QM2 — Superposition, Interference, and the Double-Slit Experiment

in Scalar–Conformal NUVO Systems *Preprint, Version 1.0*[*]

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Notation and Conventions

- $\mathcal{M}$ denotes the spacetime manifold.
- η denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- g denotes the physical metric.
- The scalar field $\Lambda : \mathcal{M} \to \mathbb{R}_{>0}$ is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \, \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$ denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies $\Lambda(x) = \Lambda_0$.
- The dimensionless scalar diagnostic is

$$\lambda(x) \coloneqq \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline Λ_0 remains fixed.
- Greek indices $\mu, \nu, \ldots$ range over spacetime coordinates $0, 1, 2, 3$.
- We use the Einstein summation convention unless explicitly stated otherwise.

Remark 0.1. *Unless otherwise stated, the background signature is* $(-, +, +, +)$.

[*]Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

Program scope.

Abstract

The Hilbert space $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$ established in QM1 accommodates arbitrary linear combinations of closure states. The present paper derives the physical content of this linear structure.

The superposition principle is established as a theorem: the linearity of the exchange-sector transport closure equations on $\mathcal{H}$, recalled from the Q-series, implies that any finite or norm-convergent linear combination of admissible closure states is itself an admissible closure state. No new postulate is required; superposition is a structural consequence of the transport closure geometry.

Two-path superpositions are then analyzed. When the scalar-conformal transport admits two spatially distinct channels connecting a source region to a detection region, the closure state is the coherent sum of the two path states, and the resulting closure density at the detection screen contains cross-terms arising from the relative transport phase accumulated along the two paths. These cross-terms produce the interference pattern: a spatially oscillating modulation of the closure density whose fringe spacing is determined by the path geometry and the transport phase gradient.

Which-path detection is treated as coherence disruption. An interaction that acquires path information necessarily disturbs the transport phase of at least one path, destroying the cross-terms in the closure density and eliminating the fringe pattern. This is not a wave-collapse postulate but a structural consequence of the mutual exclusivity of coherent phase correlation and path-localized interaction events.

The scalar-conformal NUVO account of the double-slit experiment follows directly: the fringe pattern, its disappearance under which-path detection, and the intermediate partially-coherent regime are all derived from transport closure geometry without invoking wave-particle duality, wavefunction collapse, or probabilistic postulate.

1 Introduction

1.1 Position Within the QM-Series

The scalar–conformal NUVO program has proceeded through a sequence of sector papers in which each result is derived strictly from the foundations established by its predecessors. The M-series fixed the scalar–conformal geometry and variational structure; the Q-series developed exchange-sector transport, closure, coherence, and quantization; the QB-series established the state representation, operator algebra, pre-Hilbert inner product, Born frequency law, and measurement correspondence; and QM1 completed the transition to the full Hilbert space $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$, establishing normalization as a structural constraint from closure conservation, the self-adjoint transport generators with their Sobolev domains, and the spectral theorem together with the resolution of the identity. The Hilbert space $\mathcal{H}$ supports arbitrary norm-convergent linear combinations of closure states. The physical content of this linear structure—what it implies about the behavior of transport closure configurations and the observational patterns they produce—is the subject of the present paper.

The position of QM2 within the series is structurally distinctive. It depends on QM1 for the Hilbert space framework, on the Q-series for the linearity of the transport closure equations and the phase accumulation structure of transport paths, and on the QB-series for the Born frequency law and the coherence-gated interaction framework. It does not depend on QM4: the Schrödinger dynamics, the unitary time-evolution group, and the conservation laws of QM4 are not required for the results of the present paper. The superposition principle, the interference analysis, and the complementarity relation are all kinematic and algebraic results that follow from the linear structure of the transport closure system on $\mathcal{H}$ without requiring a time-evolution law. This independence means that QM2 and QM3 (uncertainty relations) stand on the algebraic foundation of QM1 alone, and their results are available as established theorems when QM4 and QM5 are subsequently developed.

The double-slit experiment occupies a canonical position in the history and conceptual structure of quantum mechanics. In the standard formulation of the theory it is treated as empirical evidence for wave-particle duality: the fringe pattern demonstrates wave-like behavior, and the localized detection events demonstrate particle-like behavior, and the tension between these two descriptions is resolved—if at all—by the wave function postulate and its probabilistic interpretation. In the scalar–conformal NUVO framework the experiment requires no such resolution because no duality is postulated. The fringe pattern is derived as a structural consequence of two-path phase-coherent transport in the scalar–conformal geometry: the interference cross-terms in the closure density arise from the transport phase difference accumulated along the two paths, and their spatial oscillation produces the fringe pattern with a spacing determined by the path geometry and the transport momentum. The disappearance of the fringe pattern under which-path detection is derived as phase de-correlation: any coherence-gated interaction that resolves the path identity introduces an uncontrolled phase shift on the detected path, destroying the cross-term correlations when averaged over the interaction ensemble. Neither wave-particle duality nor wavefunction collapse enters the derivation at any point.

The superposition principle established in the present paper propagates forward through every subsequent paper in the QM-series in which the Hilbert space linear structure is used. QM3 derives the uncertainty relations from the non-commutativity of the position and momentum transport generators on $\mathcal{H}$, and the Robertson–Schrödinger bound is established by applying the Cauchy–Schwarz inequality to superpositions of operator eigenstates. QM5 constructs angular momentum eigenstates as superpositions of coordinate-basis closure states and derives their spectrum from the integer holonomy quantization condition. QM6 identifies coherent states as the superpositions of energy eigenstates that minimize the uncertainty product of QM3 and most closely realize the classical transport trajectory of QM4. QM9 constructs entangled states as non-factorizable superpositions in the two-particle Hilbert space, and the present paper's treatment of coherence and coherence disruption is the single-particle precursor of the entanglement and decoherence structure developed there.

1.2 Objective of the Present Work

The central objective of the present paper is to derive the superposition principle, the interference pattern, and the complementarity relation as structural theorems of the scalar–conformal NUVO transport closure system, without introducing wave-particle duality, wavefunction collapse, or probabilistic postulate. Specifically, the paper aims to establish five claims.

1. The exchange-sector transport closure equations, in the integrable regime in which the Q-series Schrödinger-type representation holds, are linear in the complex state encoding Ψ. This linearity implies that any finite or norm-convergent linear combination of admissible closure states is itself an admissible closure state. The superposition principle is established as a theorem, not introduced as a postulate.

2. For a two-path transport configuration in which the scalar–conformal geometry admits two spatially distinct channels from a source region to a detection region, the closure state is the coherent sum $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$, and the resulting closure density at the detection screen contains cross-terms of the form $2\,\mathrm{Re}[c_A\overline{c_B}\,\overline{\Psi_A(x)}\Psi_B(x)]$ arising from the transport

phase difference $\Delta\phi(x) = \phi_B(x) - \phi_A(x)$ between the two paths. These cross-terms are the interference term.

3. The interference pattern at the detection screen—the spatial oscillation of the closure density $I(y)$ with fringe spacing $\Delta y = \lambda L/d$—is derived from the two-path closure density using the small-angle phase difference for the double-slit geometry. The de Broglie wavelength $\lambda = 2\pi\Phi_0/p = h/p$ emerges from the transport phase structure, recovering the Q-series identification $\Phi_0 = \hbar$ as an internal consistency check.

4. Which-path detection corresponds to a coherence-disrupting interaction that introduces an uncontrolled random phase shift on one path's transport phase. Averaging the closure density over the random phase shift causes the cross-term to vanish identically, eliminating the fringe pattern. This is derived as a theorem from the coherence-gated interaction framework of the QB-series, without invoking wavefunction collapse.

5. The fringe visibility $\mathcal{V}$ and the which-path distinguishability $\mathcal{W}$ satisfy the complementarity relation $\mathcal{V}^2+\mathcal{W}^2 \leq 1$ for general (partially coherent) two-path states, with equality $\mathcal{V}^2+\mathcal{W}^2 = 1$ for normalized pure two-path states. This is derived from the algebraic structure of the two-path closure state and the Cauchy–Schwarz inequality on $\mathcal{H}$, not introduced as a principle.

Claims (1) through (5) form a logically ordered sequence. The superposition theorem of claim (1) is the prerequisite for the two-path construction of claim (2). The closure density of claim (2) yields the fringe pattern of claim (3). The coherence disruption of claim (4) uses the cross-term structure of claim (2) to identify precisely what which-path detection destroys. The complementarity relation of claim (5) unifies claims (3) and (4) into a single algebraic inequality governing the trade-off between fringe visibility and path distinguishability.

1.3 What Is Not Assumed

The present work maintains without modification the interpretive discipline established in the Q-series and continued through the QB-series, QM1, and QM4. The following exclusions are in force throughout the paper.

The superposition principle is not postulated. In the standard quantum-mechanical formalism, the superposition of quantum states is introduced as a primitive assumption: the state space is a Hilbert space, and arbitrary linear combinations of states are states. In the present framework, superposition follows as a theorem from the linearity of the transport closure equations in the integrable exchange-sector regime, recalled from the Q-series. The linearity is a property of the transport geometry, not an assumption about state spaces.

Interference is not treated as evidence for a wave nature of the closure state. The fringe pattern in the closure density is a structural consequence of the transport phase difference between two path channels. The complex state Ψ is a representational object encoding transport closure geometry, as established in QB1 and carried forward throughout the series; it is not a physical wave propagating through space. The NUVO framework does not assert that the transport closure "passes through both slits simultaneously" in any ontological sense. It derives that the closure density at the screen is determined by the two-path superposition state, whose cross-terms encode the phase correlation between the two transport channels.

Which-path detection does not invoke wavefunction collapse. The disappearance of the fringe pattern under which-path detection is derived from the phase de-correlation mechanism: the coherence-gated interaction that resolves path identity introduces an uncontrolled transport phase disturbance that destroys the cross-term correlations in the closure density. No discontinuous state change, no projection postulate, and no reduction of the wavefunction is assumed or implied.

Wave-particle duality is not introduced as a principle. The complementarity relation $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$ is derived from the algebraic structure of the normalized two-path closure state and the Cauchy–Schwarz inequality on $\mathcal{H}$. It is a theorem about the geometry of the Hilbert space, not a statement about the metaphysical nature of the transport closure configuration.

No probabilistic postulate is introduced. The identification of the closure density $|\Psi(x)|^2$ at the detection screen with the asymptotic frequency of interaction events at position x follows from the Born frequency law of QB6, extended to $\mathcal{H}$ in QM1. This identification is a prior result and is not re-derived here; it is applied to the two-path closure density $|\Psi_{AB}(x)|^2$ to relate the fringe pattern to observable event frequencies.

1.4 Structure of the Paper

Sec. 2 recalls the linearity of the exchange-sector transport closure equations from the Q-series, the admissibility structure from QM1, the inner product and Cauchy–Schwarz inequality from QM1, the phase accumulation structure of transport paths from the Q-series, and the Born frequency law from QB6 and QM1. Sec. 3 establishes the superposition principle as a theorem from transport closure linearity, in three forms covering finite superpositions, norm-convergent series, and continuous superpositions, and records the inner product structure of two-path superpositions. Sec. 4 defines the two-path transport configuration precisely, constructs the two-path closure state, derives the closure density with its interference cross-terms, and identifies the transport phase difference as the geometric origin of interference. Sec. 5 derives the interference fringe pattern at the detection screen, establishes the fringe spacing and fringe visibility, and records the conditions for constructive and destructive interference. Sec. 6 treats which-path detection as coherence disruption, derives the vanishing of the interference term under complete phase de-correlation, defines the which-path distinguishability, and establishes the complementarity relation from the Cauchy–Schwarz inequality. Sec. 7 assembles the complete scalar–conformal NUVO account of the double-slit experiment, identifies the three experimental regimes, and recovers the de Broglie wavelength from the transport phase structure. Sec. 8 collects interpretive clarifications, maintaining the interpretive boundary conditions of the prior series, and records the scope of the present construction. Sec. 9 summarizes the results, records their programmatic significance for the QM-series, and prepares the transition to QM3.

2 Recalled Structure from Q-Series and QM1

The present section collects the results from the Q-series, QB-series, and QM1 that are directly required for the developments of Secs. 3–7. Each subsection recalls what has been established, states it in the form in which it will be used, and provides the relevant citations. No result in this section is new; the section serves to make the logical dependencies of the paper explicit and to fix notation.

2.1 Linearity of the Transport Closure Equations

The Q-series established the exchange-sector transport closure system as a deterministic coupled evolution for the closure density $\rho : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}_{\geq 0}$ and the transport phase $\phi : \mathbb{R}^3 \times \mathbb{R} \to \mathbb{R}$. The full (ρ, ϕ) system, recalled in QM1 Sec. **??**, is nonlinear: the continuity equation $\partial_t \rho + \nabla \cdot (\rho\, v) = 0$

and the phase transport equation $\partial_t \phi + v \cdot \nabla \phi = \mathcal{E}(x, t)$ both involve the product $\rho\, v$, which is nonlinear in the pair (ρ, ϕ).

The linearity that underlies the superposition principle operates at a different level. The Q-series established that in the integrable exchange-sector regime — the regime in which the holonomic closure condition is satisfied and the Schrödinger-type representation of Q3 is valid — the transport closure law takes a form that is linear in the complex state encoding $\Psi = \sqrt{\rho}\, e^{i\phi/\Phi_0}$. Specifically, the transport closure operator $\mathcal{L}$ of the Q-series satisfies

$$\mathcal{L}[\Psi] = 0 \tag{1}$$

where $\mathcal{L}$ is a linear differential operator in Ψ in the integrable regime [1]. Linearity of $\mathcal{L}$ is the key property: if $\mathcal{L}[\Psi_1] = 0$ and $\mathcal{L}[\Psi_2] = 0$, then for any $c_1, c_2 \in \mathbb{C}$,

$$\mathcal{L}[c_1\Psi_1 + c_2\Psi_2] = c_1\mathcal{L}[\Psi_1] + c_2\mathcal{L}[\Psi_2] = 0,$$

so the linear combination is also a solution.

Remark 2.1. *The linearity of $\mathcal{L}$ holds in the integrable exchange-sector regime and is not a universal property of the full (ρ, ϕ) system. The distinction is important. An arbitrary linear combination $c_1\Psi_1 + c_2\Psi_2$ of two complex state encodings need not correspond to a pair $(\tilde{\rho}, \tilde{\phi})$ with $\tilde{\rho} \geq 0$ everywhere, since the real part of the encoding may be negative for the superposition state in spatial regions where the two path states have opposite signs. The closure density of the superposition is $|\Psi_{AB}|^2$, which is non-negative by definition, and the cross-term structure of $|\Psi_{AB}|^2$ is derived in Sec. 4 without requiring a non-negative closure density for each of $c_1\Psi_1$ and $c_2\Psi_2$ separately. The admissibility of the superposition as a closure state in $\mathcal{H}$ follows from the linearity of $\mathcal{L}$; the interpretation of its closure density is addressed in Sec. 3.4.*

2.2 Admissible Closure States in $\mathcal{H}$

QM1 Sec. **??** established that not every element of $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$ arises from a transport closure configuration in the strict geometric sense, but that the full Hilbert space $\mathcal{H}$ is the natural completion of the space of admissible states and that working in $\mathcal{H}$ is justified on three grounds: norm-preserving limits of admissible sequences remain in $\mathcal{H}$, superpositions of admissible states are admissible (the content of the present paper, established in Sec. 3), and the spectral and functional-analytic theory required for the QM-series is fully developed for operators on $\mathcal{H}$.

The relevant property for the present paper is that the set of admissible closure states is closed under the linear operations of the Hilbert space: finite linear combinations, norm-convergent series, and norm-convergent integrals of admissible states are admissible. This closure property is the superposition principle and is derived in Sec. 3 from the linearity of the transport closure operator $\mathcal{L}$ recalled in Sec. 2.1. It is recorded here as the principal structural consequence of transport closure linearity that the present section prepares.

2.3 The Inner Product and Cauchy–Schwarz Inequality

The following results from QM1 are used directly in Secs. 3–6.

The closure inner product (QM1 Definition 4.1). For $\Psi_1, \Psi_2 \in \mathcal{H}$,

$$\langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}} := \int_{\mathbb{R}^3} \overline{\Psi_1(x)}\, \Psi_2(x)\, d^3x. \tag{2}$$

The induced norm is $\|\Psi\|_{\mathcal{H}} = \langle \Psi, \Psi \rangle_{\mathcal{H}}^{1/2}$, and the normalization condition $\|\Psi\|_{\mathcal{H}} = 1$ is equivalent to $\int_{\mathbb{R}^3} |\Psi(x)|^2 \, \mathrm{d}^3 x = 1$.

The Cauchy–Schwarz inequality (QM1 Lemma 4.2, property (iv)). For any $\Psi_1, \Psi_2 \in \mathcal{H}$,

$$|\langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}}|^2 \le \|\Psi_1\|_{\mathcal{H}}^2 \, \|\Psi_2\|_{\mathcal{H}}^2 . \tag{3}$$

Equality holds if and only if Ψ_1 and Ψ_2 are proportional. The Cauchy–Schwarz inequality is used in Sec. 4 to bound the interference cross-term, in Sec. 5 to bound the fringe visibility, and in Sec. 6 to derive the complementarity relation.

Expansion of the squared norm of a sum (QM1 Lemma 4.2, derived). For $\Psi_1, \Psi_2 \in \mathcal{H}$ and $c_1, c_2 \in \mathbb{C}$,

$$\|c_1\Psi_1 + c_2\Psi_2\|_{\mathcal{H}}^2 = |c_1|^2 \, \|\Psi_1\|_{\mathcal{H}}^2 + |c_2|^2 \, \|\Psi_2\|_{\mathcal{H}}^2 + 2\,\mathrm{Re}\big[c_1\overline{c_2} \, \langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}}\big]. \tag{4}$$

The third term on the right is the *global interference term*: its pointwise analogue $2\,\mathrm{Re}[c_1\overline{c_2}\,\overline{\Psi_1(x)}\Psi_2(x)]$ will appear in the local closure density in Sec. 4.

2.4 Phase Accumulation Along Transport Paths

The Q-series established that the transport phase $\phi(x,t)$ accumulated by an exchange-sector closure state along an admissible transport path is determined by the exchange rate $\mathcal{E}(x,t)$ along that path [1]. For a transport path γ from a source point x_0 to a field point x, the phase accumulated is

$$\phi_\gamma(x) = \int_\gamma \mathcal{E}(x', t') \, \mathrm{d}t', \tag{5}$$

where the integral is taken along the path γ in the (x,t) spacetime and $\mathcal{E}$ is the local exchange rate of the scalar–conformal transport system.

Two distinct paths γ_A and γ_B connecting the same source point to the same field point x accumulate in general different phases $\phi_A(x)$ and $\phi_B(x)$. The phase difference

$$\Delta\phi(x) \coloneqq \phi_B(x) - \phi_A(x) \tag{6}$$

is determined by the exchange rate along each path and the geometric length of each path. It is the central geometric quantity underlying interference: as derived in Sec. 4, the interference cross-term in the two-path closure density is proportional to $\cos(\Delta\phi(x)/\Phi_0)$, so the spatial variation of $\Delta\phi(x)$ across the detection screen produces the fringe pattern.

In the free transport sector (uniform scalar capacity $\Lambda = \Lambda_0$), the exchange rate is determined by the transport momentum p through the Q-series kinematic relation, and the phase accumulated along a straight path of length ℓ is

$$\phi = \frac{p\,\ell}{\Phi_0}, \tag{7}$$

identifying Φ_0 as the action quantum and recovering the de Broglie phase relation. For the double-slit geometry, the phase difference between two straight paths of lengths ℓ_A and ℓ_B is

$$\Delta\phi = \frac{p\,(\ell_B - \ell_A)}{\Phi_0}, \tag{8}$$

which drives the fringe pattern of Sec. 7.

Remark 2.2. *The phase difference* $\Delta\phi(x)$ *is a purely geometric quantity: it is determined by the scalar–conformal transport geometry, the path lengths, and the exchange rate structure. It does not depend on any probabilistic interpretation of the closure state. The interference pattern that* $\Delta\phi(x)$ *produces in the closure density at the screen is accordingly a geometric feature of the two-path transport configuration, not a probabilistic effect. This is the precise sense in which the NUVO account of interference is derivation-complete: the fringe pattern is a structural consequence of the scalar–conformal geometry and the linearity of the transport closure equations, with no wave ontology or probabilistic postulate required.*

2.5 The Born Frequency Law and Its Extension

The connection between the closure density $|\Psi(x)|^2$ and observable interaction-event frequencies is provided by the Born frequency law, established in QB6 and extended to the full Hilbert space $\mathcal{H}$ in QM1. Two results are recalled here.

The Born frequency law (QB6, extended in QM1 Remark 7.5). For a normalized closure state $\Psi \in \mathcal{H}$ and a spatial region $B \subset \mathbb{R}^3$, the asymptotic relative frequency of coherence-gated interaction events localized in B is

$$\lim_{N\to\infty} \frac{N_B}{N_{\mathrm{tot}}} = \int_B |\Psi(x)|^2 \, \mathrm{d}^3x, \tag{9}$$

where N_B is the number of events in B and N_{tot} is the total number of events. This is an asymptotic event-frequency law derived from the coherence-gated interaction structure of the transport system; it is not a probabilistic postulate.

Application to the two-path closure density. In the present paper, the Born frequency law is applied to the two-path closure state Ψ_{AB} at the detection screen. The asymptotic frequency of interaction events at screen position y is $|\Psi_{AB}(y)|^2 = \rho_{AB}(y)$, the two-path closure density derived in Sec. 4. The fringe pattern of Sec. 5 is therefore not merely a pattern in the closure density but simultaneously a prediction for the asymptotic spatial distribution of detection events, via the Born law. No new postulate is required for this identification; it follows from the extension of QB6 to $\mathcal{H}$ already established in QM1.

Remark 2.3. *The logical structure of the connection between the closure density and the observed fringe pattern is as follows. The interference cross-term* $2\,\mathrm{Re}[c_A\overline{c_B}\,\overline{\Psi_A(x)}\Psi_B(x)]$ *in the two-path closure density* $|\Psi_{AB}(x)|^2$ *is a structural consequence of the superposition principle (derived in Sec. 3) and the two-path transport geometry (analyzed in Sec. 4). The identification of* $|\Psi_{AB}(x)|^2$ *with the asymptotic event frequency at position* x *is provided by the Born frequency law of QB6 (recalled in this subsection). These are two logically independent results that together yield the prediction: the observed detection-event distribution exhibits fringes with the spacing and visibility derived in Sec. 5. Neither result depends on the other, and neither introduces a probabilistic postulate.*

3 The Superposition Principle

The complete Hilbert space $\mathcal{H}$ established in QM1 supports arbitrary norm-convergent linear combinations of closure states as elements of $\mathcal{H}$. Whether such linear combinations are *admissible*—in the sense of corresponding to transport closure configurations in the scalar–conformal exchange-sector system—is a separate question, answered by the transport closure linearity recalled in Sec. 2.1. The present section derives the superposition principle from that linearity, establishes the inner product structure of two-path superpositions, and records the interpretive constraints that govern the use of superposition states throughout the remainder of the paper.

3.1 Linearity of the Transport Closure Encoding

The central structural fact underlying the superposition principle is that the transport closure operator $\mathcal{L}$ of the Q-series, in the integrable exchange-sector regime, is linear in the complex state encoding Ψ. This fact was recalled in Sec. 2.1; it is now used to derive the admissibility of superposition states.

Lemma 3.1 (Linearity of the transport closure encoding)**.** *In the integrable exchange-sector regime in which the Q-series Schrödinger-type representation holds, the transport closure operator $\mathcal{L}$ of Eq.* (1) *is linear in the complex state encoding Ψ. Consequently, if $\Psi_1, \Psi_2 \in \mathcal{H}$ are admissible closure states satisfying $\mathcal{L}[\Psi_1] = 0$ and $\mathcal{L}[\Psi_2] = 0$, then for any $c_1, c_2 \in \mathbb{C}$, the linear combination $c_1\Psi_1 + c_2\Psi_2$ satisfies*

$$\mathcal{L}[c_1\Psi_1 + c_2\Psi_2] = c_1\,\mathcal{L}[\Psi_1] + c_2\,\mathcal{L}[\Psi_2] = 0, \tag{10}$$

and is therefore an admissible closure state.

Proof. The Q-series established that in the integrable exchange-sector regime, the transport closure law takes the form $\mathcal{L}[\Psi] = 0$ where $\mathcal{L}$ is a linear differential operator in Ψ [1]. Linearity of $\mathcal{L}$ means:

$$\mathcal{L}[c_1\Psi_1 + c_2\Psi_2] = c_1\,\mathcal{L}[\Psi_1] + c_2\,\mathcal{L}[\Psi_2]$$

for all $c_1, c_2 \in \mathbb{C}$ and all Ψ_1, Ψ_2 in the domain of $\mathcal{L}$. If $\mathcal{L}[\Psi_1] = 0$ and $\mathcal{L}[\Psi_2] = 0$, then Eq. (10) gives $\mathcal{L}[c_1\Psi_1 + c_2\Psi_2] = 0$, so the linear combination satisfies the transport closure law and is admissible. □

Remark 3.2. *The linearity established in Lemma 3.1 is a property of the transport closure operator $\mathcal{L}$ at the Ψ-level in the integrable regime. It is not a property of the full nonlinear (ρ, ϕ) system of the Q-series, as clarified in Remark 2.1. This distinction is important for the correct interpretation of the superposition state $c_1\Psi_1 + c_2\Psi_2$: the closure density of the superposition is $|c_1\Psi_1 + c_2\Psi_2|^2$, which is non-negative everywhere by virtue of being a squared modulus, even in spatial regions where the individual path states Ψ_1 and Ψ_2 have opposite complex phases. The superposition state is a well-defined element of $\mathcal{H}$ and an admissible closure state; its closure density is analyzed in Sec. 4.*

3.2 The Superposition Principle as a Theorem

Lemma 3.1 establishes the admissibility of finite linear combinations of two admissible states. The full superposition principle extends this to arbitrary finite combinations, norm-convergent series, and norm-convergent integral superpositions, drawing on both the transport closure linearity and the completeness of $\mathcal{H}$ established in QM1 Theorem 4.3.

Theorem 3.3 (Superposition principle)**.** *The set of admissible closure states in $\mathcal{H}$ is a linear subspace of $\mathcal{H}$. Specifically:*

(i) Finite superpositions: *For any admissible closure states $\Psi_1, \ldots, \Psi_N \in \mathcal{H}$ and coefficients $c_1, \ldots, c_N \in \mathbb{C}$, the state*

$$\Psi = \sum_{n=1}^{N} c_n \Psi_n$$

is admissible.

(ii) Norm-convergent series: *For any sequence of admissible states $\{\Psi_n\}_{n\geq 1}$ and coefficients $\{c_n\}_{n\geq 1} \in \ell^2(\mathbb{C})$ such that the partial sums $S_N = \sum_{n=1}^{N} c_n\Psi_n$ converge in the $\mathcal{H}$-norm to a limit Ψ, the limit Ψ is admissible.*

(iii) Continuous superpositions: *For any measurable family* $\{\Psi_k\}_{k\in\mathbb{R}}$ *of admissible states and coefficient function* $c \in L^2(\mathbb{R}, \mathbb{C})$ *such that the Bochner integral* $\Psi = \int_{\mathbb{R}} c(k)\, \Psi_k \, \mathrm{d}k$ *converges in the* $\mathcal{H}$*-norm, the integral superposition* Ψ *is admissible.*

Proof. Part (i): The base case $N = 2$ is Lemma 3.1. The case of general N follows by induction: if $\sum_{n=1}^{N-1} c_n\Psi_n$ is admissible (inductive hypothesis), then $c_N\Psi_N$ is admissible (by Lemma 3.1 with $c_1 = 1$, $\Psi_1 = \sum_{n=1}^{N-1} c_n\Psi_n$, $c_2 = c_N$, $\Psi_2 = \Psi_N$), and the sum of two admissible states is admissible by Lemma 3.1.

Part (ii): Each partial sum $S_N = \sum_{n=1}^{N} c_n\Psi_n$ is admissible by part (i). Since $\{c_n\} \in \ell^2$ and the Ψ_n are normalized, the partial sums form a Cauchy sequence in $\mathcal{H}$:

$$\|S_M - S_N\|_{\mathcal{H}} \le \sum_{n=N+1}^{M} |c_n|\; \|\Psi_n\|_{\mathcal{H}} \le \left(\sum_{n=N+1}^{M} |c_n|^2\right)^{1/2} \left(\sum_{n=N+1}^{M} \|\Psi_n\|_{\mathcal{H}}^2\right)^{1/2} \to 0$$

as $M, N \to \infty$, by the ℓ^2 assumption on $\{c_n\}$. By QM1 Theorem 4.3, $\mathcal{H}$ is complete, so the Cauchy sequence $\{S_N\}$ converges to a limit $\Psi \in \mathcal{H}$. Admissibility of Ψ follows from the closure of the set of admissible states under $\mathcal{H}$-norm limits: if $\mathcal{L}[S_N] = 0$ for all N and $S_N \to \Psi$ in $\mathcal{H}$-norm, then $\mathcal{L}[\Psi] = 0$ by the continuity of $\mathcal{L}$ in the appropriate Sobolev topology [1].

Part (iii): The Bochner integral $\Psi = \int_{\mathbb{R}} c(k)\Psi_k \, \mathrm{d}k$ is defined as the $\mathcal{H}$-norm limit of Riemann sums $\sum_j c(k_j)\Psi_{k_j}\Delta k_j$, each of which is admissible by part (i). The norm-convergence assumption guarantees the limit exists in $\mathcal{H}$ by QM1 Theorem 4.3, and admissibility of the limit follows from the same continuity argument as in part (ii) [3]. ☐

Remark 3.4. *The three parts of Theorem 3.3 establish the superposition principle at three levels of generality that are all used in the QM-series. Part (i) is used in the two-path interference analysis of the present paper and in QM5 for finite sums of angular momentum eigenstates. Part (ii) is used in QM6 for discrete superpositions of energy eigenstates (coherent states) and in QM9 for Schmidt decompositions of entangled two-particle states. Part (iii) is used in QM3 for superpositions of momentum eigenstates (wavepackets) and in QM10 for scattering state expansions over the continuous spectrum. In each case the admissibility of the superposition follows from the transport closure linearity of Lemma 3.1 and the completeness of* $\mathcal{H}$*, not from a separate postulate.*

3.3 The Inner Product Structure of Superpositions

For a two-path superposition $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$, the norm $\|\Psi_{AB}\|_{\mathcal{H}}$ and the closure density $|\Psi_{AB}(x)|^2$ both contain cross-terms that depend on the inner product $\langle\Psi_A, \Psi_B\rangle_{\mathcal{H}}$ and its pointwise analogue $\overline{\Psi_A(x)}\Psi_B(x)$. These cross-terms are the global and local forms of the interference term. The present subsection records their structure at the global (norm) level; the pointwise (closure density) level is treated in Sec. 4.

Proposition 3.5 (Norm of a two-path superposition)**.** *For admissible closure states* $\Psi_A, \Psi_B \in \mathcal{H}$ *and* $c_A, c_B \in \mathbb{C}$*, the two-path state* $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ *satisfies*

$$\|\Psi_{AB}\|_{\mathcal{H}}^2 = |c_A|^2\, \|\Psi_A\|_{\mathcal{H}}^2 + |c_B|^2\, \|\Psi_B\|_{\mathcal{H}}^2 + 2\,\mathrm{Re}\big[c_A\overline{c_B}\; \langle\Psi_A, \Psi_B\rangle_{\mathcal{H}}\big]. \tag{11}$$

The third term, $2\,\mathrm{Re}[c_A\overline{c_B}\; \langle\Psi_A, \Psi_B\rangle_{\mathcal{H}}]$*, is the* global interference term*. It satisfies*

$$\big|2\,\mathrm{Re}\big[c_A\overline{c_B}\; \langle\Psi_A, \Psi_B\rangle_{\mathcal{H}}\big]\big| \le 2|c_A||c_B|\; \|\Psi_A\|_{\mathcal{H}}\, \|\Psi_B\|_{\mathcal{H}} \tag{12}$$

by the Cauchy–Schwarz inequality Eq. (3)*, and vanishes if and only if* $\Psi_A \perp \Psi_B$ *in* $\mathcal{H}$*.*

Proof. Expand $\|\Psi_{AB}\|^2_{\mathcal{H}} = \langle \Psi_{AB}, \Psi_{AB}\rangle_{\mathcal{H}}$ using the linearity of the inner product in the second argument and conjugate linearity in the first:

$$\begin{aligned}\langle c_A\Psi_A + c_B\Psi_B, c_A\Psi_A + c_B\Psi_B\rangle_{\mathcal{H}} = |c_A|^2 \langle \Psi_A, \Psi_A\rangle_{\mathcal{H}} + c_A\overline{c_B}\,\langle \Psi_B, \Psi_A\rangle_{\mathcal{H}} \\ + \overline{c_A}c_B\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}} + |c_B|^2 \langle \Psi_B, \Psi_B\rangle_{\mathcal{H}}\,.\end{aligned}$$

The cross-terms combine as $c_A\overline{c_B}\,\langle \Psi_B, \Psi_A\rangle_{\mathcal{H}} + \overline{c_A}c_B\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}} = c_A\overline{c_B}\,\overline{\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}} + \overline{c_A}c_B\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}} = 2\,\mathrm{Re}[c_A\overline{c_B}\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}]$, using the conjugate symmetry $\langle \Psi_B, \Psi_A\rangle_{\mathcal{H}} = \overline{\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}}$ (QM1 Lemma 4.2(i)). Substituting $\|\Psi_A\|^2_{\mathcal{H}} = \langle \Psi_A, \Psi_A\rangle_{\mathcal{H}}$ and $\|\Psi_B\|^2_{\mathcal{H}} = \langle \Psi_B, \Psi_B\rangle_{\mathcal{H}}$ yields Eq. (11). The bound Eq. (12) follows from the Cauchy–Schwarz inequality Eq. (3): $|c_A\overline{c_B}\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}| = |c_A||c_B||\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}\,| \leq |c_A||c_B|\,\|\Psi_A\|_{\mathcal{H}}\,\|\Psi_B\|_{\mathcal{H}}$. Vanishing of the interference term if and only if $\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}} = 0$ is immediate. □

Remark 3.6. *For normalized path states $\|\Psi_A\|_{\mathcal{H}} = \|\Psi_B\|_{\mathcal{H}} = 1$ and coefficients satisfying $|c_A|^2 + |c_B|^2 = 1$, the normalization of Ψ_{AB} is*

$$\|\Psi_{AB}\|^2_{\mathcal{H}} = 1 + 2\,\mathrm{Re}\big[c_A\overline{c_B}\;\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}\big].$$

This equals unity if and only if the global interference term vanishes, i.e., if and only if $\Psi_A \perp \Psi_B$ in $\mathcal{H}$. When the path states Ψ_A and Ψ_B are not orthogonal—as is generically the case for two-path transport configurations whose path states overlap in the detection region—the normalization of Ψ_{AB} requires the coefficients c_A and c_B to account for the cross-term. For the symmetric two-path configuration with equal weights, the appropriate choice satisfying $\|\Psi_{AB}\|_{\mathcal{H}} = 1$ is $c_A = c_B = 1/\sqrt{2 + 2\,\mathrm{Re}\,\langle \Psi_A, \Psi_B\rangle_{\mathcal{H}}}$. In the double-slit geometry analyzed in Sec. 7, the path states Ψ_A and Ψ_B have spatially separated support in the source-to-barrier region and are approximately orthogonal, making $c_A = c_B = 1/\sqrt{2}$ the appropriate normalization in that regime.

3.4 Interpretive Note: Superposition Without Wave Ontology

The superposition state $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ is a closure state in $\mathcal{H}$, admissible by Theorem 3.3 (i). As established in QB1 and carried forward throughout the NUVO series, the complex state encoding Ψ is a representational object: it encodes the transport closure geometry compactly through the relation $\rho = |\Psi|^2$, $\phi = \Phi_0 \arg\Psi$, and carries no independent ontological status as a physical wave or oscillatory medium. The superposition state Ψ_{AB} inherits this representational character.

Several common locutions are therefore not available in the NUVO framework and are excluded from the present paper. The statement that the transport closure "passes through both slits simultaneously" is not a NUVO statement; the NUVO statement is that the admissible transport closure configurations at the detection screen are encoded by the state Ψ_{AB}, which carries phase information from both path channels. The statement that the superposition "is a wave" is not a NUVO statement; the NUVO statement is that $\Psi_{AB} \in \mathcal{H}$ is a complex-valued square-integrable function whose squared modulus is the closure density at the screen. The statement that "the particle goes through slit A or slit B with amplitudes c_A and c_B" conflates the closure state with a particle trajectory and is not adopted here.

What the superposition state does encode, in a precise and derivation-complete way, is the phase correlation between the two transport channels. This phase correlation is expressed in the cross-term $\overline{\Psi_A(x)}\Psi_B(x) = \sqrt{\rho_A(x)\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\Delta\phi(x)/\Phi_0}$ of the closure density, which is the geometric quantity derived in Sec. 4 and shown to produce the interference fringe pattern in Sec. 5. The interference is a consequence of this phase correlation; the phase correlation is a consequence of the transport geometry; and the transport geometry is a consequence of the scalar–conformal structure of the exchange sector established in the M-series. The derivation chain is complete without any appeal to wave ontology.

4 Two-Path Transport and the Closure Density

The superposition principle established in Sec. 3 guarantees that the linear combination of two admissible path states is itself an admissible closure state in $\mathcal{H}$. The present section turns from this structural result to the physical content it carries: when two transport channels are simultaneously open, what does the closure density at the detection screen look like, and how does it depend on the geometry of the two paths? The answer is given by Proposition 4.3, which derives the pointwise closure density of the two-path state and identifies the interference cross-term as a function of the local transport phase difference $\Delta\phi(x)$ between the two channels.

4.1 The Two-Path Transport Configuration

The geometric setting for the two-path analysis is established precisely before the closure density is computed.

Definition 4.1 (Two-path transport configuration)**.** *A* two-path transport configuration *consists of the following elements.*

(i) *A* source region $\mathcal{S} \subset \mathbb{R}^3$ *from which the transport closure originates, and a* detection region $\mathcal{D} \subset \mathbb{R}^3$ *in which the closure density is observed, with $\mathcal{S}$ and $\mathcal{D}$ spatially separated by a barrier.*

(ii) *Two* path channels *A and B: distinct spatial corridors through or around the barrier connecting $\mathcal{S}$ to $\mathcal{D}$.*

(iii) *Two* single-path closure states $\Psi_A, \Psi_B \in \mathcal{H}$*, each normalized to unity, with Ψ_A supported primarily along channel A and Ψ_B supported primarily along channel B in the source-to-barrier region. In the detection region $\mathcal{D}$, both Ψ_A and Ψ_B have non-negligible support, allowing the two path states to interfere there.*

(iv) *Complex coefficients $c_A, c_B \in \mathbb{C}$ representing the relative amplitude and phase of transport through each channel, satisfying*

$$|c_A|^2 + |c_B|^2 = 1 \tag{13}$$

in the regime where the path states are approximately orthogonal in $\mathcal{H}$ (see Remark 3.6).

The two-path closure state *is*

$$\Psi_{AB}(x) := c_A\Psi_A(x) + c_B\Psi_B(x), \tag{14}$$

which is admissible by Theorem 3.3 (i).

Remark 4.2. *The two path channels of Definition 4.1 are abstract: they represent any two spatially distinct transport routes from source to detector, whether defined by apertures in a physical barrier, by distinct arms of an interferometer, or by any other geometric configuration that admits two coherent transport channels. The double-slit experiment of Sec. 7 is the canonical realization, in which the two channels are the two slit apertures and the detection region is the observation screen. The formalism of the present section applies without modification to any two-channel transport configuration.*

4.2 The Closure Density of the Two-Path State

The closure density of the two-path state Ψ_{AB} is computed pointwise from the squared modulus $|\Psi_{AB}(x)|^2$. The polar decomposition of each path state, $\Psi_A(x) = \sqrt{\rho_A(x)}\,\mathrm{e}^{\mathrm{i}\phi_A(x)/\Phi_0}$ and $\Psi_B(x) = \sqrt{\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\phi_B(x)/\Phi_0}$, makes the dependence on the transport phases ϕ_A and ϕ_B explicit.

Proposition 4.3 (Closure density of the two-path state)**.** *Let $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ be the two-path closure state of Definition 4.1, with single-path states in polar form $\Psi_A(x) = \sqrt{\rho_A(x)}\,\mathrm{e}^{\mathrm{i}\phi_A(x)/\Phi_0}$ and $\Psi_B(x) = \sqrt{\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\phi_B(x)/\Phi_0}$. The closure density of Ψ_{AB} at position x is*

$$\rho_{AB}(x) \coloneqq |\Psi_{AB}(x)|^2 = |c_A|^2\rho_A(x) + |c_B|^2\rho_B(x) + \mathcal{I}(x), \tag{15}$$

where the local interference term *is*

$$\mathcal{I}(x) \coloneqq 2\,\mathrm{Re}\big[c_A\overline{c_B}\,\overline{\Psi_A(x)}\,\Psi_B(x)\big] = 2|c_A||c_B|\sqrt{\rho_A(x)\,\rho_B(x)}\,\cos\!\left(\frac{\Delta\phi(x)}{\Phi_0} + \arg(c_A) - \arg(c_B)\right), \tag{16}$$

and $\Delta\phi(x) \coloneqq \phi_B(x) - \phi_A(x)$ is the local phase difference *between the two path states at position x.*

Proof. Compute $|\Psi_{AB}(x)|^2$ directly:

$$\begin{aligned}
|\Psi_{AB}(x)|^2 &= |c_A\Psi_A(x) + c_B\Psi_B(x)|^2 \\
&= (c_A\Psi_A(x) + c_B\Psi_B(x))\overline{(c_A\Psi_A(x) + c_B\Psi_B(x))} \\
&= |c_A|^2|\Psi_A(x)|^2 + |c_B|^2|\Psi_B(x)|^2 + c_A\overline{c_B}\,\Psi_A(x)\overline{\Psi_B(x)} + \overline{c_A}c_B\,\overline{\Psi_A(x)}\Psi_B(x).
\end{aligned}$$

Since $|\Psi_A(x)|^2 = \rho_A(x)$ and $|\Psi_B(x)|^2 = \rho_B(x)$, and since $c_A\overline{c_B}\Psi_A(x)\overline{\Psi_B(x)} = \overline{c_A\overline{c_B}\overline{\Psi_A(x)}\Psi_B(x)}$ is the complex conjugate of the last term, the two cross-terms combine as $2\,\mathrm{Re}[c_A\overline{c_B}\,\overline{\Psi_A(x)}\Psi_B(x)]$, giving Eq. (15).

To obtain the explicit form Eq. (16), substitute the polar representations:

$$\begin{aligned}
\overline{\Psi_A(x)}\,\Psi_B(x) &= \sqrt{\rho_A(x)}\,\mathrm{e}^{-\mathrm{i}\phi_A(x)/\Phi_0}\cdot\sqrt{\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\phi_B(x)/\Phi_0} \\
&= \sqrt{\rho_A(x)\,\rho_B(x)}\,\mathrm{e}^{\mathrm{i}(\phi_B(x)-\phi_A(x))/\Phi_0} \\
&= \sqrt{\rho_A(x)\,\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\Delta\phi(x)/\Phi_0}.
\end{aligned}$$

Writing $c_A\overline{c_B} = |c_A||c_B|\,\mathrm{e}^{\mathrm{i}(\arg c_A - \arg c_B)}$ and taking the real part:

$$\mathcal{I}(x) = 2\,\mathrm{Re}\Big[|c_A||c_B|\sqrt{\rho_A\rho_B}\,\mathrm{e}^{\mathrm{i}(\Delta\phi/\Phi_0 + \arg c_A - \arg c_B)}\Big] = 2|c_A||c_B|\sqrt{\rho_A\rho_B}\,\cos\!\left(\frac{\Delta\phi(x)}{\Phi_0} + \arg c_A - \arg c_B\right),$$

which is Eq. (16). □

Remark 4.4. *Three features of Eq.* (15) *and Eq.* (16) *are central to the subsequent analysis.*

(a) The incoherent baseline. *The first two terms $|c_A|^2\rho_A(x) + |c_B|^2\rho_B(x)$ represent the closure density that would be obtained if the two path states were statistically independent—if no phase correlation existed between them. This is the incoherent sum, and it is the closure density that results from which-path detection, as shown in Sec. 6.*

(b) The interference term as cosine modulation. *The local interference term $\mathcal{I}(x)$ is a cosine of the local phase difference $\Delta\phi(x)/\Phi_0$, modulated in amplitude by $2|c_A||c_B|\sqrt{\rho_A(x)\rho_B(x)}$. As x varies across the detection region, $\Delta\phi(x)$ varies continuously, producing a spatially oscillating pattern in* *the total closure density $\rho_{AB}(x)$. This spatial oscillation is the interference fringe pattern derived in Sec. 5.*

(c) Dependence on path geometry alone. *The interference term depends on $\Delta\phi(x)$, which is determined entirely by the transport phase accumulated along each path from the source to x—a purely geometric quantity. It does not depend on the overall phase of either path state (which cancels in the phase difference) or on any probabilistic feature of the closure density. The fringe pattern is therefore a direct readout of the path geometry encoded in the scalar–conformal transport structure.*

4.3 The Phase Difference as the Geometric Source of Interference

The local phase difference $\Delta\phi(x) = \phi_B(x) - \phi_A(x)$ is the quantity that controls the spatial structure of the interference pattern. Its form in the free transport sector is established here, connecting the abstract phase accumulation of Sec. 2.4 to the explicit fringe-spacing formula of Sec. 5.

In the free transport sector ($\Lambda = \Lambda_0$ uniform), the phase accumulated by a closure state of definite transport momentum p along a straight path of length ℓ from aperture to detection point is given by Eq. (7): $\phi = p\ell/\Phi_0$. For the two-path configuration with path channels A and B having path lengths $\ell_A(x)$ and $\ell_B(x)$ to detection point x, the phase difference is

$$\Delta\phi(x) = \frac{p}{\Phi_0}\big(\ell_B(x) - \ell_A(x)\big). \tag{17}$$

The local interference term Eq. (16) therefore takes the form

$$\mathcal{I}(x) = 2|c_A||c_B|\sqrt{\rho_A(x)\,\rho_B(x)}\,\cos\!\left(\frac{p}{\Phi_0}\big(\ell_B(x) - \ell_A(x)\big) + \arg c_A - \arg c_B\right). \tag{18}$$

Lemma 4.5 (Phase difference in the small-angle approximation)**.** *For a two-path configuration with path channels separated by distance d at the barrier and a detection screen at distance L from the barrier, in the small-angle approximation $y \ll L$ where y is the lateral screen coordinate, the path-length difference is*

$$\ell_B(y) - \ell_A(y) \approx \frac{d\,y}{L}, \tag{19}$$

and the phase difference is

$$\Delta\phi(y) \approx \frac{p\,d\,y}{\Phi_0\,L} = \frac{2\pi d\,y}{\lambda\,L}, \tag{20}$$

where $\lambda := 2\pi\Phi_0/p = h/p$ is the de Broglie wavelength associated with transport momentum p, and $h = 2\pi\Phi_0$ is the Planck constant recovered from the Q-series identification $\Phi_0 = \hbar$.

Proof. Place channel A at lateral position $+d/2$ and channel B at $-d/2$ at the barrier plane. The path length from channel A to screen position y at distance L is $\ell_A(y) = \sqrt{L^2 + (y - d/2)^2}$, and from channel B is $\ell_B(y) = \sqrt{L^2 + (y + d/2)^2}$. In the small-angle limit $y, d \ll L$, expand to first order:

$$\begin{aligned}\ell_B(y) - \ell_A(y) &\approx L\left[1 + \frac{(y + d/2)^2}{2L^2}\right] - L\left[1 + \frac{(y - d/2)^2}{2L^2}\right] \\ &= \frac{(y + d/2)^2 - (y - d/2)^2}{2L} = \frac{2yd}{2L} = \frac{d\,y}{L},\end{aligned}$$

which is Eq. (19). Substituting into Eq. (17) gives Eq. (20), with $\lambda = 2\pi\Phi_0/p$ identified as the de Broglie wavelength. □

Remark 4.6. *The de Broglie wavelength $\lambda = 2\pi\Phi_0/p = h/p$ emerges in Lemma 4.5 as the natural length scale of the phase difference in the free transport sector. It is not introduced as an additional assumption: it follows from the transport phase relation $\phi = p\ell/\Phi_0$ of Eq. (7) and the Q-series identification $\Phi_0 = \hbar$. The fringe spacing $\Delta y = \lambda L/d$, derived in Sec. 5, is therefore a direct consequence of the scalar–conformal transport phase structure, not a postulate about the wave nature of the closure state. This constitutes an internal consistency check on the NUVO framework: the double-slit fringe spacing predicted by the transport closure geometry agrees with the de Broglie formula derived from the hydrogenic correspondence of the Q-series.*

5 The Interference Pattern

The two-path closure density of Proposition 4.3 contains a spatially oscillating interference term $\mathcal{I}(x)$ whose argument is the local phase difference $\Delta\phi(x)/\Phi_0$. The present section makes this oscillation explicit as a fringe pattern at the detection screen, derives its spatial period and amplitude, defines the fringe visibility as a measure of fringe contrast, and records the conditions on the phase difference for constructive and destructive interference. All results follow directly from Proposition 4.3 and Lemma 4.5; no new physical input is required.

5.1 The Fringe Pattern at the Detection Screen

The symmetric two-path configuration—equal coefficients $|c_A| = |c_B| = 1/\sqrt{2}$, equal single-path envelope densities $\rho_A(y) = \rho_B(y) =: \rho_0(y)$, and real equal-phase coefficients $\arg c_A = \arg c_B = 0$—is the case that produces maximum fringe contrast and corresponds to the standard double-slit geometry with a symmetric source. The general case with unequal coefficients is treated in Sec. 5.2.

Theorem 5.1 (Interference fringe pattern)**.** *For the symmetric two-path configuration with $c_A = c_B = 1/\sqrt{2}$, equal single-path closure densities $\rho_A(y) = \rho_B(y) =: \rho_0(y)$, and the small-angle phase difference of Lemma 4.5, the two-path closure density at screen position y is*

$$\rho_{AB}(y) = 2\rho_0(y)\left[1 + \cos\!\left(\frac{2\pi d\, y}{\lambda\, L}\right)\right], \tag{21}$$

where $\lambda = 2\pi\Phi_0/p$ is the de Broglie wavelength of Lemma 4.5. The fringe pattern is a spatially oscillating modulation of the single-path envelope $\rho_0(y)$, with spatial period

$$\Delta y = \frac{\lambda\, L}{d}. \tag{22}$$

Proof. Substitute $c_A = c_B = 1/\sqrt{2}$, $\rho_A = \rho_B = \rho_0$, and $\arg c_A - \arg c_B = 0$ into Eq. (15) and Eq. (16):

$$\begin{aligned}\rho_{AB}(y) &= \tfrac{1}{2}\rho_0(y) + \tfrac{1}{2}\rho_0(y) + 2\cdot\tfrac{1}{\sqrt{2}}\cdot\tfrac{1}{\sqrt{2}}\cdot\sqrt{\rho_0(y)\,\rho_0(y)}\,\cos\!\left(\frac{\Delta\phi(y)}{\Phi_0}\right)\\ &= \rho_0(y) + \rho_0(y)\,\cos\!\left(\frac{\Delta\phi(y)}{\Phi_0}\right)\\ &= \rho_0(y)\left[1 + \cos\!\left(\frac{\Delta\phi(y)}{\Phi_0}\right)\right].\end{aligned}$$

Substituting the small-angle phase difference $\Delta\phi(y)/\Phi_0 = 2\pi d\, y/(\lambda\, L)$ from Eq. (20) gives Eq. (21). The fringe spacing Eq. (22) is the period of the cosine factor, obtained by setting the argument equal to 2π: $2\pi d\,\Delta y/(\lambda\, L) = 2\pi$, giving $\Delta y = \lambda\, L/d$. ☐

Remark 5.2. *The factor $\rho_0(y)$ in Eq. (21) is the single-path diffraction envelope: the spatial distribution of the closure density that would be observed if only one path channel were open. In the double-slit geometry it is determined by the diffraction of the closure state through a single slit aperture of finite width and decays away from the forward direction. The interference pattern Eq. (21) is the product of this diffraction envelope with the two-path cosine modulation, so the fringes are visible only within the spatial extent of $\rho_0(y)$. The diffraction envelope itself is a single-path effect and does not involve the interference cross-term; it is inherited from the single-path closure states Ψ_A and Ψ_B and is not derived in the present paper. The interference modulation factor $[1+\cos(2\pi d\, y/\lambda L)]$ is the two-path effect and is the content of the present section.*

Remark 5.3. *The fringe pattern Eq. (21) depends on three geometric quantities: the de Broglie wavelength λ, the channel separation d, and the screen distance L. All three are determined by the scalar–conformal transport geometry and the momentum of the closure state. The pattern does not depend on any probabilistic feature of the closure density, on the identification of $|\Psi|^2$ with a probability density, or on any wave-ontological assumption about the closure state. It is a structural consequence of the two-path phase difference $\Delta\phi(y)$, which is a geometric quantity derived in Lemma 4.5 from the path-length difference in the small-angle approximation. The Born frequency law of QB6, recalled in Sec. 2.5, then identifies the fringe pattern as the asymptotic distribution of detection events at the screen, without introducing any additional postulate.*

5.2 Fringe Visibility

The fringe visibility quantifies the contrast of the interference pattern: the degree to which the two-path closure density oscillates relative to its mean value. It depends on both the amplitude coefficients $|c_A|$ and $|c_B|$ and the degree of overlap between the single-path closure densities ρ_A and ρ_B at the screen.

Definition 5.4 (Fringe visibility)**.** *For a two-path closure density $I(y) = \rho_{AB}(y)$ with spatial maximum $I_{\max}$ and spatial minimum $I_{\min}$ over the detection region, the* fringe visibility *is*

$$\mathcal{V} := \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}. \tag{23}$$

The visibility satisfies $\mathcal{V} \in [0,1]$, with $\mathcal{V} = 1$ corresponding to maximum fringe contrast (the minimum closure density reaches zero) and $\mathcal{V} = 0$ corresponding to no fringe pattern (the closure density is spatially uniform).

Proposition 5.5 (Fringe visibility of the two-path pattern)**.** *For the two-path closure density of Proposition 4.3 with equal single-path envelope densities $\rho_A(y) = \rho_B(y) =: \rho_0(y)$ at the screen, the fringe visibility is*

$$\mathcal{V} = 2|c_A||c_B|. \tag{24}$$

Under the normalization constraint $|c_A|^2 + |c_B|^2 = 1$, the visibility satisfies $\mathcal{V} \in [0,1]$, with:

(i) $\mathcal{V} = 1$ *(maximum visibility) when $|c_A| = |c_B| = 1/\sqrt{2}$, i.e., equal-weight superposition;*

(ii) $\mathcal{V} = 0$ *(no fringes) when $|c_A| = 1, |c_B| = 0$ or vice versa, i.e., single-path transport.*

Proof. Substitute $\rho_A = \rho_B = \rho_0$ into Eqs. (15) and (16):

$$I(y) = (|c_A|^2 + |c_B|^2)\rho_0(y) + 2|c_A||c_B|\rho_0(y)\,\cos\!\left(\frac{\Delta\phi(y)}{\Phi_0}\right) = \rho_0(y)\left[1 + 2|c_A||c_B|\cos\!\left(\frac{\Delta\phi(y)}{\Phi_0}\right)\right],$$

using $|c_A|^2 + |c_B|^2 = 1$ from Eq. (13). The maximum and minimum of $I(y)$ occur where the cosine equals $+1$ and -1 respectively:

$$I_{\max} = \rho_0(y)\,(1 + 2|c_A||c_B|), \qquad I_{\min} = \rho_0(y)\,(1 - 2|c_A||c_B|).$$

Substituting into Definition 5.4:

$$\mathcal{V} = \frac{(1 + 2|c_A||c_B|) - (1 - 2|c_A||c_B|)}{(1 + 2|c_A||c_B|) + (1 - 2|c_A||c_B|)} = \frac{4|c_A||c_B|}{2} = 2|c_A||c_B|,$$

which is Eq. (24). Parts (i) and (ii) follow by substituting the respective coefficient values and using the AM-GM inequality $|c_A||c_B| \leq (|c_A|^2+|c_B|^2)/2 = 1/2$, with equality if and only if $|c_A| = |c_B|$. □

Remark 5.6. *The visibility bound $\mathcal{V} \leq 1$ is a consequence of the Cauchy–Schwarz inequality on $\mathcal{H}$. From Eq.* (12)*, the global interference term satisfies $|2\,\mathrm{Re}[c_A\overline{c_B}\,\langle\Psi_A, \Psi_B\rangle_{\mathcal{H}}]| \leq 2|c_A||c_B|\,\|\Psi_A\|_{\mathcal{H}}\,\|\Psi_B\|_{\mathcal{H}} = 2|c_A||c_B|$ for normalized path states. The local pointwise bound $|\mathcal{I}(y)| \leq 2|c_A||c_B|\rho_0(y)$ follows by the same argument applied pointwise. These bounds guarantee that the closure density remains non-negative: $I(y) \geq \rho_0(y)(1 - 2|c_A||c_B|) \geq 0$ for all y, with the minimum reaching zero precisely when $\mathcal{V} = 1$ and the cosine equals -1. The non-negativity of $I(y)$ is structurally guaranteed by the representation $I(y) = |\Psi_{AB}(y)|^2 \geq 0$; the Cauchy–Schwarz bound provides the quantitative upper limit on the oscillation amplitude.*

5.3 Constructive and Destructive Interference

The conditions for maximum and minimum closure density at the screen follow directly from the cosine factor in the fringe pattern. They are recorded as a corollary of Theorem 5.1 and stated in terms of the path-length difference, connecting to the geometric picture of Sec. 4.3.

Corollary 5.7 (Conditions for constructive and destructive interference)**.** *For the two-path closure density of Theorem 5.1, closure density maxima (*constructive interference*) occur at screen positions y satisfying*

$$\ell_B(y) - \ell_A(y) = n\lambda, \qquad n \in \mathbb{Z}, \tag{25}$$

*and closure density minima (*destructive interference*), at which $I(y) = 0$ for $\mathcal{V} = 1$, occur at positions satisfying*

$$\ell_B(y) - \ell_A(y) = \big(n + \tfrac{1}{2}\big)\lambda, \qquad n \in \mathbb{Z}. \tag{26}$$

Proof. Constructive interference corresponds to $\cos(\Delta\phi/\Phi_0) = 1$, i.e., $\Delta\phi/\Phi_0 = 2n\pi$ for $n \in \mathbb{Z}$. Substituting $\Delta\phi = p(\ell_B - \ell_A)/\Phi_0$ from Eq. (17): $p(\ell_B - \ell_A)/\Phi_0^2 = 2n\pi$, i.e., $\ell_B - \ell_A = 2n\pi\Phi_0/p = n\lambda$. Destructive interference corresponds to $\cos(\Delta\phi/\Phi_0) = -1$, i.e., $\Delta\phi/\Phi_0 = (2n+1)\pi$, giving $\ell_B - \ell_A = (2n+1)\pi\Phi_0/p = (n + \frac{1}{2})\lambda$. □

Remark 5.8. *Equations* (25) *and* (26) *state that constructive interference occurs when the path-length difference is an integer multiple of the de Broglie wavelength, and destructive interference when it is a half-integer multiple. These conditions are the direct transport-closure analogues of the classical optical path-difference conditions for wave interference, obtained here from the transport phase difference $\Delta\phi = p(\ell_B - \ell_A)/\Phi_0$ without any reference to wave optics. The de Broglie wavelength $\lambda = 2\pi\Phi_0/p = h/p$ appears as the natural unit of path-length difference not because the closure state is a wave but because h/p is the length scale over which the transport phase completes a full cycle of 2π, as determined by the scalar–conformal transport geometry and the Q-series identification $\Phi_0 = \hbar$.*

5.4 The General (Asymmetric) Fringe Pattern

For completeness, the fringe pattern for the general two-path configuration with unequal coefficients and unequal single-path envelope densities is recorded. This generalization is needed for the partially coherent regime of Sec. 6 and the three-regime analysis of Sec. 7.3.

Corollary 5.9 (General two-path fringe pattern)**.** *For the two-path closure density of Proposition 4.3 with general coefficients c_A, c_B and single-path envelope densities $\rho_A(y)$ and $\rho_B(y)$ (not necessarily equal), the fringe pattern is*

$$I(y) = |c_A|^2 \rho_A(y) + |c_B|^2 \rho_B(y) + 2|c_A||c_B|\sqrt{\rho_A(y)\,\rho_B(y)}\,\cos\!\left(\frac{2\pi d\,y}{\lambda L} + \arg c_A - \arg c_B\right), \tag{27}$$

and the fringe visibility at position y is

$$\mathcal{V}(y) = \frac{2|c_A||c_B|\sqrt{\rho_A(y)\,\rho_B(y)}}{|c_A|^2 \rho_A(y) + |c_B|^2 \rho_B(y)}. \tag{28}$$

The fringe visibility is maximized when $\rho_A(y) = \rho_B(y)$ and $|c_A| = |c_B|$, reducing to Eq. (24) *in that case.*

Proof. Equation (27) is Proposition 4.3 with the small-angle phase difference of Lemma 4.5 substituted. For the general visibility Eq. (28): the maximum of $I(y)$ at fixed y is achieved when the cosine equals $+1$ and the minimum when it equals -1. Substituting into Definition 5.4 and simplifying gives Eq. (28). The AM-GM inequality applied to $2\sqrt{\rho_A \rho_B} \le \rho_A + \rho_B$ and $2|c_A||c_B| \le |c_A|^2 + |c_B|^2$ confirms $\mathcal{V}(y) \le 1$ with equality under the stated conditions. □

Remark 5.10. *The term* $\arg c_A - \arg c_B$ *in Eq.* (27) *is a constant phase offset that shifts the entire fringe pattern laterally without affecting the fringe spacing or visibility. It reflects the relative phase of the transport amplitude through each channel, which is determined by the geometry of the source and the channel openings. For a symmetric source with equal-phase amplitudes* $\arg c_A = \arg c_B$*, the offset vanishes and the central fringe (at $y = 0$, where $\Delta\phi = 0$) is a maximum. A phase offset can arise physically when the two channels are at different distances from the source or when the scalar capacity field Λ differs between the two paths in the source region.*

6 Which-Path Detection as Coherence Disruption

The interference pattern derived in Sec. 5 depends on the coherence function $\gamma_{AB}(x) = \overline{\Psi_A(x)}\Psi_B(x)$: the pointwise product of the two path states that generates the oscillating cross-term in the closure density. The present section analyzes what happens to this cross-term when a which-path detecting interaction is introduced. The central result is that any interaction that acquires information about which channel the transport closure passed through necessarily introduces a phase disturbance on at least one path, and that when this disturbance is uncontrolled—as it must be for a complete which-path measurement—the cross-term averages to zero over the ensemble of interaction outcomes, eliminating the fringe pattern.

This analysis is carried out entirely within the NUVO transport closure framework. No wavefunction collapse, no projection postulate, and no measurement axiom is invoked. The mechanism is phase de-correlation: the coherent phase relationship between Ψ_A and Ψ_B that generates the cross-term is disrupted by the which-path interaction, and the disruption is structural rather than phenomenological. The section closes by deriving the complementarity relation $\mathcal{V}^2 + \mathcal{W}^2 \le 1$ from the Cauchy–Schwarz inequality on $\mathcal{H}$, establishing wave-particle complementarity as an algebraic theorem rather than a principle.

6.1 The Coherence Function and Its Role

The local interference term $\mathcal{I}(x)$ of Eq. (16) is determined pointwise by the product $\overline{\Psi_A(x)}\Psi_B(x)$, which encodes the amplitude and phase of the correlation between the two path states at x. This quantity is now defined as the coherence function of the two-path configuration.

Definition 6.1 (Coherence function and degree of coherence)**.** *For a two-path state with single-path states $\Psi_A, \Psi_B \in \mathcal{H}$, the* coherence function *at position x is*

$$\gamma_{AB}(x) \coloneqq \overline{\Psi_A(x)}\,\Psi_B(x) = \sqrt{\rho_A(x)\,\rho_B(x)}\,\mathrm{e}^{\mathrm{i}\Delta\phi(x)/\Phi_0}, \tag{29}$$

where the second expression uses the polar decomposition of Ψ_A and Ψ_B established in the proof of Proposition 4.3. The degree of coherence *at x is*

$$\mu_{AB}(x) \coloneqq \frac{|\gamma_{AB}(x)|}{\sqrt{\rho_A(x)\,\rho_B(x)}} = 1, \tag{30}$$

which equals unity for all x where both single-path densities are non-zero, reflecting the fact that the two-path closure state Ψ_{AB} of Definition 4.1 is a pure coherent superposition with fully defined relative phase $\Delta\phi(x)$ everywhere in the detection region.

Remark 6.2. *The coherence function $\gamma_{AB}(x)$ is a purely geometric quantity: it is determined by the closure densities $\rho_A(x)$ and $\rho_B(x)$ and the phase difference $\Delta\phi(x)$, all of which are fixed by the scalar–conformal transport geometry of the two-path configuration. For a pure two-path closure state as in Definition 4.1, the degree of coherence is identically one wherever both path states have non-zero support. The local interference term of Eq.* (16) *can be written as*

$$\mathcal{I}(x) = 2\,\mathrm{Re}\big[c_A\overline{c_B}\,\gamma_{AB}(x)\big],$$

making the dependence on the coherence function explicit. The fringe pattern exists because $\gamma_{AB}(x) \neq 0$ and its phase $\Delta\phi(x)/\Phi_0$ varies across the screen. Which-path detection disrupts $\gamma_{AB}(x)$, reducing the degree of coherence from one toward zero and correspondingly reducing the fringe visibility from $\mathcal{V}$ toward zero.

6.2 Which-Path Detection as Phase De-Correlation

A which-path detecting interaction is one that acquires information about whether the transport closure proceeded via channel A or channel B. In the NUVO framework, such an interaction is a coherence-gated interaction event, as established in QB5, that is sufficiently localized to one channel that its outcome resolves the path identity. The key consequence of path-resolving interactions for the closure density is established in the following theorem.

The mechanism is as follows. A path-resolving interaction localized to channel A (or B) is, by the coherence-gated interaction framework of QB5, an event that distinguishes the transport closure configuration of channel A from that of channel B. Such an event necessarily involves a coupling of the interaction system to the transport phase of the channel it monitors. This coupling introduces a phase shift $\delta\phi$ on the monitored channel whose value is determined by the details of the interaction but is not controllable from outside—it is, in the language of the transport closure system, an uncontrolled modulation of the local exchange rate along that channel. When the which-path detector operates and a path-resolving event occurs, the phase of the monitored path state acquires a contribution $\delta\phi$ that varies across the ensemble of interaction events.

Theorem 6.3 (Which-path detection destroys interference)**.** *Let a which-path detecting interaction introduce an uncontrolled phase shift $\delta\phi$ on channel B, uniformly distributed over $[0, 2\pi)$ across the ensemble of interaction events. Then the ensemble-averaged closure density at the screen is*

$$\big\langle \rho_{AB}(x) \big\rangle_{\delta\phi} = |c_A|^2 \rho_A(x) + |c_B|^2 \rho_B(x), \tag{31}$$

with the interference term completely absent. The ensemble-averaged fringe visibility is $\langle \mathcal{V} \rangle = 0$.

Proof. The which-path phase shift modifies the path state Ψ_B to $\tilde{\Psi_B}(x) = \mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}\Psi_B(x)$, where $\delta\phi$ is the uncontrolled phase shift acquired during the path-resolving interaction. The modified two-path closure density is

$$\begin{aligned}\widetilde{\rho}_{AB}(x) &= |c_A\Psi_A(x) + c_B\tilde{\Psi_B}(x)|^2 \\ &= |c_A|^2\rho_A(x) + |c_B|^2\rho_B(x) + 2\,\mathrm{Re}\big[c_A\overline{c_B}\,\overline{\Psi_A(x)}\,\mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}\Psi_B(x)\big].\end{aligned}$$

The interference term for the modified state is

$$\widetilde{\mathcal{I}}(x) = 2\,\mathrm{Re}\big[c_A\overline{c_B}\,\mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}\,\gamma_{AB}(x)\big] = 2\,\mathrm{Re}\big[c_A\overline{c_B}\,|\gamma_{AB}(x)|\,\mathrm{e}^{\mathrm{i}(\Delta\phi(x)/\Phi_0 + \delta\phi/\Phi_0)}\big].$$

Averaging over $\delta\phi$ uniformly distributed on $[0, 2\pi)$:

$$\big\langle \mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0} \big\rangle_{\delta\phi} = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}\,\mathrm{d}(\delta\phi) = 0,$$

since the integral of a complex exponential over a full period vanishes. Therefore $\langle\widetilde{\mathcal{I}}(x)\rangle_{\delta\phi} = 0$, and the ensemble-averaged closure density reduces to Eq. (31). □

Remark 6.4. *Theorem 6.3 does not invoke wavefunction collapse, a projection postulate, or any measurement axiom. The disappearance of the interference term is a consequence of phase averaging over the ensemble of interaction outcomes: the which-path interaction introduces a random phase shift whose distribution over the ensemble has zero mean complex exponential, causing the cross-term to vanish in the ensemble average. The individual closure densities $\rho_A(x)$ and $\rho_B(x)$ are unaffected; only their coherent cross-term is disrupted. This is the precise sense in which which-path detection is a* coherence disruption *rather than a wavefunction collapse: the transport closure densities of the two paths are not altered by the detection, but their phase correlation is destroyed.*

Remark 6.5. *Theorem 6.3 treats the limiting case of complete which-path detection, in which $\delta\phi$ is uniformly distributed over the full $[0, 2\pi)$ interval. For a partial which-path detector—one that acquires incomplete path information and introduces a phase shift $\delta\phi$ with a distribution of variance $\sigma^2 < \infty$—the ensemble average of $\mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}$ does not vanish but takes the value $|\langle \mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}\rangle| = \mathrm{e}^{-\sigma^2/(2\Phi_0^2)} \in (0, 1)$ for a Gaussian phase distribution. The interference term is attenuated by this factor rather than eliminated, giving a partially coherent fringe pattern with reduced visibility $\mathcal{V}' = \mathcal{V} \cdot \mathrm{e}^{-\sigma^2/(2\Phi_0^2)} < \mathcal{V}$. This partial coherence regime is the continuously variable intermediate case between full interference ($\sigma = 0$) and no interference ($\sigma \to \infty$), and it connects naturally to the three-regime analysis of Sec. 7.3.*

6.3 The Which-Path Distinguishability

The fringe visibility $\mathcal{V}$ quantifies how well the interference pattern is visible; an orthogonal measure is needed to quantify how well the two paths can be distinguished from each other. The which-path distinguishability is defined as the maximum probability of correctly identifying the path using an optimal measurement on the coefficient structure of the two-path state.

Definition 6.6 (Which-path distinguishability)**.** *For a normalized two-path closure state with coefficients c_A, c_B satisfying $|c_A|^2 + |c_B|^2 = 1$, the* which-path distinguishability *is*

$$\mathcal{W} := \big||c_A|^2 - |c_B|^2\big|, \tag{32}$$

which is the absolute difference of the single-path closure contents of the two-path state. The distinguishability satisfies $\mathcal{W} \in [0, 1]$, with $\mathcal{W} = 1$ when the transport is entirely via one channel (complete path information available) and $\mathcal{W} = 0$ when $|c_A| = |c_B| = 1/\sqrt{2}$ (no path information available from the coefficient weights alone).

Remark 6.7. *The which-path distinguishability $\mathcal{W}$ of Definition 6.6 measures the asymmetry in the closure content assigned to each channel by the two-path state. When $\mathcal{W} = 1$ one channel carries all the closure content and the other carries none; an optimal measurement can identify the path with certainty. When $\mathcal{W} = 0$ the closure content is equally shared between the two channels; an optimal measurement cannot do better than random guessing. For intermediate values, $\mathcal{W}$ quantifies the degree to which the closure content asymmetry provides path information, independently of any which-path detector introduced into the experiment. This is the* a priori *path information encoded in the coefficient structure of the two-path state, as distinct from the* a posteriori *path information acquired by a detector.*

6.4 The Complementarity Relation

The fringe visibility $\mathcal{V} = 2|c_A||c_B|$ of Eq. (24) and the which-path distinguishability $\mathcal{W} = ||c_A|^2 - |c_B|^2|$ of Eq. (32) are both functions of the same pair $(|c_A|, |c_B|)$ subject to the normalization $|c_A|^2 + |c_B|^2 = 1$. Their algebraic relationship constitutes the complementarity relation.

Theorem 6.8 (Wave-particle complementarity)**.** *For a normalized two-path closure state with fringe visibility $\mathcal{V} = 2|c_A||c_B|$ and which-path distinguishability $\mathcal{W} = ||c_A|^2 - |c_B|^2|$,*

$$\mathcal{V}^2 + \mathcal{W}^2 = 1. \tag{33}$$

More generally, for a partially coherent two-path configuration in which the coherence function has degree of coherence $\mu_{AB} \in [0, 1]$ (as arises from partial which-path detection, Remark 6.5), the effective visibility satisfies $\mathcal{V}_{\text{eff}} \le 2|c_A||c_B|$ and the complementarity relation becomes the inequality

$$\mathcal{V}_{\text{eff}}^2 + \mathcal{W}^2 \le 1, \tag{34}$$

with the inequality a consequence of the Cauchy–Schwarz inequality on $\mathcal{H}$.

Proof. Pure state case, Eq. (33)*:* Compute directly:

$$\begin{aligned}\mathcal{V}^2 + \mathcal{W}^2 &= (2|c_A||c_B|)^2 + \big(|c_A|^2 - |c_B|^2\big)^2 \\ &= 4|c_A|^2|c_B|^2 + |c_A|^4 - 2|c_A|^2|c_B|^2 + |c_B|^4 \\ &= |c_A|^4 + 2|c_A|^2|c_B|^2 + |c_B|^4 \\ &= \big(|c_A|^2 + |c_B|^2\big)^2 = 1,\end{aligned}$$

using $|c_A|^2 + |c_B|^2 = 1$ from Eq. (13).

Partially coherent case, Eq. (34)*:* For a partially coherent configuration with degree of coherence $\mu_{AB}(x) \in [0, 1]$, the effective local interference term is $\mathcal{I}_{\text{eff}}(x) = 2|c_A||c_B|\sqrt{\rho_A\rho_B}\,\mu_{AB}(x)\cos(\Delta\phi(x)/\Phi_0 + \vartheta)$ for some phase ϑ. The effective visibility at screen position y is then $\mathcal{V}_{\text{eff}}(y) = \mu_{AB}(y)\cdot 2|c_A||c_B| \le 2|c_A||c_B|$, since $\mu_{AB} \le 1$. The Cauchy–Schwarz inequality on $\mathcal{H}$, Eq. (3), bounds the coherence function: $|\gamma_{AB}(x)| = |\langle \Psi_A, \cdot \rangle| \le \|\Psi_A\|_{\mathcal{H}}\, \|\Psi_B\|_{\mathcal{H}} = 1$ applied pointwise, giving $\mu_{AB}(x) \le 1$. Therefore $\mathcal{V}_{\text{eff}}^2 + \mathcal{W}^2 \le (2|c_A||c_B|)^2 + \mathcal{W}^2 = 1$, which is Eq. (34). □

Remark 6.9. *Theorem 6.8 is the NUVO derivation of wave-particle complementarity. In the standard quantum-mechanical formalism, complementarity is introduced as a principle: wave-like and particle-like behavior are mutually exclusive descriptions, and the experimental setup determines which description applies. In the NUVO framework, complementarity is not a principle but a theorem. For a pure two-path closure state, it is the algebraic identity* $(2|c_A||c_B|)^2+(|c_A|^2-|c_B|^2)^2 = (|c_A|^2 + |c_B|^2)^2 = 1$*, which is a consequence of the binomial identity applied to the normalization constraint. For partially coherent states, it is the inequality* $\mathcal{V}_{\text{eff}}^2 + \mathcal{W}^2 \leq 1$*, which is a consequence of the Cauchy–Schwarz inequality on* $\mathcal{H}$*. Both forms express a structural constraint on the two-path closure state: the sum of squared visibility and squared distinguishability cannot exceed unity, because both quantities are bounded by the same normalization of the closure state. No wave-particle duality, no measurement disturbance principle, and no new physical assumption enters the derivation.*

Remark 6.10. *The complementarity relation Eq.* (33) *expresses a trade-off that is continuously variable, not binary. As* $|c_A|$ *increases from* 0 *to* 1 *(with* $|c_B|$ *decreasing correspondingly), the visibility* $\mathcal{V} = 2|c_A||c_B|$ *traces the arc* $\mathcal{V} = 2|c_A|\sqrt{1-|c_A|^2}$*, which peaks at* $\mathcal{V} = 1$ *when* $|c_A| = 1/\sqrt{2}$ *and falls to* $\mathcal{V} = 0$ *at* $|c_A| = 0$ *or* 1*. Simultaneously* $\mathcal{W} = |2|c_A|^2 - 1|$ *traces the complementary arc from* $\mathcal{W} = 1$ *(at* $|c_A| = 0$ *or* 1*) to* $\mathcal{W} = 0$ *(at* $|c_A| = 1/\sqrt{2}$*). The point* $(\mathcal{V}, \mathcal{W})$ *traces the unit circle arc in the first quadrant as* $|c_A|$ *varies, with every intermediate point on the arc achievable by choosing the appropriate coefficient ratio. This continuous variability is the hallmark of a structural algebraic constraint rather than a binary classical incompatibility.*

7 The Double-Slit Experiment

The results of Secs. 3–6 are now assembled into a complete scalar–conformal NUVO account of the double-slit experiment. The experiment is the canonical demonstration of quantum interference, and its three principal features—the fringe pattern when both slits are open, the disappearance of fringes under which-path detection, and the continuous trade-off between fringe visibility and path information—all follow from results already established in the present paper. The present section introduces no new formal results; it records how the preceding theorems apply to the specific double-slit geometry, derives the fringe pattern in that geometry with the de Broglie wavelength identified explicitly, and establishes the three experimental regimes as corollaries of the complementarity relation.

7.1 The Experimental Geometry

The double-slit geometry is a special case of the two-path transport configuration of Definition 4.1, realized by two apertures in an otherwise opaque barrier.

Definition 7.1 (Double-slit geometry)**.** *The* double-slit geometry *consists of:*

(i) *A* source *at the origin, producing a closure state* Ψ_S *that is approximately spatially uniform over the barrier plane and has definite transport momentum* p *in the forward direction.*

(ii) *A* barrier *at distance* z_0 *from the source, opaque everywhere except for two apertures (slits): slit* A *centered at lateral position* $+d/2$ *and slit* B *centered at* $-d/2$*, each of width* $w \ll d$*.*

(iii) *A* detection screen *at distance* L *beyond the barrier (total distance* $z_0 + L$ *from the source), with lateral coordinate* y *measured from the forward axis.*

The two-path channels of Definition 4.1 are identified with the two slit apertures: channel A is transport through slit A and channel B is transport through slit B. The single-path closure states Ψ_A and Ψ_B are the closure states produced by transport through each slit in isolation, approximated as spherical transport fronts originating from the slit positions in the far-field regime $L \gg d$.

Remark 7.2. *The single-path closure states Ψ_A and Ψ_B in the double-slit geometry are admissible by construction: each is the closure state produced by a single-slit transport configuration, which is an admissible transport closure of the scalar–conformal exchange sector. The two-path state $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ is admissible by Theorem 3.3 (i). In the far-field regime, the spatial support of Ψ_A and Ψ_B at the detection screen overlaps extensively, making the inner product $\langle \Psi_A, \Psi_B \rangle_{\mathcal{H}}$ non-negligible and the interference term significant across the entire illuminated region of the screen. The small-angle approximation of Lemma 4.5 is valid in the far-field regime $L \gg d$, which is the regime of practical interest for double-slit interference.*

7.2 The Double-Slit Closure Density and Fringe Pattern

The closure density at the detection screen follows directly from Proposition 4.3 with the double-slit geometry substituted. The symmetric case with equal slit widths and a spatially uniform source gives $\rho_A(y) = \rho_B(y) =: \rho_0(y)$, where $\rho_0(y)$ is the single-slit diffraction envelope.

Proposition 7.3 (Double-slit closure density and fringe pattern)**.** *For the double-slit geometry of Definition 7.1 in the symmetric configuration $c_A = c_B = 1/\sqrt{2}$ and far-field regime $L \gg d$, the closure density at screen position y is*

$$\rho_{AB}(y) = 2\rho_0(y)\left[1 + \cos\!\left(\frac{2\pi d\, y}{\lambda\, L}\right)\right], \tag{35}$$

where the de Broglie wavelength *is*

$$\lambda := \frac{2\pi\Phi_0}{p} = \frac{h}{p}, \tag{36}$$

$h = 2\pi\Phi_0$ is the Planck constant recovered from the Q-series identification $\Phi_0 = \hbar$, and $\rho_0(y)$ is the single-slit diffraction envelope. The fringe spacing is

$$\Delta y = \frac{\lambda\, L}{d} = \frac{h\, L}{p\, d}, \tag{37}$$

and the fringe visibility is $\mathcal{V} = 1$.

Proof. This is Theorem 5.1 applied to the double-slit geometry of Definition 7.1. The symmetry conditions $c_A = c_B = 1/\sqrt{2}$ and $\rho_A = \rho_B = \rho_0$ are satisfied by the symmetric source and equal slit widths. The small-angle phase difference $\Delta\phi(y) = 2\pi d\, y/(\lambda\, L)$ is given by Lemma 4.5. Substituting into Theorem 5.1 yields Eq. (35). The fringe spacing Eq. (37) is Eq. (22) with $\lambda = 2\pi\Phi_0/p$. The visibility $\mathcal{V} = 2|c_A||c_B| = 2 \cdot (1/\sqrt{2}) \cdot (1/\sqrt{2}) = 1$ follows from Proposition 5.5. $\square$

Remark 7.4. *The de Broglie wavelength $\lambda = h/p$ appearing in Eq. (36) is not introduced as a new assumption. It emerges from two prior results: the transport phase relation $\phi = p\ell/\Phi_0$ of Eq. (7), derived from the Q-series free transport structure, and the identification $\Phi_0 = \hbar$ (equivalently $h = 2\pi\hbar = 2\pi\Phi_0$) established through the hydrogenic correspondence of the Q-series. The fringe spacing formula $\Delta y = hL/(pd)$ is therefore a direct consequence of the scalar–conformal transport phase structure and requires no independent wave postulate. This constitutes an internal consistency check of the NUVO program: the double-slit fringe spacing predicted by the transport closure geometry with the Q-series phase relation agrees exactly with the empirically established de Broglie formula $\lambda = h/p$.*

Remark 7.5. *By the Born frequency law of QB6, recalled in Sec. 2.5, the asymptotic relative frequency of detection events at screen position y in an interval $[y, y + \mathrm{d}y]$ is $\rho_{AB}(y)\,\mathrm{d}y$. The fringe pattern Eq. (35) therefore predicts the spatial distribution of detection events: events accumulate preferentially at positions of constructive interference (Corollary 5.7, Eq. (25)) and are suppressed at positions of destructive interference (Eq. (26)). This prediction is a structural consequence of the transport closure geometry and the Born frequency law, neither of which introduces a probabilistic postulate. The "build-up" of the fringe pattern from individual detection events—the feature that makes the double-slit experiment appear paradoxical in the standard formulation—is, in the NUVO framework, the accumulation of coherence-gated interaction events whose asymptotic frequency distribution follows the closure density $\rho_{AB}(y)$ by the Born law of QB6.*

7.3 The Three Experimental Regimes

The complementarity relation of Theorem 6.8 identifies a continuously variable family of experimental configurations parametrized by $(|c_A|, |c_B|)$ and the degree of coherence μ_{AB}. Three qualitatively distinct regimes stand out and correspond to the canonical experimental scenarios. All three are unified by the complementarity relation and follow as corollaries of the results established in Secs. 5 and 6.

Corollary 7.6 (Three experimental regimes of the double-slit experiment)**.** *The following three regimes are distinguished by their values of fringe visibility $\mathcal{V}$ and which-path distinguishability $\mathcal{W}$, each satisfying $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$ by Theorem 6.8.*

(i) Full interference regime: $\mathcal{V} = 1$, $\mathcal{W} = 0$. *Both slits open, no which-path detection, symmetric coefficients $|c_A| = |c_B| = 1/\sqrt{2}$, full coherence $\mu_{AB} = 1$. The closure density is the fringe pattern of Proposition 7.3:*

$$\rho_{AB}(y) = 2\rho_0(y)\left[1 + \cos\left(\frac{2\pi d\, y}{\lambda\, L}\right)\right].$$

Detection events accumulate at positions of constructive interference and are absent at positions of destructive interference. No path information is available from the coefficient structure ($\mathcal{W} = 0$) or from the detection pattern (fringes are symmetric, not path-specific).

(ii) No-interference regime: $\mathcal{V} = 0$, $\mathcal{W} \in [0, 1]$. *This regime arises in two physically distinct ways.* (a) One slit closed: $|c_A| = 1$, $|c_B| = 0$ *(or vice versa), giving $\mathcal{W} = 1$. The closure density is the single-slit pattern $\rho_{AB}(y) = \rho_A(y)$; complete path information is available.* (b) Complete which-path detection: $|c_A|$ *and* $|c_B|$ *arbitrary, but $\mu_{AB} = 0$ due to complete phase de-correlation by the which-path detector (Theorem 6.3). The ensemble-averaged closure density is the incoherent sum $\rho_{AB}(y) = |c_A|^2\rho_A(y) + |c_B|^2\rho_B(y)$, with no fringes. In case (b) with $|c_A| = |c_B| = 1/\sqrt{2}$, $\mathcal{W} = 0$ from the coefficient structure, but the detector itself provides the path information by recording which channel each event traversed.*

(iii) Partial-interference regime: $0 < \mathcal{V} < 1$, $0 < \mathcal{W} \leq 1$, $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$. *Both slits open with a partial which-path detector, or asymmetric coefficients $|c_A| \neq |c_B|$, or partial phase de-correlation $\mu_{AB} \in (0, 1)$. The closure density is the general fringe pattern of Corollary 5.9 with effective visibility $\mathcal{V}_{\text{eff}} = \mu_{AB} \cdot 2|c_A||c_B| \in (0, 1)$. Both partial fringes and partial path information are simultaneously present, with the degree of each constrained by the complementarity relation.*

Proof. Regime (i) is Proposition 7.3 with $\mathcal{V} = 1$ from Proposition 5.5 and $\mathcal{W} = ||c_A|^2 - |c_B|^2| = 0$ from Definition 6.6. Regime (ii)(a) follows by setting $|c_B| = 0$ in Eq. (15): the cross-term vanishes and $\rho_{AB} = \rho_A$. Regime (ii)(b) is Theorem 6.3, with $\mu_{AB} = 0$ produced by the complete phase de-correlation of a which-path detector with uniform phase distribution. Regime (iii) follows from the general visibility Eq. (28) and the partial coherence analysis of Remark 6.5, with the complementarity inequality Eq. (34) holding by Theorem 6.8. □

7.4 Recovery of the De Broglie Relation and Internal Consistency

The fringe spacing formula Eq. (37) provides an internal consistency check on the scalar–conformal NUVO program that is worth recording explicitly.

Proposition 7.7 (Internal consistency: double-slit fringe spacing)**.** *The fringe spacing $\Delta y = \lambda L/d$ predicted by the scalar–conformal transport closure geometry is consistent with the de Broglie relation $\lambda = h/p$ in the following sense. The Q-series established the phase constant $\Phi_0 = \hbar$ through the hydrogenic correspondence. The free transport phase relation $\phi = p\ell/\Phi_0$, derived from the Q-series exchange-sector transport structure, then yields the fringe spacing*

$$\Delta y = \frac{2\pi\Phi_0 \, L}{p\, d} = \frac{h\, L}{p\, d},$$

which is the empirically established double-slit fringe spacing formula, with $h = 2\pi\hbar$ the Planck constant. This agreement is not a coincidence or a circular argument: the Q-series fixed $\Phi_0 = \hbar$ from the hydrogenic energy spectrum, and the present paper derives the fringe spacing from the transport phase geometry without any reference to the empirical double-slit result.

Proof. The derivation is the chain: Q-series identifies $\Phi_0 = \hbar$ from the hydrogenic spectrum $E_n = -me^4/(2\Phi_0^2 n^2)$ and the correspondence $E_n = -me^4/(2\hbar^2 n^2)$ [2]. The free transport phase relation $\phi = p\ell/\Phi_0$ follows from the Q-series exchange-sector transport structure in the uniform scalar capacity sector [1]. Lemma 4.5 then gives the phase difference $\Delta\phi(y) = pd\, y/(\Phi_0 L)$. Theorem 5.1 gives the fringe spacing $\Delta y = 2\pi\Phi_0 L/(pd)$. Substituting $h = 2\pi\Phi_0 = 2\pi\hbar$ gives $\Delta y = hL/(pd)$. □

Remark 7.8. *Proposition 7.7 completes the NUVO account of the double-slit experiment. The following derivation chain is now fully established from the scalar–conformal geometry:*

- Superposition of path states: *Theorem 3.3, from transport closure linearity (Q-series).*
- Interference cross-term: *Proposition 4.3, from the squared modulus of the two-path state and the QB1 polar decomposition.*
- Phase difference and fringe spacing: *Lemma 4.5 and Theorem 5.1, from the Q-series free transport phase relation and path geometry.*
- Fringe visibility and complementarity: *Propositions 5.5 and Theorem 6.8, from the coefficient structure and Cauchy–Schwarz on $\mathcal{H}$ (QM1).*
- Which-path detection destroys interference: *Theorem 6.3, from the phase de-correlation mechanism of the QB5 coherence-gated interaction framework.*
- Detection event distribution: *Born frequency law (QB6, extended in QM1), connecting $\rho_{AB}(y)$ to the asymptotic event frequency.*

At no point in this chain is wave-particle duality postulated, wavefunction collapse invoked, a probabilistic axiom assumed, or the closure state interpreted as a physical wave. The double-slit experiment is accounted for entirely within the scalar–conformal NUVO transport closure framework.

8 Interpretive Clarifications and Scope

The present section collects the interpretive constraints that have been applied throughout the paper and states them explicitly as a unified set of boundary conditions on the NUVO account of superposition, interference, and which-path detection. These constraints are not incidental; they are the precise statements that distinguish the NUVO derivation from the standard formulation and that protect the logical integrity of the series. Four items are addressed: the representational character of the superposition state, the geometric rather than wave-ontological account of interference, the coherence-disruption rather than collapse account of which-path detection, and the scope of the present construction relative to the remainder of the QM-series.

8.1 Superposition Without Wave Ontology

The superposition state $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ is an element of the Hilbert space $\mathcal{H}$ established in QM1. As an element of $\mathcal{H}$, it is a complex-valued square-integrable function on $\mathbb{R}^3$; as an admissible closure state in the sense of Theorem 3.3, it is a solution of the transport closure equation $\mathcal{L}[\Psi] = 0$ in the integrable exchange-sector regime. Its squared modulus $|\Psi_{AB}(x)|^2 = \rho_{AB}(x)$ is the normalized closure density at position x, as established in QB1 and carried forward through the series.

The superposition state is a representational object. It encodes the phase correlation between the two transport channels in the cross-term $\overline{\Psi_A(x)}\Psi_B(x) = \sqrt{\rho_A(x)\rho_B(x)}\,\mathrm{e}^{\,\mathrm{i}\Delta\phi(x)/\Phi_0}$, which appears in the closure density at the screen. It does not represent a physical wave propagating through space, a particle simultaneously present in both channels, or a superposition of distinct classical configurations. The NUVO framework makes none of these ontological claims.

The specific locutions excluded from the present paper and from the NUVO treatment of superposition more generally are recorded here for completeness. The statement that "the closure state passes through both slits simultaneously" is not adopted: the NUVO statement is that the two-path closure state encodes phase information from both transport channels, with the two-path character expressed in the cross-term of the closure density rather than in any simultaneous spatial occupancy. The statement that "the superposition is a wave" is not adopted: $\Psi_{AB} \in \mathcal{H}$ is a complex-valued function whose mathematical structure includes oscillatory cross-terms, but the function itself is a representational encoding of transport closure geometry rather than a physical field. The statement that "the particle goes through slit A with amplitude c_A and slit B with amplitude c_B" conflates the closure state with a particle trajectory and is not available in the NUVO framework, which does not adopt particle trajectories as primitive objects.

What the superposition does encode, precisely and without additional ontological commitment, is the following: the transport closure configurations at the detection screen are described by a state Ψ_{AB} whose closure density $|\Psi_{AB}(x)|^2$ carries the interference cross-term $\mathcal{I}(x)$ as a geometric consequence of the phase difference $\Delta\phi(x)$ between the two transport channels. This is a statement about the geometry of the scalar–conformal transport system; it requires no wave ontology and no particle trajectory.

8.2 Interference Without Wave-Particle Duality

Wave-particle duality is the conventional resolution of the tension between the interference behavior exhibited by quantum systems (which suggests a wave description) and the localized detection events they produce (which suggests a particle description). In the standard formulation of quantum mechanics, duality is introduced as a principle: whether a quantum system exhibits wave-like or particle-like behavior depends on the experimental arrangement, and no single description captures both aspects simultaneously.

In the NUVO framework, this tension does not arise because neither description—wave nor particle—is adopted as primitive. The transport closure state Ψ_{AB} is neither a wave nor a particle; it is a representational encoding of the scalar–conformal exchange-sector transport geometry. The interference fringe pattern of Eq. (35) is a structural consequence of the two-path phase difference $\Delta\phi(y)$, which is a geometric property of the path configuration. The localized detection events at the screen are coherence-gated interaction events, as established in QB5 and QB6, whose asymptotic frequency distribution follows the Born frequency law rather than any classical particle trajectory.

The complementarity relation $\mathcal{V}^2+\mathcal{W}^2 \leq 1$ of Theorem 6.8 replaces wave-particle duality as the organizing principle of the double-slit phenomenology. In the standard formulation, the statement that "you cannot observe both wave-like and particle-like behavior simultaneously" is a qualitative principle whose precise content depends on the experimental context. In the NUVO framework, the corresponding statement is the quantitative algebraic theorem $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$: the sum of squared fringe visibility and squared path distinguishability cannot exceed unity, as a consequence of the Cauchy–Schwarz inequality on $\mathcal{H}$ and the normalization of the two-path coefficient structure. The duality principle is subsumed by a derivation-complete inequality that specifies precisely how much of each quantity is available for any given two-path configuration.

Remark 8.1. *The conceptual shift from wave-particle duality as a principle to the complementarity relation as a theorem is characteristic of the NUVO derivation program. In both cases, the physical content—that interference and which-path information are mutually exclusive in the extreme cases and trade off continuously in intermediate cases—is the same. The difference is logical status: in the standard formulation the content is assumed; in the NUVO framework it is derived from the algebraic structure of the two-path closure state in $\mathcal{H}$ and the Cauchy–Schwarz inequality. The derivation makes explicit what the principle leaves implicit: the precise quantitative form of the trade-off, its continuous character (Remark 6.10), and the conditions under which equality holds.*

8.3 Which-Path Detection Without Collapse

The disappearance of the fringe pattern under which-path detection is, in the standard quantum-mechanical formulation, typically explained by wavefunction collapse: the act of measurement "collapses" the superposition state onto one of its components, destroying the interference. This explanation requires a collapse postulate—a discontinuous state change upon measurement that is not described by the Schrödinger equation—and has been the source of extensive interpretive difficulty in the foundations of quantum mechanics.

In the NUVO framework, no collapse postulate is available or needed. The disappearance of the fringe pattern under which-path detection is derived in Theorem 6.3 by a completely different mechanism: phase de-correlation. The which-path detecting interaction, as a coherence-gated interaction event in the sense of QB5, introduces an uncontrolled phase shift $\delta\phi$ on the monitored transport channel. The phase shift is uncontrolled because the which-path interaction, by design, is sensitive to which channel the closure is in—and this sensitivity is precisely what makes the interaction phase-disturbing as well as path-resolving. When the closure density is averaged over the ensemble of interaction outcomes (over the distribution of $\delta\phi$), the cross-term $2\,\mathrm{Re}[c_A\overline{c_B}e^{i\delta\phi/\Phi_0}\gamma_{AB}(x)]$ averages to zero, as shown in the proof of Theorem 6.3.

The mechanism differs from collapse in three ways that are worth stating precisely. First, the closure densities $\rho_A(x)$ and $\rho_B(x)$ of the individual path states are *not* altered by the which-path interaction: the transport closure in each channel continues undisturbed. What is destroyed is not the closure content of either channel but the phase correlation between them. Second, the destruction of interference is an *ensemble* statement: for any particular which-path interaction event with a definite phase shift $\delta\phi$, the closure density is a fringe pattern shifted by $\delta\phi/\Phi_0$; it is the averaging over events with different $\delta\phi$ values that washes out the fringes. Third, no discontinuous state change occurs: the transport closure evolution remains deterministic throughout, and the ensemble averaging is a consequence of the statistical distribution of interaction outcomes across the ensemble, not a fundamental indeterminism in the individual evolution.

Remark 8.2. *The phase de-correlation mechanism of Theorem 6.3 is conceptually related to, but distinct from, the decoherence framework of the standard quantum-mechanical literature. Both approaches explain the disappearance of interference as a consequence of entanglement between the system and an environment (in the standard framework) or between the closure state and the interaction system (in the NUVO framework), rather than as a collapse postulate. The NUVO treatment is, however, more elementary: it does not require the formalism of entanglement or reduced density matrices, which are developed only in QM9, but operates directly at the level of the phase shift distribution and its effect on the ensemble-averaged closure density. The connection to the full decoherence structure, including the identification of the which-path detector as an environment and the derivation of the pointer basis, is a topic that extends beyond the scope of the present paper and is deferred to QM9.*

8.4 Scope of the Present Construction

The present paper establishes the superposition principle, two-path interference, which-path detection as coherence disruption, the complementarity relation, and the scalar–conformal account of the double-slit experiment. It is equally important to record what it does not establish, so that the logical dependencies of subsequent papers are transparent.

The paper does not derive the uncertainty relations. The canonical commutation relation $[\hat{x}^j, \hat{p}_k] = \mathrm{i}\Phi_0\, \delta^j{}_k$ established in QM1 Proposition 5.4 governs the algebraic structure of the transport generators on the superposition states of the present paper. The Robertson–Schrödinger uncertainty inequality $\Delta\hat{x}^j \cdot \Delta\hat{p}_k \geq (\Phi_0/2)\, \delta^j{}_k$ follows from this commutation relation by an application of the Cauchy–Schwarz inequality on $\mathcal{H}$ to superpositions of eigenstates. This derivation is the subject of QM3, which uses the Hilbert space structure of QM1 and the superposition structure of the present paper as its foundational inputs.

The paper does not treat multi-path interference. The two-path formalism developed here extends naturally to N-path configurations—a sum $\sum_{n=1}^{N} c_n \Psi_n$ of N path states, with pairwise interference terms $2\,\mathrm{Re}[c_j \overline{c_k} \overline{\Psi_j} \Psi_k]$ for each pair (j, k)—and further to path-integral representations of arbitrary transport configurations as continuous superpositions of path states. The multi-path and path-integral extensions are deferred; the two-path case is sufficient for the double-slit account and establishes the structural pattern that multi-path configurations follow.

The paper does not treat entanglement. The complementarity relation of Theorem 6.8 and the phase de-correlation of Theorem 6.3 involve a single-particle two-path configuration. The analogous phenomena for two particles—in particular, the two-particle interference that arises from entangled states and the connection between entanglement, decoherence, and the which-path mechanism—require the multi-particle Hilbert space $\mathcal{H}^{(2)} = L^2(\mathbb{R}^6, \mathbb{C})$ of QM7 and the entanglement structure of QM9. The present paper provides the single-particle precursor, but the multi-particle generalization requires the additional tensor-product structure developed in QM7.

The paper does not treat the time-dependent dynamics of the interference pattern. The fringe pattern Eq. (35) is derived as a static closure density at the screen, without reference to the time evolution of the two-path state. The time-dependent spreading of the closure distribution, the evolution of a wavepacket through the double-slit geometry, and the temporal structure of the detection-event arrival distribution all require the Schrödinger dynamics of QM4. The present paper establishes the static structural result; QM4 provides the dynamical framework needed for the time-dependent extension.

The paper does not treat the Mach-Zehnder interferometer or other multi-element interferometric configurations. The two-path formalism of Sec. 4 applies directly to any configuration in which two transport channels connect a source to a detector, including beam-splitter configurations, optical path-difference arrangements, and neutron interferometers. The specific analysis of such configurations—including the effect of phase plates, beam splitters as partial which-path detectors, and the recovery of fringes by erasing path information—is deferred as it requires additional apparatus from the measurement theory developed in the QB-series. The two-path closure density of Proposition 4.3 and the coherence disruption of Theorem 6.3 are the formal inputs required for all such analyses; the present paper makes them available.

Remark 8.3. *The logical position of QM2 within the QM-series can be summarized as follows. QM2 establishes the structural consequence of the Hilbert space linear structure of QM1 for transport configurations with two coherent channels. It does not require the dynamical framework of QM4 (Schrödinger equation and time evolution) and is therefore available as a structural result from the moment QM1 is established. It feeds forward into QM3 (uncertainty relations, which use superpositions of incompatible eigenstates), QM5 (angular momentum eigenstates as superpositions), QM6 (coherent states as optimally localized superpositions), QM9 (entangled states as non-factorizable two-particle superpositions), and QM10 (scattering as a continuous superposition of momentum eigenstates). In each case, the superposition principle of Theorem 3.3 and the inner product structure of Proposition 3.5 are the specific QM2 results that are used. The interference and which-path results are specific to the two-path sector and are used directly in QM9 and, in the covariant extension, in QM11.*

9 Conclusion

9.1 Summary of Results

The present paper has derived the superposition principle, the two-path interference pattern, the which-path detection mechanism, the complementarity relation, and the complete scalar–conformal NUVO account of the double-slit experiment as structural theorems of the exchange-sector transport closure system, without introducing wave-particle duality, wavefunction collapse, or probabilistic postulate. The principal results are as follows.

The superposition principle (Lemma 3.1 and Theorem 3.3). In the integrable exchange-sector regime, the transport closure operator $\mathcal{L}$ is linear in the complex state encoding Ψ. This linearity, established from the Q-series exchange-sector transport structure, implies that any finite linear combination, any norm-convergent series, and any norm-convergent Bochner integral of admissible closure states is itself admissible. The superposition principle is a theorem derived from transport closure linearity and the completeness of $\mathcal{H}$ established in QM1, not a postulate about the structure of the state space. All three forms of the theorem—finite, series, and continuous superpositions—are used in subsequent papers of the QM-series.

Two-path closure density and the interference term (Definition 4.1 and Proposition 4.3). For a two-path transport configuration with path states $\Psi_A, \Psi_B \in \mathcal{H}$ and coefficients $c_A, c_B \in \mathbb{C}$, the closure density of the two-path state $\Psi_{AB} = c_A\Psi_A + c_B\Psi_B$ at the detection screen is

$$\rho_{AB}(x) = |c_A|^2\rho_A(x) + |c_B|^2\rho_B(x) + 2|c_A||c_B|\sqrt{\rho_A(x)\,\rho_B(x)}\,\cos\left(\frac{\Delta\phi(x)}{\Phi_0} + \arg c_A - \arg c_B\right),$$

where $\Delta\phi(x) = \phi_B(x) - \phi_A(x)$ is the local transport phase difference. The interference cross-term is a structural consequence of the polar decomposition of the path states established in QB1; it arises from the squared modulus of the two-path state and requires no wave-ontological assumption.

Phase difference and fringe spacing (Lemma 4.5 and Theorem 5.1). In the free transport sector with definite momentum p and the small-angle approximation appropriate to the far-field double-slit geometry, the path-length difference $\ell_B(y) - \ell_A(y) \approx d\,y/L$ produces a phase difference $\Delta\phi(y) = 2\pi d\,y/(\lambda\,L)$ where $\lambda = 2\pi\Phi_0/p = h/p$ is the de Broglie wavelength. The symmetric two-path closure density is $\rho_{AB}(y) = 2\rho_0(y)[1 + \cos(2\pi d\,y/\lambda L)]$, with fringe spacing $\Delta y = \lambda L/d$. The de Broglie wavelength $\lambda = h/p$ emerges from the Q-series transport phase relation and the identification $\Phi_0 = \hbar$ from the hydrogenic correspondence, providing an internal consistency check on the scalar–conformal program.

Fringe visibility and its Cauchy–Schwarz bound (Definition 5.4 and Proposition 5.5). The fringe visibility $\mathcal{V} = 2|c_A||c_B|$ is bounded above by one, with maximum achieved at equal weights $|c_A| = |c_B| = 1/\sqrt{2}$ and minimum (no fringes) at single-path transport. The upper bound $\mathcal{V} \leq 1$ is a consequence of the Cauchy–Schwarz inequality on $\mathcal{H}$. The general asymmetric fringe pattern with unequal path densities and arbitrary coefficient phases is recorded in Corollary 5.9.

Which-path detection as phase de-correlation (Definition 6.1 and Theorem 6.3). A which-path detecting interaction introduces an uncontrolled phase shift $\delta\phi$ on the monitored transport channel. When $\delta\phi$ is uniformly distributed over $[0, 2\pi)$ across the interaction ensemble, the ensemble average of $\mathrm{e}^{\mathrm{i}\delta\phi/\Phi_0}$ vanishes, the interference cross-term averages to zero, and the fringe pattern is eliminated. The result is the incoherent closure density $\langle\rho_{AB}\rangle = |c_A|^2\rho_A + |c_B|^2\rho_B$ with no fringe modulation. No wavefunction collapse, no projection postulate, and no discontinuous state change enters the derivation; the mechanism is phase averaging over the interaction ensemble.

The complementarity relation (Definition 6.6 and Theorem 6.8). For a normalized two-path closure state with fringe visibility $\mathcal{V} = 2|c_A||c_B|$ and which-path distinguishability $\mathcal{W} = ||c_A|^2 - |c_B|^2|$, the identity $\mathcal{V}^2 + \mathcal{W}^2 = 1$ holds algebraically. For partially coherent states with degree of coherence $\mu_{AB} \in [0, 1]$, the inequality $\mathcal{V}_{\text{eff}}^2 + \mathcal{W}^2 \leq 1$ holds as a consequence of the Cauchy–Schwarz inequality on $\mathcal{H}$. Wave-particle complementarity is thereby established as a derivation-complete theorem rather than a principle: the complementarity relation is the algebraic consequence of the normalization constraint on the two-path coefficient structure.

The double-slit experiment (Proposition 7.3 and Corollary 7.6). The scalar–conformal NUVO account of the double-slit experiment encompasses all three experimental regimes within a single unified framework: full interference ($\mathcal{V} = 1$, $\mathcal{W} = 0$, both channels open without detection), no interference ($\mathcal{V} = 0$, complete which-path detection or one channel closed), and partial interference ($0 < \mathcal{V} < 1$, $0 < \mathcal{W} \leq 1$, partial detection or asymmetric coefficients). All three regimes are corollaries of the complementarity relation and require no additional assumptions beyond the transport closure geometry, the Born frequency law of QB6, and the superposition principle established in the present paper.

9.2 Programmatic Significance

The present paper establishes two results of broad programmatic significance for the scalar–conformal NUVO series.

The first is the derivation of the superposition principle as a theorem. In the standard formulation of quantum mechanics, the state space is postulated to be a Hilbert space and linear combinations of states are states by definition. In the NUVO framework, the Hilbert space $\mathcal{H}$ was constructed in QM1 as the completion of the space of transport closure encodings, and the present paper derives that the set of admissible closure states within $\mathcal{H}$ is closed under the linear operations of $\mathcal{H}$. The derivation is from the linearity of the transport closure operator $\mathcal{L}$ in the integrable regime, a property of the Q-series exchange-sector geometry. This means that superposition—the most distinctively quantum feature of the theory—is not assumed but derived, and its scope and limitations are precisely characterized: it holds in the integrable regime where $\mathcal{L}[\Psi] = 0$ is linear in Ψ, and its validity is tied to the validity of that regime. Every subsequent use of superposition in the QM-series—in QM3 through QM11—rests on this derivation rather than on a postulate.

The second result of broad significance is the derivation of wave-particle complementarity as a Cauchy–Schwarz inequality. The complementarity relation $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$ is, in the NUVO framework, a consequence of the inner product structure of $\mathcal{H}$ and the normalization of the two-path closure state. It requires no appeal to the uncertainty principle, no Heisenberg microscope argument, and no claim about the disturbance caused by measurement. Its derivation in the present paper reveals the precise algebraic content of wave-particle duality—a content that the principle formulation leaves implicit—and connects it to the Hilbert space geometry established in QM1. This connection will be deepened in QM9, where the two-particle analogue of the complementarity relation connects the single-particle which-path mechanism to the two-particle entanglement structure.

The superposition principle and the complementarity relation together constitute the foundational results of the present paper for the QM-series. The interference analysis, the which-path detection theorem, and the double-slit account all follow from these two results combined with the phase accumulation structure of the Q-series transport geometry. No result in the paper requires QM4 or any subsequent paper; the logical dependencies run only backward through QM1, QB1–QB7, and the Q-series.

9.3 Transition to QM3

The canonical commutation relation $[\hat{x}^j, \hat{p}_k] = \mathrm{i}\Phi_0\, \delta^j{}_k$ established in QM1 Proposition 5.4 governs the algebraic relationship between the position and momentum transport generators on the superposition states of the present paper. The next paper, QM3, exploits this commutation relation to derive the uncertainty relations as structural theorems of the scalar–conformal transport system.

The derivation in QM3 proceeds as follows. For any self-adjoint operators A and B on $\mathcal{H}$ with $[A, B] = \mathrm{i}C$ for some self-adjoint C, the Robertson inequality $\Delta A \cdot \Delta B \geq \frac{1}{2}|\langle C\rangle|$ follows from the Cauchy–Schwarz inequality applied to the superposition state $(\hat{A} - \langle A\rangle)\Psi + \mathrm{i}\lambda(\hat{B} - \langle B\rangle)\Psi$ for optimal λ. Applied to $A = \hat{x}^j$, $B = \hat{p}_k$, $C = \Phi_0\, \delta^j{}_k$, this yields $\Delta\hat{x}^j \cdot \Delta\hat{p}_k \geq (\Phi_0/2)\, \delta^j{}_k$, the position-momentum uncertainty relation. The minimum-uncertainty states for this bound—Gaussian wavepacket closure states—are identified in QM3 and shown to be the single-particle precursors of the coherent states developed in QM6. The Cauchy–Schwarz inequality used in QM3 is the same inequality used in the present paper to establish the complementarity bound $\mathcal{V}^2 + \mathcal{W}^2 \leq 1$, making QM2 and QM3 algebraically parallel: both derive fundamental quantum constraints from the Hilbert space inner product structure of QM1 rather than from separate physical principles.

References

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