projectors provides a structural description of outcome classes within the hydrogenic sector.

- Events correspond to classes of closure-compatible configurations.
- Orthogonality encodes incompatibility of outcome classes.
- Decompositions of the identity represent complete outcome resolutions.

Importantly, this structure is derived from closure partitions and the induced inner product, rather than assumed as part of a measurement postulate.

9.7 Preparation for Weight Assignment

The event structure constructed here provides the domain on which weight assignments may be defined. In particular, a weight assignment is a map

$$\mu : \{\text{projectors}\} \to [0, 1]$$

satisfying consistency conditions with respect to event decompositions.

The existence and form of such assignments will be the subject of the next stage of the program, where we will show that consistency requirements strongly constrain the possible forms of μ.

9.8 Summary

The projector algebra on $\mathcal{V}_H^{\text{fin}}$ defines a complete event structure for the hydrogenic sector. This structure is:

- derived from closure partitions and representational geometry,
- independent of probabilistic assumptions,
- and sufficiently rich to support general consistency constraints on weight assignments.

10 Interpretation

The results of this paper establish that the hydrogenic sector of scalar–conformal NUVO systems admits a well-defined event structure derived entirely from closure and coherence principles. It is important to clarify the meaning and scope of this construction.

10.1 Events as Structural Objects

Events have been identified with orthogonal projectors on the representational space $\mathcal{V}_H^{\text{fin}}$. These projectors are not introduced as primitive measurement objects, but arise from:

- closure-class partitions induced by interaction constraints,
- the orthogonal decomposition of the representational space,
- and the inner product structure derived from holonomic coherence.

Thus, events correspond to structurally defined classes of admissible configurations, rather than externally imposed measurement outcomes.

10.2 Independence from Measurement Postulates

No measurement postulates have been assumed in this construction. In particular:

- events are not associated with experimental apparatus,
- projectors are not introduced as observables,
- and no probabilistic interpretation has been assigned.

Instead, the event structure is derived from intrinsic properties of the hydrogenic closure hierarchy and its representational geometry.

10.3 Geometric Meaning of Exclusivity

The condition of exclusivity,

$$PQ = 0,$$

is interpreted as incompatibility of closure-compatible configurations. Two events are exclusive if their corresponding subspaces share no common component under the induced comparison structure.

This interpretation is purely geometric and reflects the structure of the closure hierarchy under interaction constraints.

10.4 Context Independence

The context-independence result established in Section 7 ensures that events are determined solely by the induced closure-class partition, not by the specific form of the interaction that realizes it.

This implies that the event structure is intrinsic to the system and does not depend on the details of how it is probed.

10.5 Role of the Full Projector Algebra

While selector-induced partitions generate a distinguished class of projectors, the full projector algebra arises from the inner product structure of the representational space.

This extension reflects the representational completeness of the inner product structure rather than the direct enumeration of closure-induced outcome classes. In addition, this extension also reflects the fact that the comparison structure supports all orthogonal decompositions consistent with the underlying geometry. The resulting event space is therefore complete at the level of the representational structure.

10.6 Summary

The central conclusion of this paper is that the hydrogenic sector admits a projector-based event structure derived from closure and coherence principles alone. This structure:

- encodes outcome classes as subspaces,
- represents exclusivity through orthogonality,
- and is independent of probabilistic or measurement assumptions.

It provides the structural framework required for the introduction of consistent weight assignments in subsequent work.

11 Outlook

The construction of a projector-based event structure completes the second stage of the program: the derivation of outcome structure from closure and coherence principles. We now outline the next stage, in which a consistent weighting scheme will be introduced.

11.1 Weight Assignments on Event Structure

Given the event structure defined by the projector algebra on $\mathcal{V}_H^{\text{fin}}$, one may consider maps of the form

$$\mu : \{\text{projectors}\} \to [0, 1],$$

assigning a weight to each event.

Such assignments must satisfy consistency conditions reflecting the structural properties of the event space. In particular, for any decomposition of the identity

$$\sum_\alpha P_\alpha = I,$$

it is natural to require

$$\sum_{\alpha} \mu(P_\alpha) = 1.$$

In addition to additivity and noncontextuality, the applicability of Gleason-type arguments requires that the representational space admit a sufficiently rich family of orthogonal decompositions. In particular, this condition is satisfied in sectors of dimension three or greater, where nontrivial decompositions beyond basis-aligned partitions exist.

At this stage, no specific form of μ is assumed.

11.2 Consistency and Noncontextuality

The context-independence of projectors established in Section 7 ensures that any admissible weight assignment must depend only on the projector itself and not on the particular decomposition in which it appears.

This noncontextuality condition, together with additivity over orthogonal decompositions, imposes strong constraints on the possible forms of μ.

11.3 Emergence of a Unique Weighting Rule

In sufficiently rich finite-dimensional sectors of $\mathcal{V}_H^{\mathrm{fin}}$, the above consistency conditions are known to determine the form of admissible weight assignments uniquely.

In the next stage of the program, we will show that these conditions lead to a representation of the form

$$\mu(P) = \mathrm{Tr}(\rho P),$$

for some positive operator ρ with unit trace.

This establishes a direct connection between the event structure constructed here and the standard trace-based weighting rule.

11.4 Connection to Holonomic Coherence

The trace representation obtained above will be interpreted within the NUVO framework in terms of closure density and holonomic coherence. In particular, the weighting rule will be shown to arise naturally from the comparison structure induced by the invariant coherence functional.

This provides a route by which Born-type weighting emerges as a consequence of structural consistency, rather than as a postulate.

11.5 Scope of the Present Construction

The analysis in this paper has been restricted to the hydrogenic stationary sector and to finite-dimensional representational spaces. Extension to more general systems, including dynamical regimes and multi-particle configurations, will require additional development.

However, the hydrogenic case provides a minimal setting in which the essential structural features can be isolated and rigorously developed.

11.6 Final Remarks

With the completion of this paper, we have established:

- a pre-Hilbert structure from holonomic coherence (Paper 1),

- a projector-based event structure from closure partitions (this work).

The next step is to show that consistency conditions on weight assignments over this event structure lead to a unique probabilistic rule. This will complete the derivation of Born-type weighting from geometric and coherence principles within the NUVO framework.

References

[1] Rickey W. Austin. QB3: Holonomic coherence and the emergence of pre-hilbert structure in scalar–conformal nuvo systems. NUVO QB-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[2] Rickey W. Austin. QB5: Nuvo qb-series paper 5. NUVO QB-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[3] Rickey W. Austin. Q2: Closure law and admissible return structure in scalar–conformal nuvo systems. NUVO Q-series. St Claire Scientific Research, Development, and Publishing. Zenodo DOI to be assigned., 2025.

[4] Garrett Birkhoff and John von Neumann. The logic of quantum mechanics. *Annals of Mathematics*, 37(4):823–843, 1936.