QB6 – Coherence-Gated Event Frequencies and the Realization of Quadratic Weighting in Scalar–Conformal NUVO Systems

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Abstract

We establish the dynamical realization of the quadratic weighting rule derived in QB5 within the deterministic transport framework of scalar–conformal NUVO systems. Building on the coherence-gated interaction structure introduced in Q8 and the projector-based event algebra of QB4, interaction events are defined as discrete occurrences along deterministic worldlines governed by coherence admissibility.

We introduce a coherence-flux decomposition principle, according to which the total interaction intensity decomposes uniquely into projector-resolved channels determined by the projected coherence content of the transported state. This yields a deterministic event-counting measure along the worldline, without the introduction of probabilistic postulates or equilibrium assumptions.

Under admissible transport regimes, we show that the asymptotic relative frequency of interaction events associated with a projector P satisfies

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)} = \mathrm{Tr}(\rho P),$$

where ρ is the operator uniquely determined in QB5. In the pure-state case, this reduces to

$$|\langle \Phi, \Psi \rangle_H|^2.$$

The quadratic weighting rule is thus identified with the deterministic frequency law of coherence-gated interaction, providing a direct structural connection between closure-based dynamics and the statistical framework of quantum mechanics.

1 Introduction

In QB3–QB5 [1, 2, 3], we established a structural reconstruction of the state and event framework associated with scalar–conformal NUVO systems. In QB4, interaction events were identified with projectors on a finite-dimensional representational space, and in QB5 it was shown that any assignment of weights to such events satisfying normalization, additivity, and noncontextuality must take the quadratic form

$$\mu(P) = \mathrm{Tr}(\rho P),$$

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

with ρ a positive operator of unit trace.

These results determine the unique admissible weighting structure compatible with the event algebra. However, they do not by themselves establish how such weights arise in the context of deterministic dynamics. In particular, QB5 identifies the form of the weighting rule, but does not connect this rule to the realized distribution of interaction events along a worldline.

The purpose of the present work is to establish this connection. Specifically, we show that the quadratic weighting rule is realized as the asymptotic frequency of interaction events generated by deterministic transport and coherence-gated interaction.

The analysis proceeds by introducing a deterministic event-counting framework along transported worldlines. Interaction events are defined as discrete occurrences determined by coherence admissibility, as developed in Q8 [4]. These events are partitioned into channels associated with projectors, following the event structure established in QB4 [5].

We establish a coherence-flux decomposition principle, which determines how the total interaction intensity along the worldline resolves into contributions associated with each projector channel. This decomposition is shown to be uniquely fixed by the structural constraints of the event algebra and the transported state.

Using this framework, we define channel-resolved event counts and corresponding relative frequencies. Under admissible transport regimes, we show that the asymptotic frequency of interaction events associated with a projector P is given by [6]

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\text{tot}}(T)} = \mathrm{Tr}(\rho P),$$

in agreement with the structural weighting rule derived in QB5.

The quadratic form is thus identified both as the unique admissible weight assignment and as the realized frequency law of deterministic coherence-gated interaction. No probabilistic assumptions or equilibrium hypotheses are introduced in this derivation.

This paper is organized as follows. Section 2 reviews the structural inputs from QB3–QB5. Section 3 introduces deterministic transport of represented states. Section 4 defines projector-resolved coherence channels. Section 5 establishes the coherence-flux decomposition principle. Section 6 introduces event-counting measures and relative frequencies. Section 7 proves the coherence-gated frequency law. Section 8 demonstrates consistency with the structural weighting result of QB5. Sections 9–11 discuss interpretation, limitations, and outlook.

2 Structural Inputs from Closure and Coherence

This section summarizes the structural elements from QB3–QB5 that will be used in the present analysis. No new assumptions are introduced here; we recall only those features required for the formulation of deterministic event frequencies.

2.1 Representational Space

Following QB3, the state of a system is represented by a unit vector

$$\Psi \in \mathcal{V}_H^{\text{fin}},$$

where $\mathcal{V}_H^{\text{fin}}$ is a finite-dimensional complex inner product space associated with a fixed closure configuration. The inner product

$$\langle \cdot, \cdot \rangle_H$$

encodes coherence relations between admissible configurations.

The normalization condition

$$\langle \Psi, \Psi \rangle_H = 1$$

is preserved under admissible transport.

2.2 Projector Algebra

As established in QB4, interaction events are represented by projectors

$$P \in \mathsf{P}(\mathcal{V}_H^{\mathrm{fin}}),$$

where $\mathsf{P}(\mathcal{V}_H^{\mathrm{fin}})$ denotes the set of orthogonal projectors on the representational space.

Projectors correspond to subspaces of closure-compatible configurations. Orthogonality of projectors encodes mutual exclusivity of events, while completeness of a set $\{P_\alpha\}$ satisfying

$$\sum_\alpha P_\alpha = I$$

ensures that all admissible interaction outcomes are accounted for.

2.3 Event Interpretation

Within this framework, each projector P defines an event channel corresponding to the set of configurations compatible with the associated subspace. Interaction events, as defined in Q8, occur at discrete points along the worldline where coherence admissibility conditions are satisfied. Each such event is associated with exactly one projector channel.

The projector algebra therefore provides a complete and mutually exclusive classification of interaction events.

2.4 Structural Weight Assignment

In QB5, it was shown that any map

$$\mu : \mathsf{P}(\mathcal{V}_H^{\mathrm{fin}}) \to [0, 1]$$

satisfying the following conditions:

- normalization over orthogonal decompositions,
- additivity for orthogonal projectors,
- noncontextuality,

must take the form

$$\mu(P) = \mathrm{Tr}(\rho P),$$

for a unique positive operator ρ with unit trace.

In the case of a pure state $\rho = |\Psi\rangle \langle\Psi|$, this reduces to

$$\mu(P_\Phi) = |\langle \Phi, \Psi \rangle_H|^2.$$

This result establishes the quadratic form as the unique admissible assignment of weights on the projector algebra.

2.5 Summary of Inputs

The analysis in the present work will rely on the following structural elements:

- a finite-dimensional representational space $\mathcal{V}_H^{\text{fin}}$ with inner product $\langle \cdot, \cdot \rangle_H$,
- a projector algebra $\mathsf{P}(\mathcal{V}_H^{\text{fin}})$ representing interaction channels,
- a deterministic notion of interaction events associated with projectors,
- and the unique quadratic form $\mu(P) = \text{Tr}(\rho P)$ as the admissible weight assignment.

These elements will be used to construct a deterministic event-counting framework and to determine the resulting distribution of interaction events along transported worldlines.

Origin in scalar-modulated closure structure. The representational space and its associated projector algebra ultimately arise from the scalar-modulated return condition established in Q2 [7],

$$k \oint_{\gamma} \lambda_{\text{eff}}(x, u)\, ds = L_{\gamma}.$$

This condition determines the admissible closure-compatible configurations and thereby fixes the structure of coherence channels and interaction events considered in the present work.

3 Deterministic Transport of Represented States

We now introduce the notion of deterministic transport of represented states along a worldline. This provides the dynamical framework within which interaction events and their frequencies will be analyzed.

3.1 Transported States

Definition 3.1 (Transported state)**.** *A transported state is a map*

$$\Psi : \tau \mapsto \Psi(\tau) \in \mathcal{V}_H^{\text{fin}},$$

defined along a worldline parameterized by τ*, such that*

$$\langle \Psi(\tau), \Psi(\tau) \rangle_H = 1$$

for all τ*.*

The parameter τ may be taken as the proper-time parameter associated with the underlying transport process.

3.2 Deterministic Evolution

We assume that the evolution of $\Psi(\tau)$ is deterministic, in the sense that $\Psi(\tau)$ is uniquely determined by its initial value and the transport geometry. No stochastic elements are introduced in the evolution law.

The precise form of the evolution need not be specified for the present analysis. It is sufficient that $\Psi(\tau)$ varies continuously in τ and that its evolution preserves the inner product structure of $\mathcal{V}_H^{\text{fin}}$.

3.3 Coherence Preservation

The normalization condition

$$\langle \Psi(\tau), \Psi(\tau) \rangle_H = 1$$

implies that transport preserves total coherence. This reflects the fact that the represented state remains within the admissible configuration space throughout its evolution.

More generally, the inner product

$$\langle \Phi, \Psi(\tau) \rangle_H$$

provides a measure of coherence between the transported state and any fixed reference configuration Φ. The evolution of these quantities encodes the redistribution of coherence across projector-defined channels.

3.4 Phase Structure

As developed in Q8 [4] and subsequent papers in the Q-series, the transported state may carry an intrinsic phase structure that evolves along the worldline. This phase evolution plays a central role in determining the occurrence of interaction events through coherence-gating conditions.

For the purposes of the present work, it is sufficient to note that this phase evolution is deterministic and contributes to the time dependence of the coherence content relative to different projector channels.

3.5 Interpretive Boundary

We emphasize that the transported state $\Psi(\tau)$ is not introduced as a probabilistic object. It represents a deterministic configuration evolving within the representational space associated with a given closure structure.

All statistical features that arise in subsequent sections will be derived from the interaction structure and coherence properties of this deterministic evolution.

3.6 Summary

We have introduced a deterministic transport framework in which represented states evolve continuously along worldlines while preserving normalization and coherence structure. This framework provides the basis for defining interaction events and their distribution across projector channels in the sections that follow.

4 Projector-Resolved Coherence Channels

We now introduce the projector-resolved decomposition of coherence associated with a transported state. This provides the structural basis for the channel-wise distribution of interaction events.

4.1 Coherence Content

Definition 4.1 (Coherence content). *Let $\Psi(\tau)$ be a transported state and let $P \in \mathsf{P}(\mathcal{V}_H^{\text{fin}})$ be a projector. The coherence content of $\Psi(\tau)$ relative to the channel defined by P is given by*

$$C_P(\tau) := \langle \Psi(\tau), P\Psi(\tau) \rangle_H.$$

The quantity $C_P(\tau)$ measures the compatibility of the transported configuration with the subspace associated with P. It depends deterministically on the transported state and varies along the worldline.

4.2 Basic Properties

The coherence content satisfies the following structural properties.

Lemma 4.2 (Positivity)**.** *For all projectors P and all τ,*

$$C_P(\tau) \geq 0.$$

Proof. Since P is a positive operator, $\langle \Psi(\tau), P\Psi(\tau)\rangle_H \geq 0$ follows immediately from the properties of the inner product. □

Lemma 4.3 (Additivity)**.** *Let P and Q be orthogonal projectors, $PQ = 0$. Then*

$$C_{P+Q}(\tau) = C_P(\tau) + C_Q(\tau).$$

Proof. Since P and Q are orthogonal projectors, we have

$$(P + Q)\Psi(\tau) = P\Psi(\tau) + Q\Psi(\tau),$$

and

$$\langle \Psi(\tau), (P + Q)\Psi(\tau)\rangle_H = \langle \Psi(\tau), P\Psi(\tau)\rangle_H + \langle \Psi(\tau), Q\Psi(\tau)\rangle_H,$$

which establishes the result. □

Lemma 4.4 (Completeness)**.** *Let $\{P_\alpha\}$ be an orthogonal decomposition of the identity,*

$$\sum_\alpha P_\alpha = I.$$

Then

$$\sum_\alpha C_{P_\alpha}(\tau) = 1$$

for all τ.

Proof. Using completeness,

$$\sum_\alpha C_{P_\alpha}(\tau) = \sum_\alpha \langle \Psi(\tau), P_\alpha\Psi(\tau)\rangle_H = \langle \Psi(\tau), \left(\sum_\alpha P_\alpha\right)\Psi(\tau)\rangle_H = \langle \Psi(\tau), \Psi(\tau)\rangle_H = 1.$$

□

4.3 Channel Decomposition

The preceding properties show that the coherence content defines a decomposition of the total coherence of the transported state across projector-defined channels.

For any orthogonal decomposition $\{P_\alpha\}$, the collection $\{C_{P_\alpha}(\tau)\}$ forms a normalized additive family satisfying

$$C_{P_\alpha}(\tau) \geq 0, \qquad \sum_\alpha C_{P_\alpha}(\tau) = 1.$$

Thus, the transported state admits a resolution into mutually exclusive coherence channels, each associated with a projector.

4.4 Relation to Structural Weights

The functional form of $C_P(\tau)$ coincides with the quadratic expression identified in QB5 as the unique admissible weight assignment on the projector algebra. However, in the present context, $C_P(\tau)$ is interpreted as a deterministic coherence quantity evaluated along a worldline, rather than as a probability or weight.

The role of $C_P(\tau)$ in the present work is to determine how interaction intensity is distributed across channels, as will be established in the next section.

4.5 Interpretive Boundary

We emphasize that the quantities $C_P(\tau)$ are not introduced as probabilities. They arise solely from the inner product structure of the representational space and the projector algebra. Their interpretation is purely geometric, reflecting the degree of coherence of the transported state relative to different closure-compatible configurations.

4.6 Summary

We have defined the projector-resolved coherence content of a transported state and established its fundamental properties of positivity, additivity, and completeness. These properties provide the structural basis for the decomposition of interaction intensity into channel-resolved contributions in the next section.

5 Coherence-Flux Decomposition Principle

We now establish the mechanism by which interaction events are distributed across the projector-defined event structure introduced in the preceding sections. The goal is to determine, from structural considerations alone, how the total interaction intensity along a deterministic worldline decomposes into contributions associated with individual projectors.

5.1 Interaction Events and Channel Structure

As established in Q8, interaction events occur at discrete points along the worldline where coherence admissibility conditions are satisfied. These events arise from the restoration of compatibility between the transported configuration and the ambient closure structure. The set of such events may be written as a discrete collection of parameter values $\{\tau_k\}$.

From QB4, each admissible event is associated with a projector $P \in \mathsf{P}(\mathcal{V}_H^{\mathrm{fin}})$, representing a subspace of closure-compatible configurations. Orthogonality of projectors encodes mutual exclusivity, while completeness of the projector algebra ensures that every admissible interaction corresponds to exactly one projector channel.

Accordingly, the set of interaction events admits a partition into disjoint channel subsets

$$\{\tau_k\} = \bigsqcup_P \{\tau_k^{(P)}\},$$

where each subset corresponds to events associated with a given projector.

5.2 Total Interaction Intensity

Let $\kappa(\tau)$ denote the total interaction intensity along the worldline, defined so that the total event-counting measure satisfies

$$dN_{\text{tot}}(\tau) = \kappa(\tau)\, d\tau.$$

The function $\kappa(\tau)$ depends on the local transport and interaction geometry, but its explicit form is not required for the present analysis. It represents the rate at which coherence-gated events occur along the trajectory.

5.3 Decomposition Requirements

We now consider how the total interaction intensity decomposes across projector channels. For each projector P, we seek a channel-specific intensity $dN_P(\tau)$ such that

$$dN_{\text{tot}}(\tau) = \sum_{\alpha} dN_{P_\alpha}(\tau)$$

for every orthogonal decomposition $\{P_\alpha\}$ of the identity.

The decomposition must satisfy the following structural requirements:

- **Positivity:** $dN_P(\tau) \geq 0$ for all P.
- **Additivity:** If P and Q are orthogonal projectors, then

$$dN_{P+Q}(\tau) = dN_P(\tau) + dN_Q(\tau).$$

- **Completeness:** For any orthogonal decomposition $\{P_\alpha\}$,

$$\sum_{\alpha} dN_{P_\alpha}(\tau) = dN_{\text{tot}}(\tau).$$

- **Noncontextuality:** The value of $dN_P(\tau)$ depends only on P and the transported state $\Psi(\tau)$, and not on the particular decomposition in which P appears.

These conditions mirror the structural constraints imposed on weight assignments in QB5.

5.4 Dependence on the Transported State

Since the only state-dependent structure available is the transported configuration $\Psi(\tau)$, the channel intensity must take the form

$$dN_P(\tau) = \kappa(\tau)\, F(P, \Psi(\tau))\, d\tau,$$

for some function F satisfying the conditions above.

The requirements of positivity, additivity, completeness, and noncontextuality imply that, for each fixed τ, the map

$$P \mapsto F(P, \Psi(\tau))$$

defines a normalized additive functional on the projector algebra.

5.5 Uniqueness of the Decomposition

By the results of QB5, the only function on the projector algebra satisfying these conditions is given by the quadratic form

$$F(P, \Psi) = \langle \Psi, P\Psi \rangle_H.$$

This establishes that the decomposition of interaction intensity is uniquely determined by the projected coherence content of the transported state.

Uniqueness of channel-resolved decomposition. The structural requirements of positivity, additivity, completeness, and noncontextuality imply that, for each fixed τ, the map

$$P \mapsto \frac{dN_P(\tau)}{dN_{\text{tot}}(\tau)}$$

defines a normalized additive functional on the projector algebra.

By the uniqueness result established in QB5, the only such functional is given by the quadratic form

$$\langle \Psi(\tau), P\Psi(\tau) \rangle_H.$$

Accordingly, the channel-resolved interaction measures are uniquely determined as

$$dN_P(\tau) = \kappa(\tau) \, \langle \Psi(\tau), P\Psi(\tau) \rangle_H \, d\tau.$$

5.6 Interpretation

The above relation expresses the fact that interaction intensity distributes across projector-defined channels in proportion to the coherence content of the transported state relative to each channel. This is not a probabilistic assignment, but a geometric decomposition of a deterministic interaction measure.

The total interaction intensity is thus resolved into mutually exclusive channels according to the structure of the projector algebra, with weights fixed uniquely by coherence compatibility.

5.7 Summary

The coherence-flux decomposition principle provides the unique, structurally consistent resolution of interaction intensity into projector channels. It follows directly from the event structure and the constraints established in QB5, and requires no additional probabilistic assumptions.

In the following sections, this decomposition will be used to derive the asymptotic frequency of interaction events associated with each projector.

6 Event Counting Measures

We now convert the channel-resolved interaction structure established in Section 5 into explicit counting measures along the worldline. The purpose of this section is to define, in a precise and non-probabilistic manner, the quantities whose asymptotic behavior will determine the observed distribution of interaction events.

6.1 Event Counting Along the Worldline

Let $\{\tau_k\}$ denote the discrete set of interaction times along the worldline, as defined in Section 5. For a given projector P, let

$$\{\tau_k^{(P)}\} \subset \{\tau_k\}$$

denote the subset of interaction events associated with channel P.

For any $T > 0$, define the counting functions

$$N_{\mathrm{tot}}(T) := \#\{\tau_k \mid 0 \le \tau_k \le T\},$$

$$N_P(T) := \#\{\tau_k^{(P)} \mid 0 \le \tau_k^{(P)} \le T\}.$$

Thus, $N_{\mathrm{tot}}(T)$ counts the total number of interaction events up to parameter value T, while $N_P(T)$ counts the number of events occurring in the channel associated with projector P.

By construction, for any orthogonal decomposition $\{P_\alpha\}$ of the identity,

$$N_{\mathrm{tot}}(T) = \sum_\alpha N_{P_\alpha}(T).$$

6.2 Measure Representation of Event Counts

The coherence-flux decomposition principle introduced in Section 5 provides a differential representation of these counting functions. Specifically, the channel-resolved event measures satisfy

$$dN_P(\tau) = \kappa(\tau)\,\langle \Psi(\tau), P\Psi(\tau)\rangle_H \, d\tau,$$

and the total event measure is

$$dN_{\mathrm{tot}}(\tau) = \kappa(\tau)\, d\tau.$$

Integrating along the worldline, we obtain

$$N_P(T) = \int_0^T \kappa(\tau)\,\langle \Psi(\tau), P\Psi(\tau)\rangle_H \, d\tau,$$

$$N_{\mathrm{tot}}(T) = \int_0^T \kappa(\tau)\, d\tau.$$

These expressions provide a continuous representation of the discrete counting functions, with the interaction intensity $\kappa(\tau)$ acting as a density of event occurrence along the trajectory.

6.3 Relative Frequencies

We now define the relative frequency of events associated with a projector P up to parameter value T by

$$f_P(T) := \frac{N_P(T)}{N_{\mathrm{tot}}(T)},$$

provided $N_{\mathrm{tot}}(T) \neq 0$.

Using the integral representation above, this may be written as

$$f_P(T) = \frac{\int_0^T \kappa(\tau)\,\langle \Psi(\tau), P\Psi(\tau)\rangle_H \, d\tau}{\int_0^T \kappa(\tau)\, d\tau}.$$

Thus, the relative frequency is expressed as a ratio of two deterministic integrals along the worldline.

6.4 Structural Properties

The relative frequency $f_P(T)$ inherits the structural properties of the coherence content:

- **Positivity:** $f_P(T) \geq 0$.
- **Additivity:** For orthogonal projectors P and Q,

$$f_{P+Q}(T) = f_P(T) + f_Q(T).$$

- **Normalization:** For any orthogonal decomposition $\{P_\alpha\}$,

$$\sum_\alpha f_{P_\alpha}(T) = 1.$$

These properties hold for every finite T and reflect the underlying projector structure of the event algebra.

6.5 Long-Time Behavior

The central object of interest is the asymptotic behavior of the relative frequency as $T \to \infty$. Formally, we consider the limit

$$\lim_{T\to\infty} f_P(T),$$

whenever this limit exists.

From the integral representation, this limit takes the form of a weighted time average:

$$\lim_{T\to\infty} \frac{\int_0^T \kappa(\tau) \, \langle \Psi(\tau), P\Psi(\tau) \rangle_H \, d\tau}{\int_0^T \kappa(\tau) \, d\tau}.$$

Thus, the asymptotic frequency is determined by the long-time average of the projected coherence content, weighted by the interaction intensity.

6.6 Interpretive Boundary

At this stage, no probabilistic interpretation has been introduced. The quantities $N_P(T)$ and $f_P(T)$ are defined purely as counts and ratios of deterministic interaction events along a worldline.

The expressions obtained above are identities following from the coherence-flux decomposition principle and the structure of the projector algebra. The question of whether the asymptotic limit exists, and how it is evaluated, will be addressed in the next section.

6.7 Summary

We have defined:

- the total and channel-resolved event counts $N_{\text{tot}}(T)$ and $N_P(T)$,
- their representation as integrals over the worldline,
- and the corresponding relative frequency $f_P(T)$.

These quantities provide the precise framework needed to analyze the long-time distribution of interaction events. In the next section, we determine the asymptotic behavior of these frequencies and establish their relation to the quadratic weighting structure derived in QB5.

7 Coherence-Gated Frequency Law

We now determine the asymptotic behavior of the relative frequencies defined in Section 6 and establish their connection to the quadratic weighting structure derived in QB5.

7.1 Preliminaries

Recall that the relative frequency associated with a projector P is given by

$$f_P(T) = \frac{\int_0^T \kappa(\tau)\, \langle \Psi(\tau), P\Psi(\tau) \rangle_H \, d\tau}{\int_0^T \kappa(\tau)\, d\tau}.$$

Thus, $f_P(T)$ is a weighted time average of the coherence content $C_P(\tau)$ with respect to the interaction intensity $\kappa(\tau)$.

7.2 Admissible Transport Regimes

To evaluate the long-time behavior of $f_P(T)$, we restrict attention to transport regimes satisfying the following condition:

Assumption 7.1 (Admissible transport)**.** *The transported state $\Psi(\tau)$ evolves such that the weighted time average of the coherence content converges:*

$$\lim_{T\to\infty} \frac{\int_0^T \kappa(\tau)\, C_P(\tau)\, d\tau}{\int_0^T \kappa(\tau)\, d\tau} = \overline{C_P},$$

for all projectors P.

This condition expresses the requirement that the long-time interaction statistics are well-defined. It does not impose any probabilistic structure, but only the existence of a stable asymptotic average under deterministic transport.

Structural basis for convergence. The admissible transport condition reflects the requirement that the transported state explores the coherence structure in a manner compatible with the underlying closure dynamics. In particular, coherence-preserving or stationary transport ensures that the distribution of coherence content across projector channels stabilizes in the long-time limit.

The convergence assumption therefore encodes a structural property of the transport dynamics rather than an external statistical hypothesis [8].

7.3 Frequency Law

Theorem 7.2 (Coherence-Gated Frequency Law)**.** *Let $\Psi(\tau)$ be a transported state satisfying the conditions of Sections 3–6 and the admissible transport assumption above. Then, for any projector P,*

$$\lim_{T\to\infty} f_P(T) = \overline{C_P}.$$

Proof. By definition,

$$f_P(T) = \frac{\int_0^T \kappa(\tau)\, C_P(\tau)\, d\tau}{\int_0^T \kappa(\tau)\, d\tau}.$$

The result follows directly from the existence of the limit specified in the admissible transport assumption. □

7.4 Quadratic Realization

We now show that the limiting value $\overline{C_P}$ coincides with the quadratic form obtained in QB5.

Theorem 7.3 (Quadratic realization of event frequencies)**.** *Under coherence-preserving or stationary transport, the asymptotic average satisfies*

$$\overline{C_P} = \mathrm{Tr}(\rho P),$$

where ρ is the operator determined in QB5. In the pure-state case $\rho = |\Psi\rangle\langle\Psi|$, this reduces to

$$\overline{C_{P_\Phi}} = |\langle\Phi, \Psi\rangle_H|^2.$$

Proof. Under stationary or coherence-preserving transport, the statistical properties of the transported state are invariant along the worldline. In this regime, the time-averaged coherence content coincides with its structural value determined by the state.

From Section 4,

$$C_P(\tau) = \langle\Psi(\tau), P\Psi(\tau)\rangle_H.$$

Under stationary or coherence-preserving transport, the time-averaged coherence content coincides with its invariant structural value, reflecting the absence of net redistribution of coherence across projector channels. Therefore,

$$\overline{C_P} = \langle\Psi, P\Psi\rangle_H.$$

From QB5, the unique weight assignment compatible with the projector structure is given by

$$\mu(P) = \mathrm{Tr}(\rho P),$$

and in the pure-state case,

$$\mu(P_\Phi) = |\langle\Phi, \Psi\rangle_H|^2.$$

Thus,

$$\overline{C_P} = \mu(P),$$

which establishes the result. □

7.5 Consequences

Combining the preceding results, we obtain

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)} = \mathrm{Tr}(\rho P),$$

and, for pure states,

$$\lim_{T\to\infty} \frac{N_{P_\Phi}(T)}{N_{\mathrm{tot}}(T)} = |\langle\Phi, \Psi\rangle_H|^2.$$

This establishes that the quadratic weighting rule derived in QB5 is realized as the asymptotic frequency of interaction events along deterministic worldlines.

7.6 Interpretation

The result shows that the quadratic form arises as a consequence of deterministic coherence transport and interaction geometry. No probabilistic assumptions or equilibrium hypotheses have been introduced. The observed distribution of events is determined entirely by the structure of coherence channels and their associated flux decomposition.

7.7 Summary

We have shown that the long-time relative frequency of interaction events associated with a projector P is given by the quadratic form $\mathrm{Tr}(\rho P)$. This completes the dynamical realization of the weight structure established in QB5.

8 Consistency with Structural Weighting

We now establish that the frequency law derived in Section 7 is fully consistent with, and uniquely determined by, the structural weighting rule obtained in QB5.

8.1 Structural Weight Assignment from QB5

In QB5, it was shown that any assignment

$$\mu : \mathsf{P}(\mathcal{V}_H^{\mathrm{fin}}) \to [0,1]$$

satisfying normalization, additivity over orthogonal projectors, and noncontextuality must take the form

$$\mu(P) = \mathrm{Tr}(\rho P),$$

for a unique positive operator ρ with unit trace.

In the pure-state case $\rho = |\Psi\rangle\langle\Psi|$, this reduces to

$$\mu(P_\Phi) = |\langle \Phi, \Psi \rangle_H|^2.$$

This result establishes the quadratic form as the unique admissible weight assignment on the projector algebra.

8.2 Frequency Assignment from QB6

In the present work, we have shown that, under deterministic transport and coherence-gated interaction,

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)} = \overline{C_P}.$$

Under admissible transport regimes, the limiting value satisfies

$$\overline{C_P} = \mathrm{Tr}(\rho P),$$

and hence

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)} = \mathrm{Tr}(\rho P).$$

Thus, the asymptotic frequency of interaction events defines a map

$$\mu_{\mathrm{dyn}}(P) := \lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)}$$

with the same functional form as the structural weight assignment.

8.3 Verification of Structural Conditions

We now verify that the dynamically defined map μ_{dyn} satisfies the conditions required in QB5.

- **Positivity:** Since $N_P(T) \geq 0$ and $N_{\text{tot}}(T) > 0$ for sufficiently large T, we have

$$\mu_{\text{dyn}}(P) \geq 0.$$

- **Normalization:** For any orthogonal decomposition $\{P_\alpha\}$,

$$\sum_\alpha N_{P_\alpha}(T) = N_{\text{tot}}(T),$$

  and hence

$$\sum_\alpha \mu_{\text{dyn}}(P_\alpha) = 1.$$

- **Additivity:** For orthogonal projectors P and Q,

$$N_{P+Q}(T) = N_P(T) + N_Q(T),$$

  which implies

$$\mu_{\text{dyn}}(P+Q) = \mu_{\text{dyn}}(P) + \mu_{\text{dyn}}(Q).$$

- **Noncontextuality:** The assignment $\mu_{\text{dyn}}(P)$ depends only on the projector P and the transported state, and not on the particular decomposition in which P appears, as follows from the coherence-flux decomposition principle.

Thus, μ_{dyn} satisfies all structural conditions required of admissible weight assignments.

8.4 Uniqueness

By the uniqueness result established in QB5, any map satisfying the conditions above must coincide with the trace form. Therefore,

$$\mu_{\text{dyn}}(P) = \text{Tr}(\rho P)$$

is not merely a compatible representation, but the only possible one.

8.5 Interpretation

The preceding argument shows that the quadratic weighting rule is not an independent postulate, but a structural necessity that is realized dynamically through coherence-gated interaction.

QB5 established that the trace form is the unique admissible assignment consistent with the event structure. QB6 shows that deterministic interaction dynamics produce event frequencies that satisfy these same structural conditions, and therefore must coincide with that unique assignment.

8.6 Summary

We conclude that the asymptotic frequency of interaction events agrees exactly with the structural weight assignment derived in QB5. The quadratic form is thus both:

- the unique admissible weight consistent with the projector algebra, and
- the realized frequency law of deterministic coherence-gated interaction.

This completes the identification of the quadratic weighting rule with the physical distribution of interaction events.

9 Interpretation

The results established in the preceding sections show that the quadratic form

$$\mathrm{Tr}(\rho P)$$

arises as the asymptotic frequency of interaction events generated by deterministic transport and coherence-gated interaction.

This identification does not rely on the introduction of probabilistic postulates. The quantities $N_P(T)$ and $N_{\mathrm{tot}}(T)$ are defined as counts of discrete events along a worldline, and the relative frequency

$$\frac{N_P(T)}{N_{\mathrm{tot}}(T)}$$

is a deterministic ratio of such counts. The quadratic form emerges as the limiting value of this ratio under admissible transport.

Within this framework, the quantities $\langle \Psi, P\Psi \rangle_H$ are not interpreted as probabilities, but as coherence contents determining how interaction intensity distributes across projector-defined channels. The resulting event frequencies reflect this distribution through the coherence-flux decomposition principle.

Thus, the statistical structure associated with the projector algebra is not imposed externally, but arises from the interaction geometry of transported states. The quadratic weighting rule is therefore identified with the realized distribution of interaction events, rather than introduced as an independent assumption.

We emphasize that no intrinsic stochastic law has been introduced at the level of transport dynamics. Statistical features arise from the structure of event counting under deterministic coherence-gated interaction. All stochastic features appear only at the level of event counting and arise from the deterministic structure of coherence-gated interaction.

10 Limitations

The derivation presented in this work relies on several structural and dynamical conditions that define its domain of applicability.

First, the analysis assumes a finite-dimensional representational space $\mathcal{V}_H^{\mathrm{fin}}$, as established in QB3. Extensions to infinite-dimensional settings are not considered here.

Second, the coherence-flux decomposition principle introduced in Section 5 provides the mechanism by which interaction intensity is resolved across projector channels. While this principle is uniquely determined by the structural constraints of the projector algebra and the transported state, its connection to underlying microscopic dynamics is not developed in detail in the present work.

Third, the evaluation of asymptotic frequencies requires the existence of a well-defined long-time limit

$$\lim_{T\to\infty} \frac{N_P(T)}{N_{\mathrm{tot}}(T)}.$$

This is ensured under the admissible transport assumption introduced in Section 7, which requires that the weighted time averages of coherence content converge. The derivation does not establish general conditions under which such convergence holds, but restricts attention to transport regimes in which stable long-time behavior is realized.

Fourth, the identification

$$\overline{C_P} = \mathrm{Tr}(\rho P)$$

relies on coherence-preserving or stationary transport. The treatment of non-stationary or strongly time-dependent regimes is not addressed here.

Finally, the present analysis is formulated for single transported states within a fixed closure configuration. Extensions to multi-particle systems, interacting configurations, and dynamically changing closure structures remain to be developed.

These limitations do not affect the internal consistency of the derivation, but define the scope within which the results should be interpreted.

11 Outlook

The results obtained in this work establish that the quadratic weighting rule derived in QB5 is realized as the asymptotic frequency of interaction events generated by deterministic coherence-gated transport. This provides a direct structural connection between closure-based dynamics and the statistical framework associated with projector-valued event descriptions.

The next stage of the program is to relate this deterministic event-frequency framework to the standard formalism of quantum mechanics. In particular, the identification of projectors with measurement outcomes, together with the dynamical realization of the quadratic weighting rule, provides the basis for connecting the present framework to the Born rule [9] and its operational interpretation.

This transition will be developed in the subsequent paper [10], where the correspondence between deterministic coherence transport and the conventional measurement framework will be made explicit.

Further directions include the extension of the present analysis to multi-particle systems, the incorporation of interaction dynamics beyond single-worldline transport, and the exploration of non-stationary regimes in which the coherence structure evolves dynamically.

These developments will clarify the role of coherence geometry in the emergence of quantum behavior and further establish the connection between scalar–conformal NUVO systems and the broader framework of quantum theory.

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