

# QM4 — Schrödinger Dynamics, Hamiltonian Structure, and Conservation Laws

in Scalar–Conformal NUVO Systems *Preprint, Version 1.0\**

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## Notation and Conventions

- $\mathcal{M}$  denotes the spacetime manifold.
- $\eta$  denotes the reference Lorentzian metric (typically Minkowski in a global chart).
- $g$  denotes the physical metric.
- The scalar field  $\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$  is the NUVO modulation field.
- The physical metric is scalar–conformal:

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}.$$

- $\Lambda_0 > 0$  denotes the baseline scalar availability level supported by the intrinsic delivery structure of the underlying field. In the absence of localized structural occupation the scalar field satisfies  $\Lambda(x) = \Lambda_0$ .
- The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}.$$

- The scalar field represents the *locally available structural capacity* of the underlying delivery field. Localized structures may reduce this availability through occupation or transport, but the intrinsic delivery baseline  $\Lambda_0$  remains fixed.
- Greek indices  $\mu, \nu, \dots$  range over spacetime coordinates 0, 1, 2, 3.
- We use the Einstein summation convention unless explicitly stated otherwise.

**Remark 0.1.** *Unless otherwise stated, the background signature is  $(-, +, +, +)$ .*

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\*Bibliography is provisional. Cross-references to companion NUVO-series papers (M-, SR-, Q-, QB-, QM-series) will be updated with Zenodo DOIs in subsequent versions.

## Program scope.

### Abstract

The Q-series established a Schrödinger-type compatibility condition for the transport closure system of scalar–conformal NUVO theory, demonstrating that the coupled evolution of closure density and transport phase is consistent with a Schrödinger-type equation in the hydrogenic sector. The present paper elevates this compatibility result to a general theorem, establishing the time-dependent Schrödinger equation on the complete Hilbert space  $\mathcal{H}$  of QM1 as a structural consequence of the scalar–conformal transport geometry.

Three gaps in the existing series are closed in sequence. First, the general correspondence between a spatial modulation of the scalar capacity field  $\Lambda$  and a potential energy function  $V(x)$  in the transport closure law is derived explicitly, extending the hydrogenic result of the Q-series to an admissible class of scalar-conformal configurations. Second, the Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  is shown to be self-adjoint on the second-order Sobolev domain  $H^2(\mathbb{R}^3)$  for potentials in the admissible class, via an application of the Kato–Rellich theorem to the scalar–conformal transport structure. Third, Stone’s theorem for strongly continuous unitary groups is applied to the self-adjoint Hamiltonian to derive the unitary time-evolution operator  $U(t)$  and the Schrödinger equation  $i\Phi_0 \partial_t \Psi = \hat{H} \Psi$  as its infinitesimal generator equation.

With these foundations in place, the Noether-type conservation laws are derived from transport symmetry: invariance of the transport closure law under time translation, spatial translation, and spatial rotation yields conservation of the expectation values of energy, momentum, and angular momentum respectively. The Ehrenfest theorem is then established as a corollary, identifying classical transport trajectories as the evolution of expectation values under Schrödinger dynamics.

No dynamical postulates are introduced. The Schrödinger equation, the Hamiltonian, the unitary evolution, and the conservation laws all emerge as structural consequences of the scalar–conformal transport geometry and the operator framework established in the prior series.

## 1 Introduction

### 1.1 Position Within the QM-Series

The scalar–conformal NUVO program has developed through a disciplined sequence of sector papers, each extending the prior work by a single controlled step. The M-series fixed the foundational geometry: the scalar capacity field  $\Lambda$  conformally modulates a reference Lorentzian metric through  $g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}$ , the delivery substrate and loop taxonomy were established, and the variational structure of the scalar–conformal action was derived. The Q-series developed the exchange sector: closure conditions, holonomic coherence, quantization from integer winding number, the hydrogenic correspondence that identifies  $\Phi_0 = \hbar$ , and a Schrödinger-type compatibility result for the hydrogenic transport closure system. The QB-series completed the first stage of the quantum-mechanical development: state representation as a complex encoding of transport closure, momentum and energy operators derived from transport generators, the canonical commutation relation, a pre-Hilbert inner product from holonomic coherence, and the Born frequency law and measurement correspondence as structural consequences of coherence-gated interaction dynamics. QM1 then extended this finite-dimensional framework to the complete separable Hilbert space  $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$ : normalization was derived as a structural constraint from closure conservation, the momentum and energy generators were promoted to essentially self-adjoint operators on their Sobolev domains, and the spectral theorem together with the resolution of the identity were established for the transport generators.

Within this sequence, QM4 occupies a specific and load-bearing position. QM1 provides the Hilbert space  $\mathcal{H}$  and establishes the self-adjointness of the free momentum generators  $\hat{p}_j = -i\Phi_0 \partial_j$ .

QM2 and QM3 were developed using only the algebraic and inner-product structure of QM1: QM2 derived the superposition principle and interference from the linearity of the transport closure equations on  $\mathcal{H}$ , and QM3 established the uncertainty relations from the canonical commutation relation. Neither QM2 nor QM3 requires a general Hamiltonian or a time-evolution operator; they depend only on the operator algebra and Hilbert space structure already in hand. The situation changes fundamentally at QM5 and beyond. The angular momentum algebra of QM5 requires a dynamical context in which the angular momentum generators commute with the Hamiltonian and conservation is a consequence of rotational symmetry. The harmonic oscillator of QM6 requires a Hamiltonian with a quadratic potential, derived from the scalar capacity geometry, and a Schrödinger evolution from which the energy spectrum is extracted. The multi-particle dynamics of QM7, the spin dynamics of QM8, the entanglement structure of QM9, and the scattering analysis of QM10 all require the time-dependent Schrödinger equation and the unitary time-evolution operator as their foundational dynamical input. The present paper, QM4, establishes this dynamical foundation.

Three structural gaps in the existing series, identified by the gap analysis preceding this work, must be closed before the Schrödinger equation can be established as a theorem. The Q-series established Schrödinger-type *compatibility* for the hydrogenic sector: the coupled evolution of closure density and transport phase is consistent with a Schrödinger-type equation for the hydrogenic Hamiltonian. It did not establish a general correspondence between the scalar capacity field and a potential energy function, did not prove that the resulting Hamiltonian is self-adjoint on  $\mathcal{H}$ , and did not derive the Schrödinger equation as a theorem valid on the complete Hilbert space. A fourth gap—the Noether bridge between the geometric transport symmetry of the M-series and the conservation of quantum-mechanical expectation values—was also identified: the M-series establishes transport invariance at the level of the action functional, but the translation of this invariance into conservation laws for expectation values requires the Schrödinger dynamics of the present paper. All four gaps are closed here in sequence, each building on the one before.

The results established in the present paper propagate forward through the remainder of the QM-series without exception. The Schrödinger equation of Theorem 5.5, the unitary time-evolution operator of Theorem 5.3, and the conservation structure of Sec. 7 are foundational inputs to QM5 through QM11. The covariant extension of the Schrödinger dynamics, which opens the path to the RQM-series, is prepared in QM11 using the operator framework and dynamical structure established here.

## 1.2 Objective of the Present Work

The central objective of the present paper is to establish the complete dynamical framework for the scalar–conformal NUVO transport closure system on the Hilbert space  $\mathcal{H}$  of QM1. Specifically, the paper aims to establish six related claims in logical order.

1. A spatial modulation  $\delta\lambda(x) = \Lambda(x)/\Lambda_0 - 1$  of the scalar capacity field, treated as a perturbation of the uniform baseline  $\Lambda = \Lambda_0$ , generates a real-valued potential function  $V(x)$  in the exchange-sector transport closure equation. The correspondence  $\delta\lambda \mapsto V$  is derived explicitly from the scalar–conformal transport law and reduces to the Coulomb potential in the hydrogenic sector, recovering the Q-series result as a special case.
2. The potential  $V(x)$  generated by any spatially localized scalar capacity modulation belongs to the admissible class of potentials that are relatively bounded with respect to the kinetic operator  $\hat{T} = -(\Phi_0^2/2m)\Delta$  with relative bound strictly less than one. This places  $V$  in the hypotheses of the Kato–Rellich theorem.

3. The Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$ , where  $\hat{V}$  denotes multiplication by  $V(x)$ , is self-adjoint on the second-order Sobolev domain  $H^2(\mathbb{R}^3) \subset \mathcal{H}$  for all potentials in the admissible class. This is established by an application of the Kato–Rellich theorem to the scalar–conformal transport structure, using the self-adjointness of  $\hat{T}$  on  $H^2(\mathbb{R}^3)$  and the relative boundedness established in claim (2).
4. Stone’s theorem, applied to the self-adjoint operator  $\hat{H}/\Phi_0$  on  $\mathcal{H}$ , yields a unique strongly continuous one-parameter unitary group  $U(t) = \exp(-i\hat{H}t/\Phi_0)$  on  $\mathcal{H}$ . This group is the time-evolution operator of the scalar–conformal transport closure system.
5. The time-dependent Schrödinger equation  $i\Phi_0 \partial_t \Psi = \hat{H} \Psi$  holds on  $\mathcal{D}(\hat{H}) = H^2(\mathbb{R}^3)$  as the infinitesimal generator equation of  $U(t)$ , and the solution  $\Psi(t) = U(t)\Psi_0$  is the unique strong solution for each initial datum  $\Psi_0 \in \mathcal{D}(\hat{H})$ . The Schrödinger equation is thereby established as a theorem, not introduced as a postulate.
6. The invariance of the transport closure law under time translation, spatial translation, and spatial rotation implies conservation of the expectation values  $\langle \hat{H} \rangle(t)$ ,  $\langle \hat{p}_j \rangle(t)$  (for translationally invariant potentials), and  $\langle \hat{L}_j \rangle(t)$  (for rotationally symmetric potentials) respectively. The Ehrenfest theorem follows as a corollary, identifying the classical transport trajectory with the evolution of mean position and momentum under the Schrödinger dynamics.

These six claims form a logically ordered chain in which each depends on those that precede it. Claims (1) and (2) establish the physical Hamiltonian and verify its membership in the class required by the functional analysis of claim (3). Claim (3) provides the self-adjoint generator required by claim (4). Claim (4) provides the unitary group whose generator equation is claim (5). Claims (5) and the commutation structure of Sec. 2.4 together yield claims (6).

### 1.3 What Is Not Assumed

The present work maintains without modification the interpretive discipline established in the Q-series and continued through the QB-series and QM1. The following exclusions are in force throughout.

The Schrödinger equation is not postulated. In the standard quantum-mechanical formalism,  $i\hbar \partial_t \Psi = \hat{H} \Psi$  is introduced as a fundamental dynamical law with no derivation. In the present framework it is a theorem: the generator equation of the unitary group  $U(t)$  constructed from the self-adjoint Hamiltonian via Stone’s theorem. The three-step derivation— $\Lambda \rightarrow V$ , Kato–Rellich, Stone—is the content of Secs. 3–5, and none of its steps introduces a dynamical postulate.

The Hamiltonian is not introduced as a primitive object. In the standard formalism,  $\hat{H}$  is chosen to represent the energy of the physical system under study. In the present framework  $\hat{H} = \hat{T} + \hat{V}$  is derived: the kinetic term  $\hat{T}$  arises from the free transport closure system with uniform scalar capacity, and the potential term  $\hat{V}$  arises from the spatial modulation of the scalar capacity field. The Hamiltonian is therefore not a choice but a consequence.

The potential  $V(x)$  is not introduced by hand. Every potential treated in the QM-series—the Coulomb potential of the hydrogenic sector, the harmonic potential of QM6, the barrier potentials of QM10—arises from a specific scalar capacity modulation  $\delta\lambda(x)$  through the correspondence of Sec. 3. The admissible class of potentials is determined by the scalar–conformal geometry, not by external physical input.

The conservation laws are not assumed from classical mechanics. They are derived from the commutation structure of the transport generators with the Hamiltonian, which is in turn a consequence of the symmetry properties of the scalar–conformal transport law. No classical Noether

theorem is invoked; the quantum Noether structure of Sec. 7 is self-contained within the Hilbert space framework.

No wavefunction collapse, measurement postulate, or new probabilistic axiom is introduced. The Schrödinger dynamics established in the present paper are deterministic: given the initial closure state  $\Psi_0 \in \mathcal{D}(\hat{H})$ , the full time evolution  $\Psi(t) = U(t)\Psi_0$  is uniquely determined. The statistical interpretation of  $\Psi(t)$  is inherited from the Born frequency law of QB6 and the Parseval identity of QM1, and is not re-derived here.

## 1.4 Structure of the Paper

Sec. 2 recalls the scalar–conformal geometry, the free transport structure, the Schrödinger-type compatibility result of the Q-series, and the operator and norm-preservation results of QM1, and identifies precisely the four structural gaps that the present paper closes. Sec. 3 derives the general correspondence between a scalar capacity modulation  $\delta\lambda(x)$  and a potential energy function  $V(x)$ , defines the admissible class of scalar–conformal potentials, and verifies that the hydrogenic Coulomb potential is recovered as a special case. Sec. 4 establishes self-adjointness of the Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  on the Sobolev domain  $H^2(\mathbb{R}^3)$  via the Kato–Rellich theorem, records the essential self-adjointness on the Schwartz domain, and describes the spectral structure of  $\hat{H}$  for admissible potentials. Sec. 5 applies Stone’s theorem to construct the unitary time-evolution group  $U(t)$  and derives the Schrödinger equation as its infinitesimal generator equation, establishes uniqueness of the evolution, and recovers the Q-series hydrogenic compatibility result as a special case of the general theorem. Sec. 6 establishes unitarity of  $U(t)$ , records the consistency of this result with the norm-preservation corollary of QM1, and shows that the Schrödinger dynamics preserves the full inner product on  $\mathcal{H}$ . Sec. 7 derives the Noether-type conservation laws from the commutation structure of transport generators, establishing conservation of  $\langle \hat{H} \rangle$ ,  $\langle \hat{p}_j \rangle$ , and  $\langle \hat{L}_j \rangle$  from time-translation, spatial-translation, and rotational symmetry respectively, and records the Noether bridge as a theorem. Sec. 8 establishes the Ehrenfest theorem as a corollary of the conservation structure and identifies the classical transport trajectory with the evolution of mean position and momentum. Sec. 9 collects interpretive clarifications, maintains the interpretive boundary conditions of the prior series, and records the scope of the present construction. Sec. 10 summarizes the results, records their programmatic significance for the QM-series and the approach to RQM, and prepares the transition to QM5.

## 2 Recalled Structure from Prior Series

The present section collects the results from the M-, Q-, QB-, and QM1-series that are directly needed for the developments of Secs. 3–8. Each subsection recalls what has been established, states it in the form in which it will be used, and provides the relevant citations. The final subsection identifies the four structural gaps that the prior series leaves open and that the present paper closes.

### 2.1 Scalar-Conformal Geometry and Transport Energy

The M-series established the scalar–conformal geometric framework that underlies all subsequent sector developments. The physical metric is

$$g_{\mu\nu} = \Lambda^2 \eta_{\mu\nu}, \tag{1}$$

where  $\Lambda : \mathcal{M} \rightarrow \mathbb{R}_{>0}$  is the scalar capacity field and  $\eta$  is the reference Lorentzian metric. The base-line scalar level  $\Lambda_0 > 0$  represents the structural capacity supported by the delivery substrate in the

absence of localized occupation; in the unperturbed state  $\Lambda(x) = \Lambda_0$  uniformly. The dimensionless scalar diagnostic is

$$\lambda(x) := \frac{\Lambda(x)}{\Lambda_0}, \quad (2)$$

so that  $\lambda = 1$  in the baseline and  $\lambda(x) \neq 1$  wherever the scalar capacity field is modulated by the presence of a localized structure or exchange process.

The exchange-sector action functional, derived in the M-series and developed in the Q-series, involves the scalar capacity field through the conformal factor. In the non-relativistic limit, spatial gradients of  $\Lambda$ —equivalently, spatial gradients of  $\lambda$ —appear in the transport closure equation as restoring terms. These restoring terms play the role of potential energy in the transport dynamics: a region where  $\lambda < 1$  (reduced structural capacity due to localized occupation) presents an energy barrier to incoming transport, while a region where the gradient structure of  $\lambda$  is attractive acts as a potential well that can support bound closure modes. The explicit form of this potential correspondence is derived in Sec. 3; the present subsection records that the scalar–conformal geometry provides the geometric origin of all potential energy in the NUVO framework.

**Remark 2.1.** *The identification of potential energy with the gradient structure of the scalar capacity field  $\Lambda$  is not an assumption imported into the present paper. It is a consequence of the M-series action functional, which was derived from the scalar–conformal variational structure without reference to any external potential. The present paper makes this identification explicit and derives its functional form in Sec. 3.*

## 2.2 The Free Transport Generator and Kinetic Structure

In the baseline state  $\Lambda(x) = \Lambda_0$  (uniform scalar capacity, no spatial modulation), the exchange-sector transport closure system reduces to the free transport system. The Q-series and QB-series established that in this limit the transport generator associated with spatial displacement is the momentum operator  $\hat{p}_j = -i\Phi_0 \partial_j$ , and the energy associated with free transport is the kinetic energy. The kinetic operator is accordingly

$$\hat{T} := \frac{1}{2m} \sum_{j=1}^3 \hat{p}_j^2 = -\frac{\Phi_0^2}{2m} \Delta, \quad (3)$$

where  $\Delta = \sum_{j=1}^3 \partial_j^2$  is the spatial Laplacian on  $\mathbb{R}^3$  and  $m$  is the structural mass parameter fixed by the hydrogenic correspondence of the Q-series through the identification  $\Phi_0 = \hbar$ .

The self-adjoint properties of  $\hat{T}$  on  $\mathcal{H}$  follow from QM1. By QM1 Theorem 5.2, each momentum generator  $\hat{p}_j$  is essentially self-adjoint on the Schwartz domain  $\mathcal{S}(\mathbb{R}^3)$  with unique self-adjoint closure having domain  $H^1(\mathbb{R}^3)$ . For the kinetic operator  $\hat{T} = \hat{p}^2/(2m)$ , which involves second-order differentiation, the corresponding domain is the second-order Sobolev space:

$$\mathcal{D}(\hat{T}) = H^2(\mathbb{R}^3) := \{f \in \mathcal{H} \mid \Delta f \in \mathcal{H}\}, \quad (4)$$

where  $\Delta f$  is understood in the distributional sense. The operator  $\hat{T}$  is self-adjoint on  $H^2(\mathbb{R}^3)$ ; this is a standard consequence of the Fourier-transform characterization of the Laplacian on  $L^2(\mathbb{R}^3)$  [4] and is not re-derived here.

**Remark 2.2.** *The promotion from the first-order Sobolev domain  $H^1(\mathbb{R}^3)$  of the individual momentum operators to the second-order Sobolev domain  $H^2(\mathbb{R}^3)$  of the kinetic operator is automatic:*

since  $\hat{T} = \sum_j \hat{p}_j^2 / (2m)$  involves two derivatives, the domain condition  $\hat{T}f \in \mathcal{H}$  requires two distributional derivatives to be square-integrable, which is precisely the definition of  $H^2(\mathbb{R}^3)$ . The domain  $H^2(\mathbb{R}^3)$  will serve as the domain of the full Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  in Sec. 4, provided the potential  $\hat{V}$  does not enlarge it—a condition verified by the Kato–Rellich theorem.

### 2.3 Schrödinger-Type Compatibility from the Q-Series

The Q-series established that the exchange-sector transport closure system, in the hydrogenic configuration, is compatible with a Schrödinger-type representation. Specifically, Q3 showed that the coupled deterministic evolution of the closure density  $\rho$  and the transport phase  $\phi$  can be written in a form consistent with

$$i\Phi_0 \partial_t \Psi = \hat{H}_H \Psi, \quad (5)$$

where  $\hat{H}_H$  is the hydrogenic Hamiltonian operator and  $\Psi = \sqrt{\rho} e^{i\phi/\Phi_0}$  is the QB1 complex encoding. Q4 then identified the discrete eigenvalues of  $\hat{H}_H$  with the holonomic closure modes and recovered the hydrogenic energy spectrum  $E_n = -me^4 / (2\Phi_0^2 n^2)$  through the correspondence limit.

It is essential for the present development to state precisely what the Q-series result does and does not establish. The Q-series result is a *compatibility* statement: the transport closure evolution is consistent with the form Eq. (5) in the hydrogenic sector. It is not a derivation of Eq. (5) as a theorem on the complete Hilbert space  $\mathcal{H}$ , for three reasons.

First, the Q-series did not establish that  $\hat{H}_H$  is self-adjoint on  $\mathcal{H}$ . Compatibility with the form Eq. (5) does not require the operator to be self-adjoint; it requires only that the equation hold in a formal sense within the finite-dimensional closure mode space. Self-adjointness on  $\mathcal{H}$ , which requires the Kato–Rellich analysis of Sec. 4, was not established.

Second, the Q-series did not establish uniqueness of the evolution. Without self-adjointness of  $\hat{H}_H$  on  $\mathcal{H}$  and the application of Stone’s theorem, there is no guarantee that the compatibility relation Eq. (5) defines a unique time evolution for all initial data in  $\mathcal{H}$ .

Third, the Q-series result is confined to the hydrogenic sector. A general scalar capacity modulation  $\delta\lambda(x) \neq \delta\lambda_H(x)$  generates a potential different from the Coulomb form, and the Q-series provides no general framework for treating such configurations.

The present paper closes all three limitations. The Q-series compatibility result is recovered as a corollary of the general theorem in Proposition 5.9.

### 2.4 Norm Preservation and Operator Structure from QM1

Four results from QM1 are directly used in the present paper and are recalled here in the form in which they will be applied.

*Norm preservation* (QM1 Corollary 3.4). For any admissible transport evolution with  $\|\Psi(\cdot, 0)\|_{\mathcal{H}} = 1$ , the norm satisfies  $\|\Psi(\cdot, t)\|_{\mathcal{H}} = 1$  for all  $t \in \mathbb{R}$ . This result was derived from the divergence-free structure of the continuity equation and is independent of the Schrödinger dynamics. In Sec. 6 it will be shown that the same conclusion follows from the unitarity of  $U(t)$ , providing a second, operator-level derivation that is consistent with and independent of the geometric transport derivation of QM1.

*Hilbert space structure* (QM1 Theorem 4.3). The space  $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$  equipped with the closure inner product  $\langle \cdot, \cdot \rangle_{\mathcal{H}}$  is a separable complex Hilbert space. This is the ambient space on which the Hamiltonian of Sec. 4 acts and on which Stone’s theorem of Sec. 5 is applied.

*Essential self-adjointness of momentum generators* (QM1 Theorem 5.2). The operators  $\hat{p}_j = -i\Phi_0 \partial_j$  are essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$  with closure domain  $H^1(\mathbb{R}^3)$ . This result underlies the

self-adjointness of  $\hat{T}$  on  $H^2(\mathbb{R}^3)$  recalled in Sec. 2.2, and is the starting point for the Kato–Rellich analysis of Sec. 4.

*Canonical commutation relation on  $\mathcal{H}$*  (QM1 Proposition 5.4). On the dense domain  $\mathcal{S}(\mathbb{R}^3) \subset \mathcal{H}$ ,

$$[\hat{x}^j, \hat{p}_k] \Psi = i\Phi_0 \delta^j_k \Psi \quad (6)$$

for all  $\Psi \in \mathcal{S}(\mathbb{R}^3)$ . This relation is used in Sec. 7 to compute the commutator  $[\hat{H}, \hat{x}^j]$  and in Sec. 8 to derive the Ehrenfest equations.

## 2.5 Four Structural Gaps Addressed by the Present Paper

The prior series leaves four structural gaps that must be closed before the Schrödinger equation can be established as a theorem on  $\mathcal{H}$ . They are stated here with the precision required for the reader to follow the logical sequence of the paper.

*Gap (i): The general  $\Lambda \rightarrow V$  correspondence.* The Q-series derived the Coulomb potential  $V_H(x) = -e^2/(4\pi\epsilon_0|x|)$  from the hydrogenic scalar capacity modulation by direct correspondence with the hydrogenic spectrum. This derivation is sector-specific and does not provide a general framework for determining  $V(x)$  from an arbitrary spatial modulation  $\delta\lambda(x)$  of the scalar capacity field. Without this general correspondence, the Hamiltonian  $\hat{H}$  cannot be defined for transport sectors beyond the hydrogenic case, and the QM-series cannot progress to the harmonic oscillator (QM6), barrier potentials (QM10), or the general central force problem (QM5). Gap (i) is closed in Sec. 3.

*Gap (ii): Self-adjointness of  $\hat{H} = \hat{T} + \hat{V}$ .* QM1 established that the free kinetic operator  $\hat{T} = -(\Phi_0^2/2m)\Delta$  is self-adjoint on  $H^2(\mathbb{R}^3)$ . The addition of a potential operator  $\hat{V}$  (multiplication by  $V(x)$ ) to a self-adjoint operator does not in general preserve self-adjointness: if  $V$  is sufficiently singular, the operator  $\hat{T} + \hat{V}$  may fail to be symmetric, or may be symmetric but not self-adjoint, or may have multiple self-adjoint extensions. The Kato–Rellich theorem provides a sufficient condition—relative boundedness of  $\hat{V}$  with respect to  $\hat{T}$  with relative bound less than one—that guarantees self-adjointness of  $\hat{H}$  on the original domain  $H^2(\mathbb{R}^3)$ . Verifying this condition for the scalar–conformal potentials of Gap (i) is the content of Sec. 4. Without Gap (ii) closed, the spectral theorem cannot be applied to  $\hat{H}$  and Stone’s theorem cannot be invoked.

*Gap (iii): The Schrödinger equation as a theorem on  $\mathcal{H}$ .* Stone’s theorem (Theorem 5.1) states that a self-adjoint operator  $A$  on  $\mathcal{H}$  uniquely determines a strongly continuous unitary group  $e^{-iAt}$ , and that this group satisfies the generator equation  $i\partial_t(e^{-iAt}\Psi) = A e^{-iAt}\Psi$  for  $\Psi \in \mathcal{D}(A)$ . Applying Stone’s theorem with  $A = \hat{H}/\Phi_0$ —which requires Gap (ii) to be closed first—yields the Schrödinger equation  $i\Phi_0 \partial_t \Psi = \hat{H} \Psi$  as the generator equation of the unitary time-evolution group  $U(t)$ . This derivation is carried out in Sec. 5.

*Gap (iv): The Noether bridge.* The M-series established that the scalar–conformal action is invariant under time translation, spatial translation, and spatial rotation. At the geometric level, this invariance implies the existence of classically conserved quantities via the Noether theorem. However, the quantum-mechanical statement—that the expectation values  $\langle \hat{H} \rangle(t)$ ,  $\langle \hat{p}_j \rangle(t)$ , and  $\langle \hat{L}_j \rangle(t)$  are conserved under the Schrödinger evolution—is a separate result that requires the dynamics of Gap (iii) to be in place. The bridge from geometric transport symmetry (M-series) to conservation of quantum expectation values (present paper) is constructed in Sec. 7 using the commutation structure of the transport generators with  $\hat{H}$ . This bridge has not been explicitly constructed in any prior paper of the NUVO series.

The four gaps are not independent: Gap (ii) depends on Gap (i), Gap (iii) depends on Gap (ii), and Gap (iv) depends on Gap (iii). The present paper closes them in the order (i)–(ii)–(iii)–(iv), with each section providing the input required by the next.

### 3 The Scalar-Conformal Potential: From $\Lambda$ to $V$

The fundamental claim of the present section is that every potential energy function appearing in the QM-series is not introduced as external physical input but is derived from a spatial modulation of the scalar capacity field  $\Lambda$ . The section proceeds in four steps. First, the exchange-sector transport closure law is expanded to first order in the scalar capacity modulation, and it is shown that the first-order correction takes the form of a multiplicative potential. Second, the explicit correspondence between the modulation field  $\delta\lambda$  and the potential function  $V$  is stated as a definition and derived from the exchange-sector action structure of the M-series. Third, the hydrogenic Coulomb potential is recovered as a special case, providing an internal consistency check on the correspondence. Fourth, the admissible class of scalar-conformal potentials is defined and verified to satisfy the hypotheses of the Kato–Rellich theorem invoked in Sec. 4.

#### 3.1 Scalar Capacity Modulation and the Transport Closure Law

The baseline scalar capacity  $\Lambda = \Lambda_0$  (spatially uniform) corresponds to the free transport sector in which the exchange-sector Hamiltonian reduces to the pure kinetic operator  $\hat{T}$  of Sec. 2.2. A spatially varying scalar capacity field

$$\Lambda(x) = \Lambda_0(1 + \delta\lambda(x)), \quad |\delta\lambda(x)| \ll 1, \quad (7)$$

represents a departure from the baseline due to the presence of a localized structural configuration— a bound holonomic closure mode, a charged source, or any other exchange-sector occupation that modifies the local structural capacity. The condition  $|\delta\lambda| \ll 1$  defines the first-order (linear response) regime in which the transport closure law can be expanded in powers of  $\delta\lambda$  and the leading correction identified cleanly.

The exchange-sector action functional, derived in M1 from the scalar-conformal variational structure, takes the general form

$$S[\Psi; \Lambda] = \int \left[ \frac{i\Phi_0}{2} (\bar{\Psi} \partial_t \Psi - \Psi \partial_t \bar{\Psi}) - \mathcal{H}_{\text{ex}}[\Psi; \Lambda] \right] d^3x dt, \quad (8)$$

where  $\mathcal{H}_{\text{ex}}[\Psi; \Lambda]$  is the exchange-sector Hamiltonian density, which depends on  $\Lambda$  through the conformal factor [1]. In the baseline state  $\Lambda = \Lambda_0$ , the Hamiltonian density reduces to the free kinetic density:

$$\mathcal{H}_{\text{ex}}[\Psi; \Lambda_0] = \frac{\Phi_0^2}{2m} |\nabla \Psi|^2. \quad (9)$$

For the modulated field Eq. (7), the Hamiltonian density expands as

$$\mathcal{H}_{\text{ex}}[\Psi; \Lambda] = \frac{\Phi_0^2}{2m} |\nabla \Psi|^2 + \delta\lambda(x) \cdot \left. \frac{\delta \mathcal{H}_{\text{ex}}}{\delta(\delta\lambda)} \right|_{\delta\lambda=0} + \mathcal{O}(\delta\lambda^2). \quad (10)$$

The first-order variation  $\delta \mathcal{H}_{\text{ex}} / \delta(\delta\lambda)|_{\delta\lambda=0}$  is determined by the structure of the M-series exchange-sector action and is computed in Sec. 3.2. The critical structural claim—that this variation is a function of  $x$  alone, not a differential operator acting on  $\Psi$ —is established by the following lemma.

**Lemma 3.1** (First-order capacity modulation generates a multiplicative potential). *Let  $\Lambda(x) = \Lambda_0(1 + \delta\lambda(x))$  with  $\delta\lambda \in C_c^\infty(\mathbb{R}^3)$  a smooth, compactly supported modulation satisfying  $\|\delta\lambda\|_{L^\infty} \ll 1$ . Then the exchange-sector transport closure law, derived from the action Eq. (8) by variation with respect to  $\bar{\Psi}$ , takes the form*

$$i\Phi_0 \partial_t \Psi = \left[ -\frac{\Phi_0^2}{2m} \Delta + V(x) \right] \Psi + \mathcal{O}(\delta\lambda^2), \quad (11)$$

where  $V : \mathbb{R}^3 \rightarrow \mathbb{R}$  is a real-valued function of position alone.

*Proof.* Varying the action Eq. (8) with respect to  $\bar{\Psi}$  and using the expansion Eq. (10):

$$i\Phi_0 \partial_t \Psi = \frac{\delta}{\delta \bar{\Psi}} \int \mathcal{H}_{\text{ex}} d^3x = -\frac{\Phi_0^2}{2m} \Delta \Psi + \frac{\delta}{\delta \bar{\Psi}} \int \delta\lambda(x) \left. \frac{\delta \mathcal{H}_{\text{ex}}}{\delta \delta\lambda} \right|_{\delta\lambda=0} d^3x + \mathcal{O}(\delta\lambda^2).$$

The first term is the free kinetic operator. For the second term, the M-series exchange-sector action couples  $\delta\lambda$  to the closure state through the local structural capacity: the conformal modulation  $\delta\lambda(x)$  modifies the exchange-sector energy at position  $x$  by an amount proportional to the local closure content  $|\Psi(x)|^2$  at that point, with no derivative of  $\Psi$  appearing at first order [?,1]. This is a consequence of the fact that  $\delta\lambda$  enters the M-series action as a background field (not a dynamical variable) that couples to the closure density  $\rho = |\Psi|^2$  through the structural capacity per unit volume, which is a local (contact) interaction. The first-order variation of  $\mathcal{H}_{\text{ex}}$  with respect to  $\bar{\Psi}$  at fixed  $\delta\lambda$  is therefore a multiplication operator:

$$\frac{\delta}{\delta \bar{\Psi}} \int \delta\lambda(x) \left. \frac{\delta \mathcal{H}_{\text{ex}}}{\delta \delta\lambda} \right|_{\delta\lambda=0} d^3x = V(x) \Psi(x),$$

where  $V(x)$  is a real-valued function whose explicit form is determined by the M-series exchange-sector action in Sec. 3.2. Combining gives Eq. (11).  $\square$

**Remark 3.2.** *The multiplicative (contact) character of the first-order potential is the key structural feature established by Lemma 3.1. It reflects the fact that the scalar capacity modulation  $\delta\lambda(x)$  acts as a spatially varying energy offset for the exchange-sector closure state: at each point  $x$ , the local departure from baseline capacity contributes an energy per unit closure content equal to  $V(x)$ . This is consistent with the interpretive framework of the M-series, in which the scalar capacity field governs the energetic cost of exchange-sector occupation per unit structural volume. Higher-order corrections in  $\delta\lambda$  generate derivative couplings and non-local terms, which are systematically computable but lie outside the scope of the present first-order treatment.*

### 3.2 Derivation of the Potential from the Conformal Structure

Lemma 3.1 establishes that the first-order correction to the transport closure law is a multiplicative potential  $V(x)$ . The task of the present subsection is to identify  $V(x)$  explicitly as a function of the modulation  $\delta\lambda(x)$ .

From the M-series exchange-sector action [1], the first-order coupling between the scalar capacity modulation and the exchange-sector closure density takes the form

$$\int \mathcal{H}_{\text{ex}}^{(1)} d^3x = -\kappa_\Lambda \int \delta\lambda(x) |\Psi(x)|^2 d^3x, \quad (12)$$

where  $\kappa_\Lambda$  is a structural coupling constant with dimensions of energy, determined by the conformal factor of the M-series baseline geometry and the mass parameter  $m$  [?,1]. The negative sign reflects the convention that a positive modulation  $\delta\lambda > 0$  (increased capacity) reduces the exchange energy of the closure state, consistent with the attractive well structure of the hydrogenic sector.

Varying Eq. (12) with respect to  $\bar{\Psi}$  yields the potential operator:

**Definition 3.3** (Scalar-conformal potential). *Given a scalar capacity modulation  $\delta\lambda \in C_c^\infty(\mathbb{R}^3)$  satisfying the conditions of Lemma 3.1, the associated scalar-conformal potential is*

$$V(x) := -\kappa_\Lambda \delta\lambda(x), \quad (13)$$

where the structural coupling constant  $\kappa_\Lambda > 0$  is fixed by the  $M$ -series exchange-sector action and satisfies

$$\kappa_\Lambda = \frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{\Lambda_0 r_\Lambda}, \quad (14)$$

in which  $r_\Lambda$  is the characteristic length scale of the hydrogenic scalar capacity modulation established in  $Q_4$ , and  $e^2/(4\pi\epsilon_0)$  is the squared charge in SI units. The expression Eq. (14) is established by the hydrogenic specialization of Proposition 3.5 below.

**Remark 3.4.** The structural coupling constant  $\kappa_\Lambda$  is ultimately a derived quantity whose precise value follows from the  $M$ -series conformal factor structure and the  $Q$ -series exchange-sector action. The expression Eq. (14) records the value obtained by specializing to the hydrogenic sector and requiring consistency with the Coulomb potential established in  $Q_4$ . A first-principles derivation of  $\kappa_\Lambda$  directly from the  $M1$  action functional, without appeal to the hydrogenic specialization, is a natural next step within the *NUVO* program and is deferred as a separate item. The results of the present paper require only that  $\kappa_\Lambda$  is a well-defined positive constant, which follows from the existence of the hydrogenic correspondence.

**Proposition 3.5** (Hydrogenic specialization). *Let  $\delta\lambda_H(x)$  denote the scalar capacity modulation field generated by the hydrogenic source configuration, as established in  $Q_4$ . Then the scalar-conformal potential of Definition 3.3, evaluated for  $\delta\lambda = \delta\lambda_H$ , yields*

$$V_H(x) = -\kappa_\Lambda \delta\lambda_H(x) = -\frac{e^2}{4\pi\epsilon_0|x|}, \quad (15)$$

thereby recovering the Coulomb potential of the hydrogenic sector as a special case of the general scalar-conformal correspondence Eq. (13).

*Proof.* The  $Q$ -series established, through the hydrogenic correspondence of  $Q_4$ , that the scalar capacity modulation due to the hydrogenic source has the radial form

$$\delta\lambda_H(x) = \frac{\Lambda_0 r_\Lambda}{|x|}, \quad (16)$$

where  $r_\Lambda$  is the characteristic length set by the structural parameters of the hydrogenic configuration [2]. Substituting Eq. (16) into the general correspondence Eq. (13) and using the expression Eq. (14) for  $\kappa_\Lambda$ :

$$V_H(x) = -\kappa_\Lambda \frac{\Lambda_0 r_\Lambda}{|x|} = -\frac{e^2}{4\pi\epsilon_0} \cdot \frac{1}{\Lambda_0 r_\Lambda} \cdot \frac{\Lambda_0 r_\Lambda}{|x|} = -\frac{e^2}{4\pi\epsilon_0|x|},$$

which is the Coulomb potential. □

**Remark 3.6.** Proposition 3.5 serves as an internal consistency check on the scalar-conformal correspondence: the general formula Eq. (13) must reproduce the Coulomb potential in the hydrogenic sector, since the  $Q$ -series derived the hydrogenic spectrum from the Coulomb form. The proposition confirms this consistency and simultaneously fixes the coupling constant  $\kappa_\Lambda$  through the requirement that the two derivations agree. Any future first-principles derivation of  $\kappa_\Lambda$  from  $M1$  must reproduce Eq. (14) as a necessary condition for internal consistency of the *NUVO* program.

### 3.3 The Admissible Class of Potentials

The application of the Kato–Rellich theorem in Sec. 4 requires the potential operator  $\hat{V}$  (multiplication by  $V(x)$ ) to be relatively bounded with respect to the kinetic operator  $\hat{T}$  with relative bound strictly less than one. The present subsection defines this condition precisely and verifies that the scalar-conformal potentials of Definition 3.3 satisfy it for all physically admissible modulations.

**Definition 3.7** (Admissible potential class). *A real-valued function  $V : \mathbb{R}^3 \rightarrow \mathbb{R}$  is admissible if it is relatively bounded with respect to the kinetic operator  $\hat{T} = -(\Phi_0^2/2m)\Delta$  with relative bound  $a < 1$ : there exist constants  $a \in [0, 1)$  and  $b \geq 0$  such that*

$$\|V f\|_{\mathcal{H}} \leq a \left\| \hat{T} f \right\|_{\mathcal{H}} + b \|f\|_{\mathcal{H}} \quad (17)$$

for all  $f \in \mathcal{D}(\hat{T}) = H^2(\mathbb{R}^3)$ . The collection of all admissible potentials is denoted  $\mathcal{K}$ .

A sufficient condition for membership in  $\mathcal{K}$  is the decomposition  $V = V_1 + V_2$  with  $V_1 \in L^2(\mathbb{R}^3)$  and  $V_2 \in L^\infty(\mathbb{R}^3)$ , since such a decomposition implies Eq. (17) with  $a = 0$  in the limit of small-norm approximation [4]. This decomposition is available for all spatially localized modulation fields.

**Proposition 3.8** (Scalar-conformal potentials are admissible). *Let  $\delta\lambda \in L^2(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3)$  be a spatially localized scalar capacity modulation, and let  $V = -\kappa_\Lambda \delta\lambda$  be the associated scalar-conformal potential of Definition 3.3. Then  $V \in \mathcal{K}$ . In particular, the hydrogenic Coulomb potential  $V_{\text{H}}(x) = -e^2/(4\pi\epsilon_0|x|)$  is admissible with relative bound  $a < 1$  for all values of the coupling [4].*

*Proof.* For a general localized  $\delta\lambda \in L^2(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3)$ : since  $|V(x)| = \kappa_\Lambda |\delta\lambda(x)|$  and  $\delta\lambda$  is bounded and square-integrable,  $V$  decomposes as  $V = V_1 + V_2$  where, for any  $R > 0$ ,

$$V_1(x) = V(x) \mathbf{1}_{|x| \leq R}(x), \quad V_2(x) = V(x) \mathbf{1}_{|x| > R}(x).$$

Since  $|V|$  is bounded by  $\kappa_\Lambda \|\delta\lambda\|_{L^\infty}$  and supported within the spatial support of  $\delta\lambda$ ,  $V_1 \in L^2(\mathbb{R}^3)$  (bounded function on a bounded set) and  $V_2 \in L^\infty(\mathbb{R}^3)$  (bounded outside the ball of radius  $R$ ). A standard Sobolev interpolation argument then yields the relative bound Eq. (17) with constants  $a$  and  $b$  depending on  $R$ ,  $\kappa_\Lambda$ , and  $\|\delta\lambda\|_{L^\infty}$  [4]. Since  $V$  is bounded in  $L^\infty$  for bounded  $\delta\lambda$ , one can choose  $R$  large enough so that  $a < 1$ .

For the Coulomb potential  $V_{\text{H}}(x) = -e^2/(4\pi\epsilon_0|x|)$ : this is a classical result. The Coulomb potential in  $\mathbb{R}^3$  is in the Kato class  $\mathcal{K}^3$  and satisfies the relative bound Eq. (17) with respect to  $-(\Phi_0^2/2m)\Delta$  with  $a < 1$  for any value of the coupling  $e^2/(4\pi\epsilon_0)$  [4]. This is Theorem X.11 of Reed and Simon, cited here for orientation.  $\square$

**Remark 3.9.** *The admissibility condition of Definition 3.7 is satisfied by all scalar-conformal potentials generated by physically localized modulation fields. In particular, it covers the potentials needed for the physical sectors of the QM-series: the Coulomb potential of QM5 (central force, hydrogenic), the harmonic potential  $V(x) = \frac{1}{2}k|x|^2$  of QM6 (bounded from below, in  $L_{\text{loc}}^\infty$  but growing at infinity, which is treated separately via the Kato class for polynomially bounded potentials [4]), the piecewise constant barrier potentials of QM10, and multi-particle Coulomb interactions of QM7.*

### 3.4 Scope and Generalization

The correspondence  $\delta\lambda \mapsto V = -\kappa_\Lambda \delta\lambda$  established in the present section is a first-order result in the perturbative expansion of the exchange-sector transport law in powers of  $\delta\lambda$ . The scope of this result and the directions of its generalization are recorded here.

Within scope: the correspondence applies to all smooth, spatially localized modulations  $\delta\lambda \in L^2(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3)$  in the first-order regime  $\|\delta\lambda\|_{L^\infty} \ll 1$ ; it covers time-independent configurations  $\Lambda = \Lambda(x)$  (static potentials); and it produces potentials  $V \in \mathcal{K}$  for all such modulations, as established in Proposition 3.8. Every potential treated in QM5 through QM10 lies within this class.

Outside scope: time-dependent modulations  $\delta\lambda(x, t)$ , which generate time-dependent potentials of the form  $V(x, t)$  (relevant for the electromagnetic interaction sector, treated in the RQM-series); strong modulations with  $\|\delta\lambda\|_{L^\infty} \sim 1$ , for which the first-order expansion Eq. (11) is insufficient and higher-order terms in  $\delta\lambda$  contribute to the Hamiltonian; and singular potentials outside  $\mathcal{K}$ , which would require a separate self-adjoint extension analysis beyond the scope of the Kato–Rellich theorem.

**Remark 3.10.** *The harmonic oscillator potential  $V(x) = \frac{1}{2}k|x|^2$ , treated in QM6, grows unboundedly at spatial infinity and does not belong to the  $L^2 \cap L^\infty$  class of the preceding analysis. Its admissibility with respect to  $\hat{T}$  is established separately: the operator  $\hat{T} + \hat{V}_{\text{harm}}$  is essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$  by a direct argument using the explicit eigenbasis (the Hermite functions), independently of the Kato–Rellich theorem [4]. The scalar–conformal origin of the harmonic potential—a quadratically growing modulation  $\delta\lambda(x) \propto |x|^2$  of the scalar capacity field, corresponding to a confining structural configuration—is consistent with the general correspondence of Definition 3.3 and is developed in QM6.*

## 4 The Hamiltonian as a Self-Adjoint Operator on $\mathcal{H}$

With the scalar-conformal potential  $V \in \mathcal{K}$  established in Sec. 3, the present section constructs the Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  as a genuine self-adjoint operator on the Hilbert space  $\mathcal{H}$  of QM1. Self-adjointness is not a formal nicety: it is the precise condition required by Stone’s theorem in Sec. 5 to guarantee that  $\hat{H}$  generates a unique unitary time-evolution group. Without it, the Schrödinger equation either fails to define a unique evolution or generates an evolution that does not preserve the  $\mathcal{H}$ -norm, both of which would be inconsistent with the closure conservation established in QM1.

The section proceeds in four steps. The Hamiltonian is first defined on the natural initial domain  $\mathcal{S}(\mathbb{R}^3)$ , where it is well-defined as a differential operator. The Kato–Rellich theorem is then stated as a standalone result so that its hypotheses can be verified explicitly before the theorem is applied. Self-adjointness of  $\hat{H}$  on the Sobolev domain  $H^2(\mathbb{R}^3)$  is then established as the main theorem of this section. Finally, essential self-adjointness on  $\mathcal{S}(\mathbb{R}^3)$  is recorded as a corollary, and the spectral structure of  $\hat{H}$  is described.

### 4.1 The Formal Hamiltonian on the Schwartz Domain

The scalar-conformal Hamiltonian is assembled from the two components whose properties are established in the preceding sections: the kinetic operator  $\hat{T}$  of Sec. 2.2 and the potential operator  $\hat{V}$  arising from the scalar capacity modulation of Sec. 3.2.

**Definition 4.1** (Formal Hamiltonian). *The scalar-conformal Hamiltonian associated to a potential  $V \in \mathcal{K}$  is the operator*

$$\hat{H} := \hat{T} + \hat{V} = -\frac{\Phi_0^2}{2m}\Delta + V(x), \quad (18)$$

*defined initially on the dense domain  $\mathcal{D}(\hat{H})_0 = \mathcal{S}(\mathbb{R}^3) \subset \mathcal{H}$ , where  $V(x)$  acts as a multiplication operator and  $\Delta = \sum_{j=1}^3 \partial_j^2$  is the spatial Laplacian.*

That  $\hat{H}$  is well-defined on  $\mathcal{S}(\mathbb{R}^3)$  follows immediately from its component structure. The kinetic operator  $\hat{T} = -(\Phi_0^2/2m)\Delta$  maps  $\mathcal{S}(\mathbb{R}^3)$  to itself, since differentiation and multiplication by polynomials both preserve the Schwartz class. The potential operator  $\hat{V}$  acts on  $f \in \mathcal{S}(\mathbb{R}^3)$  by pointwise multiplication:  $(\hat{V}f)(x) = V(x)f(x)$ . For  $V \in \mathcal{K}$ , the product  $Vf$  is in  $\mathcal{H}$  for every  $f \in \mathcal{S}(\mathbb{R}^3)$ : since  $f$  decays faster than any polynomial and  $V$  grows at most polynomially in the admissible class (see Remark 3.9), the product is square-integrable. The operator  $\hat{H}$  is therefore a densely defined linear operator on  $\mathcal{H}$  with initial domain  $\mathcal{S}(\mathbb{R}^3)$ .

**Remark 4.2.** *The formal Hamiltonian  $\hat{H}$  is symmetric on  $\mathcal{S}(\mathbb{R}^3)$ : for any  $f, g \in \mathcal{S}(\mathbb{R}^3)$ ,  $\langle f, \hat{H}g \rangle_{\mathcal{H}} = \langle \hat{H}f, g \rangle_{\mathcal{H}}$ . Symmetry of  $\hat{T}$  on  $\mathcal{S}(\mathbb{R}^3)$  follows from the integration-by-parts argument of QM1 Lemma 5.1 applied twice (since  $\hat{T}$  involves two derivatives). Symmetry of  $\hat{V}$  follows from the fact that  $V$  is real-valued:  $\langle f, Vg \rangle_{\mathcal{H}} = \int \bar{f}Vg \, d^3x = \int \bar{g}Vf \, d^3x = \langle g, Vf \rangle_{\mathcal{H}}^* = \langle Vf, g \rangle_{\mathcal{H}}$ . Symmetry is a necessary condition for self-adjointness but is not sufficient; the Kato–Rellich theorem provides the sufficient condition.*

## 4.2 The Kato–Rellich Theorem

The Kato–Rellich theorem is the functional-analytic tool that converts the relative boundedness established in Proposition 3.8 into a self-adjointness conclusion for the full Hamiltonian. It is stated here as a standalone theorem so that its hypotheses can be verified explicitly against the operators at hand before the theorem is applied.

**Theorem 4.3** (Kato–Rellich). *Let  $A$  be a self-adjoint operator on  $\mathcal{H}$  with domain  $\mathcal{D}(A)$ , and let  $B$  be a symmetric operator satisfying  $\mathcal{D}(A) \subseteq \mathcal{D}(B)$ . Suppose  $B$  is relatively bounded with respect to  $A$  with relative bound  $a < 1$ : there exist constants  $a \in [0, 1)$  and  $b \geq 0$  such that*

$$\|Bf\|_{\mathcal{H}} \leq a \|Af\|_{\mathcal{H}} + b \|f\|_{\mathcal{H}}$$

*for all  $f \in \mathcal{D}(A)$ . Then  $A + B$  is self-adjoint on  $\mathcal{D}(A)$ , and if  $A$  is essentially self-adjoint on some core  $\mathcal{D} \subseteq \mathcal{D}(A)$  then  $A + B$  is also essentially self-adjoint on  $\mathcal{D}$  [4].*

The proof is a classical result of functional analysis, cited for orientation [4]. The condition  $a < 1$  is sharp: if  $a = 1$  the conclusion may fail, and there exist symmetric perturbations with relative bound exactly one for which  $A + B$  is not self-adjoint on  $\mathcal{D}(A)$ . The requirement  $a < 1$  is therefore not merely a technical convenience but the precise condition that distinguishes controllable perturbations from those that may destroy the self-adjoint domain structure.

**Remark 4.4.** *The relative bound condition  $\|Bf\|_{\mathcal{H}} \leq a \|Af\|_{\mathcal{H}} + b \|f\|_{\mathcal{H}}$  with  $a < 1$  has a direct physical interpretation in the transport closure setting. The kinetic operator  $\hat{T}$  controls the regularity of the closure state  $\Psi$  through its second-order derivative structure: states in  $\mathcal{D}(\hat{T}) = H^2(\mathbb{R}^3)$  have two square-integrable derivatives. The relative bound condition states that the potential energy  $\hat{V}$  cannot extract more than a fraction  $a < 1$  of the kinetic energy plus a fixed  $L^2$ -norm contribution. Physically, this means the potential does not dominate the kinetic term at short distances: the closure state remains regular on  $H^2(\mathbb{R}^3)$  even in the presence of the potential. This is the functional-analytic statement of the condition that the scalar capacity modulation  $\delta\lambda(x)$  is a controlled perturbation of the baseline geometry, consistent with the first-order expansion of Sec. 3.1.*

### 4.3 Self-Adjointness of the Hamiltonian

The hypotheses of the Kato–Rellich Theorem 4.3 are verified against the operators  $A = \hat{T}$  and  $B = \hat{V}$  explicitly before the conclusion is stated.

*Hypothesis 1:*  $A = \hat{T}$  is self-adjoint on  $\mathcal{D}(A) = H^2(\mathbb{R}^3)$ . This was established in Sec. 2.2 as a consequence of QM1 Theorem 5.2 and the standard characterization of the domain of the Laplacian on  $L^2(\mathbb{R}^3)$  [4].

*Hypothesis 2:*  $B = \hat{V}$  is symmetric with  $H^2(\mathbb{R}^3) \subseteq \mathcal{D}(\hat{V})$ . Symmetry follows from the real-valuedness of  $V$ , as recorded in Remark 4.2. The domain inclusion  $H^2(\mathbb{R}^3) \subseteq \mathcal{D}(\hat{V})$  holds because for  $f \in H^2(\mathbb{R}^3) \subset L^2(\mathbb{R}^3)$ , the product  $Vf$  is in  $\mathcal{H}$  by the relative bound Eq. (17) with  $a < 1$ : since  $\|Vf\|_{\mathcal{H}} \leq a \left\| \hat{T}f \right\|_{\mathcal{H}} + b \|f\|_{\mathcal{H}} < \infty$  for  $f \in H^2(\mathbb{R}^3)$ , the multiplication by  $V$  is a well-defined map from  $H^2(\mathbb{R}^3)$  to  $\mathcal{H}$ .

*Hypothesis 3:*  $B = \hat{V}$  is relatively bounded with respect to  $A = \hat{T}$  with relative bound  $a < 1$ . This is precisely Proposition 3.8, which establishes that all scalar-conformal potentials  $V \in \mathcal{K}$  satisfy the relative bound Eq. (17) with  $a < 1$ .

All three hypotheses are satisfied. The Kato–Rellich theorem therefore applies and yields the following result.

**Theorem 4.5** (Self-adjointness of the scalar-conformal Hamiltonian). *Let  $V \in \mathcal{K}$  be an admissible scalar-conformal potential in the sense of Definition 3.7. Then the scalar-conformal Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  of Definition 4.1 is self-adjoint on the domain*

$$\mathcal{D}(\hat{H}) = H^2(\mathbb{R}^3) = \{f \in \mathcal{H} \mid \Delta f \in \mathcal{H}\}, \quad (19)$$

where  $\Delta f$  is understood in the distributional sense.

*Proof.* The three hypotheses of the Kato–Rellich Theorem 4.3 have been verified above:  $\hat{T}$  is self-adjoint on  $H^2(\mathbb{R}^3)$ ,  $\hat{V}$  is symmetric with  $H^2(\mathbb{R}^3) \subseteq \mathcal{D}(\hat{V})$ , and  $\hat{V}$  is relatively bounded with respect to  $\hat{T}$  with relative bound  $a < 1$  by Proposition 3.8. Theorem 4.3 therefore applies with  $A = \hat{T}$  and  $B = \hat{V}$ , and yields self-adjointness of  $\hat{H} = \hat{T} + \hat{V}$  on  $\mathcal{D}(\hat{T}) = H^2(\mathbb{R}^3)$ .  $\square$

**Remark 4.6.** *A notable consequence of Theorem 4.5 is that the domain of the full Hamiltonian  $\hat{H}$  is the same as the domain of the kinetic operator  $\hat{T}$  alone: both are  $H^2(\mathbb{R}^3)$ . The addition of the potential  $\hat{V}$  does not shrink or enlarge the operator domain. This is a direct consequence of the relative bound  $a < 1$ : a potential with  $a < 1$  is controlled enough by the kinetic operator that it does not require additional regularity of the closure state beyond that imposed by  $\hat{T}$  itself. In physical terms, the scalar capacity modulation does not introduce new short-distance singularities that would restrict the class of admissible closure states beyond those already required to have square-integrable second derivatives.*

### 4.4 Essential Self-Adjointness on the Schwartz Domain

Theorem 4.5 establishes that  $\hat{H}$  is self-adjoint on the Sobolev domain  $H^2(\mathbb{R}^3)$ . An equally important result for the NUVO program is that  $\hat{H}$  is essentially self-adjoint on the Schwartz domain  $\mathcal{S}(\mathbb{R}^3)$ , which is the natural initial domain for transport closure states. Essential self-adjointness on  $\mathcal{S}(\mathbb{R}^3)$  means that  $\hat{H}$  restricted to smooth, rapidly decreasing states has a unique self-adjoint extension, and that extension is the operator of Theorem 4.5. No additional boundary conditions or extension choices are required.

**Corollary 4.7** (Essential self-adjointness on the Schwartz domain). *For  $V \in \mathcal{K}$ , the Hamiltonian  $\hat{H}$  of Definition 4.1 is essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$ . Its unique self-adjoint closure is the operator of Theorem 4.5, with domain  $H^2(\mathbb{R}^3)$ .*

*Proof.* The kinetic operator  $\hat{T}$  is essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$  by QM1 Theorem 5.2. For potentials  $V \in \mathcal{K}$ , the second part of the Kato–Rellich Theorem 4.3—which states that essential self-adjointness on a core is preserved under an admissible perturbation—applies with core  $\mathcal{D} = \mathcal{S}(\mathbb{R}^3)$  [4]. Since  $\hat{T}$  is essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$  and  $\hat{V}$  is admissible with relative bound  $a < 1$ , the full Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  is essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$ , with closure equal to the self-adjoint operator on  $H^2(\mathbb{R}^3)$  established in Theorem 4.5.  $\square$

**Remark 4.8.** *The physical significance of Corollary 4.7 within the NUVO program is the same as that recorded in QM1 Remark 5.3 for the free momentum operators: essential self-adjointness on  $\mathcal{S}(\mathbb{R}^3)$  means that the transport closure system selects a unique Hamiltonian operator from the initial domain of smooth, rapidly decreasing states, without any freedom of boundary condition. The scalar capacity geometry uniquely determines the Hamiltonian, and the Hamiltonian uniquely determines the dynamics via Stone’s theorem in Sec. 5. The derivation chain  $\delta\lambda \rightarrow V \rightarrow \hat{H} \rightarrow U(t) \rightarrow \Psi(t)$  is therefore free of choices at every stage.*

## 4.5 Spectral Properties of the Hamiltonian

With  $\hat{H}$  established as a self-adjoint operator on  $\mathcal{H}$ , the spectral theorem of QM1 (Theorem 6.1 of QM1) applies immediately, yielding a unique projection-valued measure  $E_{\hat{H}}$  and a spectral decomposition of every closure state in  $\mathcal{H}$ . The present subsection records the qualitative spectral structure for the admissible class of scalar-conformal Hamiltonians, which is needed for the interpretation of the conservation laws in Sec. 7 and for the scattering analysis of QM10.

**Proposition 4.9** (Spectral structure of the scalar-conformal Hamiltonian). *For an admissible scalar-conformal Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  with  $V \in \mathcal{K}$ , the following spectral properties hold.*

- (i) *The spectral theorem of QM1 applies to  $\hat{H}$ , yielding a unique projection-valued measure  $E_{\hat{H}}$  on  $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$  such that  $\hat{H} = \int_{\mathbb{R}} E dE_{\hat{H}}(E)$ , and a spectral decomposition  $\Psi = \int_{\sigma(\hat{H})} dE_{\hat{H}}(E) \Psi$  in the  $\mathcal{H}$ -norm for every  $\Psi \in \mathcal{H}$ .*
- (ii) *The spectrum decomposes as  $\sigma(\hat{H}) = \sigma_{\text{disc}}(\hat{H}) \cup \sigma_{\text{cont}}(\hat{H})$ , where  $\sigma_{\text{disc}}(\hat{H})$  is a discrete (possibly empty) set of negative eigenvalues with normalizable eigenfunctions in  $\mathcal{H}$ , and  $\sigma_{\text{cont}}(\hat{H})$  is the essential spectrum determined by the asymptotic behavior of  $V$ .*
- (iii) *The discrete eigenstates  $\{\psi_n\} \subset H^2(\mathbb{R}^3)$  satisfying  $\hat{H} \psi_n = E_n \psi_n$  with  $E_n < 0$  are identified with the bound holonomic closure modes of the Q-series; they form a complete orthonormal family within the discrete-spectrum subspace.*

*Proof.* Part (i) follows from Theorem 4.5 and the QM1 spectral theorem (QM1 Theorem 6.1): self-adjointness of  $\hat{H}$  on  $\mathcal{H}$  is the sole hypothesis of the spectral theorem, which is therefore applicable. Part (ii) is a standard consequence of the Weyl essential spectrum theorem [?]: for  $V \in \mathcal{K}$  with  $V(x) \rightarrow 0$  as  $|x| \rightarrow \infty$  (the case of all physically relevant scalar-conformal potentials), the essential spectrum of  $\hat{H}$  coincides with that of  $\hat{T}$ , which is  $[0, \infty)$ ; discrete eigenvalues below zero are possible and correspond to bound states. Part (iii) follows from the identification of the bound holonomic closure modes of the Q-series with the discrete eigenstates of the hydrogenic Hamiltonian, established in the Q-series and recovered via Proposition 5.9 of Sec. 5.  $\square$

**Remark 4.10.** *The spectral structure described in Proposition 4.9 is the precise form of the discrete-continuous decomposition established in QM1 Sec. ??, now applied to the physical Hamiltonian  $\hat{H}$  of the scalar-conformal transport closure system. The discrete spectrum  $\sigma_{\text{disc}}(\hat{H})$  corresponds to the bound holonomic closure modes—spatially localized, normalizable states in  $\mathcal{H}$ —that were identified in the  $Q$ -series and constitute the  $QB$ -series finite-dimensional pre-Hilbert space  $\mathcal{H}^{\text{fin}}$ . The continuous spectrum  $\sigma_{\text{cont}}(\hat{H}) = [0, \infty)$  corresponds to the unbound, scattering-type transport configurations that are the subject of QM10. With the spectral theorem now applied to the full  $\hat{H}$ , the decomposition of arbitrary closure states into bound and scattering components is available for all subsequent work in the  $QM$ -series.*

**Remark 4.11.** *For the hydrogenic Hamiltonian  $\hat{H}_{\text{H}} = \hat{T} + \hat{V}_{\text{H}}$  with Coulomb potential  $V_{\text{H}}(x) = -e^2/(4\pi\epsilon_0|x|)$ , the discrete spectrum is  $\sigma_{\text{disc}}(\hat{H}_{\text{H}}) = \{E_n\}_{n \geq 1}$  with  $E_n = -me^4/(2\Phi_0^2 n^2)$ , recovering the energy levels established in Q4. The continuous spectrum is  $\sigma_{\text{cont}}(\hat{H}_{\text{H}}) = [0, \infty)$ , corresponding to ionized configurations in which the closure transport is no longer holonomically bound. These are consistent with the general structure of Proposition 4.9 and provide an internal consistency check on the self-adjoint extension established in Theorem 4.5.*

## 5 Schrödinger Dynamics from Stone’s Theorem

The self-adjoint Hamiltonian  $\hat{H}$  established in Theorem 4.5 is the input to the present section. Stone’s theorem converts this single operator-theoretic fact into a complete dynamical framework: a unique unitary time-evolution group, a well-posed initial value problem, and the Schrödinger equation as its generator equation. The logical structure is clean and irreversible in the sense that each step has a unique output: the self-adjoint generator uniquely determines the unitary group, the unitary group uniquely determines the evolution, and the evolution uniquely determines the solution for each initial datum in  $\mathcal{D}(\hat{H})$ . No dynamical postulate enters at any stage.

### 5.1 Stone’s Theorem: Statement and Hypotheses

Stone’s theorem is the functional-analytic result that establishes the equivalence between self-adjoint operators and strongly continuous unitary groups on a Hilbert space. It is stated here in full so that its hypotheses can be identified explicitly and verified against the operators at hand.

**Theorem 5.1** (Stone’s theorem). *Let  $A$  be a self-adjoint operator on  $\mathcal{H}$  with domain  $\mathcal{D}(A)$ . Then the family of operators  $\{U(t)\}_{t \in \mathbb{R}}$  defined via the spectral functional calculus by*

$$U(t) := e^{-iAt} = \int_{\mathbb{R}} e^{-iat} dE_A(a),$$

where  $E_A$  is the projection-valued measure of  $A$ , constitutes a strongly continuous one-parameter unitary group on  $\mathcal{H}$ : it satisfies

- (i)  $U(0) = \hat{\mathbf{1}}$ ,
- (ii)  $U(t)U(s) = U(t+s)$  for all  $t, s \in \mathbb{R}$ ,
- (iii)  $U(t)^* = U(t)^{-1} = U(-t)$  for all  $t \in \mathbb{R}$ , and
- (iv) for each  $\Psi \in \mathcal{H}$ , the map  $t \mapsto U(t)\Psi$  is continuous in the  $\mathcal{H}$ -norm.

Moreover,  $A$  is recovered as the infinitesimal generator: for each  $\Psi \in \mathcal{D}(A)$ , the strong derivative  $\frac{d}{dt}U(t)\Psi|_{t=0}$  exists in  $\mathcal{H}$  and equals  $-iA\Psi$ . Conversely, every strongly continuous one-parameter unitary group on  $\mathcal{H}$  arises in this way from a unique self-adjoint generator [3].

The proof is a foundational result of functional analysis, cited for orientation [3]. The converse direction—every strongly continuous unitary group has a unique self-adjoint generator—is the part of Stone’s theorem responsible for the uniqueness claims recorded in Corollary 5.7 below.

**Remark 5.2.** The definition  $U(t) = e^{-iAt} = \int_{\mathbb{R}} e^{-iat} dE_A(a)$  uses the spectral functional calculus for self-adjoint operators, which applies to any bounded Borel-measurable function of the spectral variable. Since  $a \mapsto e^{-iat}$  is bounded for each fixed  $t \in \mathbb{R}$  (it has modulus one for all  $a \in \mathbb{R}$ ), the operator  $U(t)$  is a well-defined bounded operator on all of  $\mathcal{H}$  for each  $t$ . This is in contrast to  $\hat{H}$  itself, which is unbounded: the time-evolution operator is always bounded even when the generator is not. In the NUVO framework, this means that the full Hilbert space  $\mathcal{H}$  is the appropriate domain for the dynamics:  $U(t)$  maps  $\mathcal{H}$  to  $\mathcal{H}$  for all  $t$ , while  $\hat{H}$  acts only on the dense subspace  $H^2(\mathbb{R}^3) \subset \mathcal{H}$ .

## 5.2 The Unitary Time-Evolution Operator

Stone’s theorem is applied to the scalar-conformal Hamiltonian by identifying the self-adjoint generator  $A$  with  $\hat{H}/\Phi_0$ . The factor  $\Phi_0$  is required by dimensional consistency: the generator of time evolution must have dimensions of inverse time, while  $\hat{H}$  has dimensions of energy, so the combination  $\hat{H}/\Phi_0$  carries the correct dimensions.

*Verification of the Stone’s theorem hypothesis.* By Theorem 4.5,  $\hat{H}$  is self-adjoint on  $H^2(\mathbb{R}^3) \subset \mathcal{H}$ . Since  $\Phi_0 > 0$  is a fixed positive constant, the operator  $A := \hat{H}/\Phi_0$  is self-adjoint on the same domain  $H^2(\mathbb{R}^3)$ : self-adjointness is preserved under multiplication by a positive real scalar. The hypothesis of Stone’s theorem is therefore satisfied with  $A = \hat{H}/\Phi_0$ .

**Theorem 5.3** (Time-evolution operator). *Let  $\hat{H}$  be the scalar-conformal Hamiltonian of Theorem 4.5. Then the family of bounded operators*

$$U(t) := \exp\left(-\frac{i\hat{H}t}{\Phi_0}\right) = \int_{\mathbb{R}} e^{-iEt/\Phi_0} dE_{\hat{H}}(E), \quad t \in \mathbb{R}, \quad (20)$$

where  $E_{\hat{H}}$  is the projection-valued measure of  $\hat{H}$  established in Proposition 4.9, constitutes a strongly continuous one-parameter unitary group on  $\mathcal{H}$  satisfying properties (i)–(iv) of Stone’s Theorem 5.1. This group is the unique strongly continuous unitary group on  $\mathcal{H}$  whose self-adjoint generator is  $\hat{H}/\Phi_0$ .

*Proof.* Apply Stone’s Theorem 5.1 with  $A = \hat{H}/\Phi_0$ . Self-adjointness of  $A$  on  $H^2(\mathbb{R}^3)$  is verified above. Stone’s theorem therefore yields a strongly continuous one-parameter unitary group  $U(t) = e^{-iAt} = e^{-i\hat{H}t/\Phi_0}$  on  $\mathcal{H}$  satisfying properties (i)–(iv). The spectral integral representation in Eq. (20) follows from the functional calculus definition  $e^{-iAt} = \int_{\mathbb{R}} e^{-iat} dE_A(a)$  with  $a = E/\Phi_0$  and the projection-valued measure  $E_{\hat{H}}$  of Proposition 4.9. Uniqueness follows from the converse direction of Stone’s theorem: any two strongly continuous unitary groups on  $\mathcal{H}$  with the same self-adjoint generator must coincide [3].  $\square$

**Remark 5.4.** The spectral integral form of Eq. (20) makes the connection to the spectral decomposition of QM1 transparent. For a closure state  $\Psi_0$  expanded in the spectral basis of  $\hat{H}$  as

$\Psi_0 = \sum_n c_n \psi_n + \int c(k) \psi_k dk$  (QM1 Proposition 7.3), the time evolution acts by multiplying each spectral coefficient by a phase factor:

$$U(t)\Psi_0 = \sum_n c_n e^{-iE_n t/\Phi_0} \psi_n + \int c(k) e^{-iE(k)t/\Phi_0} \psi_k dk.$$

Each spectral component acquires an independent oscillatory phase at a rate determined by its eigenvalue. The modulus of each coefficient  $|c_n|$  and  $|c(k)|$  is preserved by  $|e^{-iEt/\Phi_0}| = 1$ , consistent with the Parseval identity and the closure conservation established in QM1.

### 5.3 The Schrödinger Equation as a Generator Equation

The Schrödinger equation is now derived as the generator equation of the unitary group  $U(t)$ . This derivation is the culmination of the logical chain that began with the scalar capacity modulation in Sec. 3: the geometry determines the potential, the potential determines the Hamiltonian, the Hamiltonian determines the unitary group, and the unitary group determines the equation of motion.

**Theorem 5.5** (The Schrödinger equation). *Let  $\hat{H}$  be the scalar-conformal Hamiltonian of Theorem 4.5 and let  $U(t)$  be the unitary group of Theorem 5.3. For each initial datum  $\Psi_0 \in \mathcal{D}(\hat{H}) = H^2(\mathbb{R}^3)$ , define the curve*

$$\Psi(t) := U(t) \Psi_0, \quad t \in \mathbb{R}. \quad (21)$$

Then:

- (i)  $\Psi(t) \in \mathcal{D}(\hat{H})$  for all  $t \in \mathbb{R}$ .
- (ii) The map  $t \mapsto \Psi(t)$  is strongly differentiable in  $\mathcal{H}$ .
- (iii)  $\Psi(t)$  satisfies the time-dependent Schrödinger equation

$$i\Phi_0 \frac{d}{dt} \Psi(t) = \hat{H} \Psi(t) \quad (22)$$

strongly in  $\mathcal{H}$  for all  $t \in \mathbb{R}$ .

*Proof. Part (i):* By Stone's Theorem 5.1, the unitary group  $U(t)$  preserves the domain of its generator. Since the generator of  $U(t)$  is  $A = \hat{H}/\Phi_0$  with domain  $\mathcal{D}(A) = H^2(\mathbb{R}^3) = \mathcal{D}(\hat{H})$ , and since unitary operators preserve the spectral structure of the generator, the domain  $H^2(\mathbb{R}^3)$  is invariant under  $U(t)$  for all  $t \in \mathbb{R}$ : if  $\Psi_0 \in H^2(\mathbb{R}^3)$  then  $U(t)\Psi_0 \in H^2(\mathbb{R}^3)$  for all  $t$  [3].

*Part (ii):* By Stone's theorem applied to  $\Psi_0 \in \mathcal{D}(A) = \mathcal{D}(\hat{H})$ , the map  $t \mapsto U(t)\Psi_0$  is strongly differentiable in  $\mathcal{H}$  and its strong derivative satisfies

$$\frac{d}{dt} U(t)\Psi_0 = -iA U(t)\Psi_0 = -\frac{i}{\Phi_0} \hat{H} U(t)\Psi_0 = -\frac{i}{\Phi_0} \hat{H} \Psi(t), \quad (23)$$

where in the last step part (i) is used to confirm that  $\hat{H} \Psi(t)$  is well-defined in  $\mathcal{H}$ .

*Part (iii):* Multiply both sides of Eq. (23) by  $i\Phi_0$ :

$$i\Phi_0 \frac{d}{dt} \Psi(t) = i\Phi_0 \cdot \left( -\frac{i}{\Phi_0} \right) \hat{H} \Psi(t) = \hat{H} \Psi(t).$$

This is Eq. (22), which holds strongly in  $\mathcal{H}$  for all  $t \in \mathbb{R}$ . □

**Remark 5.6.** Equation (22) is the time-dependent Schrödinger equation of quantum mechanics. In the standard formulation of quantum mechanics it is introduced as a fundamental postulate: a law of nature asserted without derivation. In the present framework it is a theorem, derived in three logically ordered steps from the scalar-conformal geometry. The derivation chain is:  $\delta\lambda(x)$  (scalar capacity modulation, Sec. 3.1)  $\Rightarrow V(x) \in \mathcal{K}$  (potential from correspondence, Sec. 3.2 and 3.3)  $\Rightarrow \hat{H} = \hat{T} + \hat{V}$  self-adjoint on  $H^2(\mathbb{R}^3)$  (Kato-Rellich, Theorem 4.5)  $\Rightarrow U(t) = e^{-i\hat{H}t/\Phi_0}$  unitary group (Stone's theorem, Theorem 5.3)  $\Rightarrow i\Phi_0 \partial_t \Psi = \hat{H} \Psi$  (generator equation, Theorem 5.5). No step in this chain introduces a dynamical postulate.

## 5.4 Uniqueness of the Schrödinger Evolution

The derivation above establishes that  $\Psi(t) = U(t)\Psi_0$  is a solution of Eq. (22). The converse direction of Stone's theorem establishes that it is the only solution.

**Corollary 5.7** (Uniqueness of the Schrödinger evolution). *For each  $\Psi_0 \in \mathcal{D}(\hat{H})$ , the curve  $\Psi(t) = U(t)\Psi_0$  is the unique strongly differentiable solution of Eq. (22) in  $\mathcal{H}$  satisfying  $\Psi(0) = \Psi_0$ .*

*Proof.* Suppose  $\tilde{\Psi}(t)$  is a second strongly differentiable solution of Eq. (22) with  $\tilde{\Psi}(0) = \Psi_0$  and  $\tilde{\Psi}(t) \in \mathcal{D}(\hat{H})$  for all  $t$ . Then  $\tilde{\Psi}(t)$  defines a strongly continuous map satisfying the generator equation for  $\hat{H}/\Phi_0$ , which by the converse direction of Stone's Theorem 5.1 implies that the group generated by  $\tilde{\Psi}$  must coincide with  $U(t)$  [3]. In particular,  $\tilde{\Psi}(t) = U(t)\Psi_0 = \Psi(t)$  for all  $t$ .  $\square$

**Remark 5.8.** *Corollary 5.7 is the Hilbert-space expression of the deterministic character of the scalar-conformal transport closure system. The Q-series established that the coupled  $(\rho, \phi)$  evolution is a deterministic system: given the initial closure density and transport phase, the subsequent evolution is uniquely determined by the transport closure law. Corollary 5.7 establishes the same conclusion at the level of the Schrödinger equation on  $\mathcal{H}$ : given the initial state  $\Psi_0 \in \mathcal{D}(\hat{H})$ , no other evolution is consistent with the Schrödinger dynamics generated by  $\hat{H}$ . The statistical character of quantum measurements, inherited from the Born frequency law of QB6, arises not from indeterminism in the dynamics but from the coherence-gated interaction structure: the evolution itself is deterministic and unique.*

## 5.5 Recovery of the Q-Series Compatibility Result

The Schrödinger equation of Theorem 5.5 is a general result, valid for all admissible scalar-conformal Hamiltonians  $\hat{H} \in \mathcal{K}$ . The Q-series Schrödinger-type compatibility result of Sec. 2.3 is recovered as a special case.

**Proposition 5.9** (Recovery of the Q-series result). *The time-dependent Schrödinger equation of Theorem 5.5, specialized to the hydrogenic Hamiltonian  $\hat{H}_H = \hat{T} + \hat{V}_H$  with the Coulomb potential of Proposition 3.5, extends and completes the Schrödinger-type compatibility result of the Q-series in the following respects.*

- (i)  $\hat{H}_H$  is self-adjoint on  $H^2(\mathbb{R}^3)$  and essentially self-adjoint on  $\mathcal{S}(\mathbb{R}^3)$  by Theorem 4.5 and Corollary 4.7. The Q-series did not establish self-adjointness.
- (ii) The unitary time-evolution group  $U_H(t) = e^{-i\hat{H}_H t/\Phi_0}$  is the unique strongly continuous unitary group on  $\mathcal{H}$  generated by  $\hat{H}_H/\Phi_0$ . The Q-series did not establish uniqueness of the evolution.
- (iii) The equation  $i\Phi_0 \partial_t \Psi = \hat{H}_H \Psi$  holds as a strong equation on  $\mathcal{D}(\hat{H}_H) = H^2(\mathbb{R}^3)$ , for all  $\Psi_0 \in H^2(\mathbb{R}^3)$  and all  $t \in \mathbb{R}$ . The Q-series established only a compatibility condition within the finite-dimensional span of stationary modes.

- (iv) *The stationary closure modes  $\psi_n$  of the Q-series are the discrete eigenstates of  $\hat{H}_H$  in  $\mathcal{H}$ , with eigenvalues  $E_n = -me^4/(2\Phi_0^2 n^2)$ . Under the full Schrödinger evolution,  $U_H(t)\psi_n = e^{-iE_n t/\Phi_0}\psi_n$ , so stationary modes remain stationary with time-dependent phase factors, consistent with the Q-series result.*

*Proof.* Parts (i) and (ii) follow directly from Theorems 4.5 and 5.3 applied to  $\hat{H}_H$ , whose admissibility was established in Proposition 3.8. Part (iii) follows from Theorem 5.5 applied to  $\hat{H}_H$ . For part (iv): if  $\hat{H}_H\psi_n = E_n\psi_n$  in  $\mathcal{H}$ , then by the spectral functional calculus,  $U_H(t)\psi_n = e^{-i\hat{H}_H t/\Phi_0}\psi_n = e^{-iE_n t/\Phi_0}\psi_n$ , since the exponential of a constant multiple of the identity acts as scalar multiplication on eigenstates. The eigenvalues  $E_n = -me^4/(2\Phi_0^2 n^2)$  are recovered from the Q-series hydrogenic spectrum via Remark 4.11.  $\square$

**Remark 5.10.** *Proposition 5.9 is the explicit statement of the promotion from compatibility to the theorem that was identified as Gap (iii) in Sec. 2.5. The Q-series result—that the hydrogenic transport closure system is consistent with a Schrödinger-type equation—is now understood as the restriction of Theorem 5.5 to the hydrogenic sector. Every aspect of the Q-series result that was tentative (self-adjointness not verified, uniqueness not established, scope limited to stationary modes) is now established as a rigorous consequence of the general theorem. The Q-series is thereby retroactively confirmed as correct within its stated scope, and the present paper provides the general framework within which it is embedded.*

## 6 Unitarity and Norm Preservation

The unitary group  $U(t)$  established in Theorem 5.3 carries with it, by definition, a set of norm-preservation and inner-product-preservation properties that are recorded in the present section. These properties are not new results in the sense of requiring derivation beyond what Stone’s theorem already provides; rather, they are the physical and structural consequences of unitarity that must be stated explicitly in order to close the logical arc of the present paper. Three items are addressed. First, unitarity of  $U(t)$  is recorded as a theorem and its norm-preservation consequence is stated. Second, the relationship between this operator-level norm preservation and the geometric norm preservation of QM1 Corollary 3.4 is analyzed; their logical independence and mutual consistency constitute a non-trivial internal check on the NUVO framework. Third, the preservation of the full inner product—not merely the norm—is established, and its consequences for the orthogonality structure of the transport closure states are recorded.

### 6.1 Unitarity of the Time-Evolution Operator

Unitarity of  $U(t)$  is an immediate consequence of Stone’s theorem and requires no additional argument: property (iii) of Theorem 5.1 states that  $U(t)^* = U(-t) = U(t)^{-1}$ , which is precisely the definition of unitarity. The present theorem records this fact and draws the norm-preservation consequence that is of primary physical importance.

**Theorem 6.1** (Unitarity and norm preservation). *The time-evolution operator  $U(t)$  of Theorem 5.3 is unitary for every  $t \in \mathbb{R}$ :*

$$U(t)^*U(t) = U(t)U(t)^* = \hat{\mathbf{1}}. \quad (24)$$

*Consequently, for any  $\Psi \in \mathcal{H}$  and all  $t \in \mathbb{R}$ ,*

$$\|U(t)\Psi\|_{\mathcal{H}} = \|\Psi\|_{\mathcal{H}}. \quad (25)$$

In particular, if  $\|\Psi_0\|_{\mathcal{H}} = 1$  then  $\|\Psi(t)\|_{\mathcal{H}} = 1$  for all  $t \in \mathbb{R}$ , where  $\Psi(t) = U(t)\Psi_0$  is the Schrödinger evolution of Theorem 5.5.

*Proof.* Unitarity Eq. (24) is part (iii) of Stone's Theorem 5.1, which established  $U(t)^* = U(-t)$  and  $U(t)U(-t) = U(0) = \hat{\mathbf{1}}$  by the group law. For norm preservation Eq. (25):

$$\|U(t)\Psi\|_{\mathcal{H}}^2 = \langle U(t)\Psi, U(t)\Psi \rangle_{\mathcal{H}} = \langle \Psi, U(t)^*U(t)\Psi \rangle_{\mathcal{H}} = \langle \Psi, \hat{\mathbf{1}}\Psi \rangle_{\mathcal{H}} = \|\Psi\|_{\mathcal{H}}^2,$$

where the second step uses the definition of the adjoint and the third uses Eq. (24). Taking square roots gives Eq. (25). The special case  $\|\Psi_0\|_{\mathcal{H}} = 1$  follows immediately.  $\square$

**Remark 6.2.** *The unitarity of  $U(t)$  can also be seen directly from the spectral integral representation of Eq. (20). For any  $\Psi \in \mathcal{H}$ :*

$$\|U(t)\Psi\|_{\mathcal{H}}^2 = \int_{\sigma(\hat{H})} |e^{-iEt/\Phi_0}|^2 d \langle \Psi, E_{\hat{H}}(E) \Psi \rangle_{\mathcal{H}} = \int_{\sigma(\hat{H})} 1 d \langle \Psi, E_{\hat{H}}(E) \Psi \rangle_{\mathcal{H}} = \|\Psi\|_{\mathcal{H}}^2,$$

since  $|e^{-iEt/\Phi_0}| = 1$  for all  $E \in \mathbb{R}$  and all  $t \in \mathbb{R}$ . This computation shows that norm preservation is a pointwise property of the spectral phase factors: every spectral component of  $\Psi$  acquires a phase of unit modulus under time evolution, so the total norm is unchanged.

## 6.2 Consistency with QM1 Norm Preservation

Norm preservation under Schrödinger evolution, Eq. (25), was already established in QM1 by a completely different argument. QM1 Corollary 3.4 derived the identity  $\|\Psi(\cdot, t)\|_{\mathcal{H}} = 1$  from the divergence-form continuity equation Eq. (??): the total closure  $\int_{\mathbb{R}^3} |\Psi|^2 d^3x$  is conserved under admissible transport because  $\rho$  satisfies a source-free conservation law at the geometric level. That derivation required no operator theory, no Hamiltonian, and no Stone's theorem; it followed solely from the transport closure geometry of the Q-series.

The present theorem establishes the same conclusion from a completely different starting point: the unitarity of  $U(t)$ , which is itself a consequence of the self-adjointness of  $\hat{H}$  and Stone's theorem. That two independent derivations, one geometric and one operator-theoretic, yield the same norm-preservation conclusion is a non-trivial consistency check on the internal structure of the NUVO framework.

**Proposition 6.3** (Consistency of the two norm-preservation derivations). *The norm-preservation statement  $\|\Psi(t)\|_{\mathcal{H}} = \|\Psi_0\|_{\mathcal{H}}$  for Schrödinger evolutions  $\Psi(t) = U(t)\Psi_0$  holds by two logically independent arguments:*

- (i) Geometric derivation (QM1 Corollary 3.4): *The continuity equation  $\partial_t \rho + \nabla \cdot (\rho v) = 0$  implies  $\frac{d}{dt} \int_{\mathbb{R}^3} |\Psi|^2 d^3x = 0$  by the divergence theorem. This argument uses the Q-series transport geometry and the QB1 encoding, and is independent of the Hamiltonian and the operator theory of the present paper.*
- (ii) Operator-theoretic derivation (Theorem 6.1): *The unitarity  $U(t)^*U(t) = \hat{\mathbf{1}}$ , which is a consequence of the self-adjointness of  $\hat{H}$  via Stone's theorem, implies  $\|U(t)\Psi_0\|_{\mathcal{H}} = \|\Psi_0\|_{\mathcal{H}}$  for all  $\Psi_0 \in \mathcal{H}$  and all  $t \in \mathbb{R}$ . This argument uses the operator theory of Secs. 4–5 and is independent of the continuity equation.*

*The two conclusions are identical. Their agreement is a structural consistency requirement of the NUVO program: the scalar-conformal transport geometry, when promoted to the Hilbert space*

level through the Schrödinger dynamics of Theorem 5.5, must automatically preserve the closure-conservation norm implied by the geometric transport law. The agreement confirms that this requirement is satisfied.

**Remark 6.4.** *The existence of two logically independent routes to the same norm-preservation conclusion reflects the structural coherence of the NUVO program across levels of description. At the geometric level (Q-series, QM1), norm preservation is a consequence of the divergence-free continuity equation for the closure density: it is a statement about the transport geometry. At the operator level (the present paper), norm preservation is a consequence of the unitarity of the time-evolution group: it is a statement about the operator algebra. Both levels describe the same physical system, and their agreement on norm preservation is the Hilbert-space analogue of the consistency condition that the scalar-conformal action produces both the correct transport equations (Q-series) and the correct operator algebra (QB-series). In subsequent papers of the QM-series, both routes remain available and will be used as appropriate: the geometric route for direct transport arguments, the operator-theoretic route for spectral and algebraic arguments.*

### 6.3 The Inner Product as a Conserved Quantity

Unitarity preserves not only the norm of each individual state but the full inner product between any two states evolving under the same dynamics. This is a stronger statement than norm preservation and has direct consequences for the orthogonality structure of the transport closure states.

**Corollary 6.5** (Inner product preservation). *For any  $\Psi_1, \Psi_2 \in \mathcal{H}$  and all  $t \in \mathbb{R}$ ,*

$$\langle U(t)\Psi_1, U(t)\Psi_2 \rangle_{\mathcal{H}} = \langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}}. \quad (26)$$

*Proof.* Using the definition of the adjoint and Eq. (24):

$$\langle U(t)\Psi_1, U(t)\Psi_2 \rangle_{\mathcal{H}} = \langle \Psi_1, U(t)^*U(t)\Psi_2 \rangle_{\mathcal{H}} = \langle \Psi_1, \hat{\mathbf{1}}\Psi_2 \rangle_{\mathcal{H}} = \langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}}.$$

□

The consequences of Corollary 6.5 for the transport closure state structure are recorded in the following remarks.

**Remark 6.6.** *Corollary 6.5 implies in particular that if  $\langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}} = 0$  at  $t = 0$ , then  $\langle U(t)\Psi_1, U(t)\Psi_2 \rangle_{\mathcal{H}} = 0$  for all  $t \in \mathbb{R}$ . Orthogonality is preserved by the Schrödinger evolution. In the NUVO framework this means that distinct holonomic closure classes, which were shown to be mutually orthogonal under the QB3 inner product and whose orthogonality was extended to  $\mathcal{H}$  in QM1 Proposition 4.2, remain orthogonal for all time under the Schrödinger dynamics. The spectral decomposition of any closure state into its discrete and continuous components, established in QM1 Sec. ??, is therefore time-stable: the projection of  $\Psi(t)$  onto any spectral subspace of  $\hat{H}$  evolves as the projection of  $\Psi_0$  multiplied by the corresponding spectral phase factor  $e^{-iE_n t/\Phi_0}$ , with no mixing between orthogonal spectral components.*

**Remark 6.7.** *Inner product preservation has a direct consequence for the Born frequency law of QB6, extended to  $\mathcal{H}$  in QM1. The frequency of a coherence-gated interaction event associated with projector  $\hat{P}_n$  is  $|c_n|^2 = |\langle \psi_n, \Psi \rangle_{\mathcal{H}}|^2$ . Under Schrödinger evolution:*

$$|\langle \psi_n, \Psi(t) \rangle_{\mathcal{H}}|^2 = |\langle U(t)^*\psi_n, \Psi_0 \rangle_{\mathcal{H}}|^2 = \left| \left\langle e^{iE_n t/\Phi_0} \psi_n, \Psi_0 \right\rangle_{\mathcal{H}} \right|^2 = |e^{iE_n t/\Phi_0}|^2 |\langle \psi_n, \Psi_0 \rangle_{\mathcal{H}}|^2 = |c_n|^2,$$

where the second equality uses the fact that  $\psi_n$  is an eigenstate of  $\hat{H}$  and hence of  $U(t)^* = e^{i\hat{H}t/\Phi_0}$ . The interaction frequency associated with each discrete spectral channel is therefore time-independent: the Born frequency law is preserved under the Schrödinger evolution. This is consistent with the physical interpretation of stationary states as closure modes with time-independent interaction frequencies, which was established in the Q-series and the QB-series without derivation; the present computation provides its operator-theoretic justification.

**Remark 6.8.** More generally, for any finite set of closure states  $\{\Psi_1, \dots, \Psi_N\}$ , the Gram matrix  $G_{ij}(t) = \langle U(t)\Psi_i, U(t)\Psi_j \rangle_{\mathcal{H}}$  satisfies  $G_{ij}(t) = G_{ij}(0)$  for all  $t$  by Corollary 6.5. In particular, a set of closure states that is orthonormal at  $t = 0$  remains orthonormal for all  $t \in \mathbb{R}$ . This fact will be used in QM7, where multi-particle antisymmetric states are constructed from orthonormal single-particle closure states, and the stability of the antisymmetry under time evolution requires exactly the inner product preservation established here.

## 7 Symmetries and Conservation Laws

The Schrödinger dynamics established in Sec. 5 and the unitarity results of Sec. 6 together provide the dynamical framework within which symmetry and conservation can be analyzed at the level of quantum expectation values. The present section constructs the Noether bridge identified as Gap (iv) in Sec. 2.5: the derivation connecting the geometric transport symmetry of the M-series, which operates at the level of the scalar-conformal action functional, to the conservation of expectation values of quantum-mechanical observables under Schrödinger evolution.

The structure of the section is as follows. A general conservation theorem is established first: if a self-adjoint observable  $G$  commutes with the Hamiltonian  $\hat{H}$  on a dense domain, then the expectation value  $\langle G \rangle(t)$  is conserved for all Schrödinger evolutions initialized in that domain. This general theorem is then applied to the three fundamental symmetries of the scalar-conformal transport system: time translation (yielding energy conservation), spatial translation (yielding momentum conservation for free transport, and the quantum Newton law for general potentials), and spatial rotation (yielding angular momentum conservation for rotationally symmetric potentials). The closing remark records the Noether bridge explicitly, closing Gap (iv).

### 7.1 Transport Symmetry and the Noether Structure

The classical Noether theorem, at the level of the scalar-conformal action, associates a conserved current to each continuous symmetry of the transport closure system. The quantum-mechanical analogue operates at the level of expectation values: a symmetry of the Hamiltonian, expressed as a commutation relation with a self-adjoint generator, implies that the expectation value of that generator is time-independent under Schrödinger evolution. The following theorem establishes this correspondence in full generality.

**Theorem 7.1** (Conservation law from commutation). *Let  $G$  be a self-adjoint operator on  $\mathcal{H}$  with domain  $\mathcal{D}(G)$ , and suppose that  $\mathcal{D}(\hat{H}) \cap \mathcal{D}(G)$  is dense in  $\mathcal{H}$ . Suppose further that*

$$[\hat{H}, G] \Psi = 0 \quad \text{for all } \Psi \in \mathcal{D}(\hat{H}) \cap \mathcal{D}(G). \quad (27)$$

*Then for any Schrödinger evolution  $\Psi(t) = U(t)\Psi_0$  with  $\Psi_0 \in \mathcal{D}(\hat{H}) \cap \mathcal{D}(G)$ , the expectation value*

$$\langle G \rangle(t) := \langle \Psi(t), G \Psi(t) \rangle_{\mathcal{H}} \quad (28)$$

is well-defined for all  $t \in \mathbb{R}$  and satisfies

$$\frac{d}{dt}\langle G \rangle(t) = 0. \quad (29)$$

*Proof.* Since  $\Psi_0 \in \mathcal{D}(\hat{H}) \cap \mathcal{D}(G)$  and Theorem 5.5 part (i) establishes that  $\Psi(t) \in \mathcal{D}(\hat{H})$  for all  $t$ , it remains to verify that  $\Psi(t) \in \mathcal{D}(G)$  for all  $t$  so that Eq. (28) is well-defined. This follows from the fact that  $U(t)$  preserves any domain that is invariant under  $\hat{H}$ ; since  $G$  commutes with  $\hat{H}$  on the dense domain,  $U(t)$  preserves  $\mathcal{D}(G)$  as well, a standard consequence of the spectral theory of commuting operators [3].

To establish Eq. (29), differentiate  $\langle G \rangle(t) = \langle \Psi(t), G \Psi(t) \rangle_{\mathcal{H}}$  with respect to  $t$ . Since both  $t \mapsto \Psi(t)$  and  $t \mapsto G \Psi(t)$  are strongly differentiable in  $\mathcal{H}$  (the former by Theorem 5.5 part (ii), the latter by the commutation condition and the same theorem), the product rule for inner products gives

$$\frac{d}{dt}\langle G \rangle(t) = \left\langle \frac{d\Psi}{dt}(t), G \Psi(t) \right\rangle_{\mathcal{H}} + \left\langle \Psi(t), G \frac{d\Psi}{dt}(t) \right\rangle_{\mathcal{H}}.$$

Substituting the Schrödinger equation  $\frac{d}{dt}\Psi(t) = -\frac{i}{\Phi_0}\hat{H}\Psi(t)$  from Theorem 5.5:

$$\frac{d}{dt}\langle G \rangle(t) = \left\langle -\frac{i}{\Phi_0}\hat{H}\Psi(t), G \Psi(t) \right\rangle_{\mathcal{H}} + \left\langle \Psi(t), G \left(-\frac{i}{\Phi_0}\hat{H}\Psi(t)\right) \right\rangle_{\mathcal{H}}.$$

Pulling the scalar factors out and using conjugate linearity in the first argument:

$$= \frac{i}{\Phi_0} \left\langle \hat{H}\Psi(t), G \Psi(t) \right\rangle_{\mathcal{H}} - \frac{i}{\Phi_0} \left\langle \Psi(t), G \hat{H}\Psi(t) \right\rangle_{\mathcal{H}}.$$

Since  $\hat{H}$  is self-adjoint,  $\left\langle \hat{H}\Psi, G \Psi \right\rangle_{\mathcal{H}} = \left\langle \Psi, \hat{H} G \Psi \right\rangle_{\mathcal{H}}$ , so the expression becomes

$$\frac{d}{dt}\langle G \rangle(t) = \frac{i}{\Phi_0} \left\langle \Psi(t), \hat{H} G \Psi(t) \right\rangle_{\mathcal{H}} - \frac{i}{\Phi_0} \left\langle \Psi(t), G \hat{H} \Psi(t) \right\rangle_{\mathcal{H}} = \frac{i}{\Phi_0} \left\langle \Psi(t), [\hat{H}, G] \Psi(t) \right\rangle_{\mathcal{H}}.$$

By the commutation condition Eq. (27) and the fact that  $\Psi(t) \in \mathcal{D}(\hat{H}) \cap \mathcal{D}(G)$ :

$$\frac{d}{dt}\langle G \rangle(t) = \frac{i}{\Phi_0} \langle \Psi(t), 0 \rangle_{\mathcal{H}} = 0.$$

□

**Remark 7.2.** The identity  $\frac{d}{dt}\langle G \rangle(t) = \frac{i}{\Phi_0}\langle [\hat{H}, G] \rangle(t)$  established in the proof of Theorem 7.1 holds for any self-adjoint  $G$  (not only those commuting with  $\hat{H}$ ) and is the Schrödinger-picture form of the Heisenberg equation of motion for the observable  $G$ . In the NUVO framework it is not a separate postulate but a direct consequence of the Schrödinger dynamics of Theorem 5.5 and the self-adjointness of  $\hat{H}$ . Theorem 7.1 is the special case  $[\hat{H}, G] = 0$ , which yields  $\frac{d}{dt}\langle G \rangle(t) = 0$ . The general identity  $\frac{d}{dt}\langle G \rangle(t) = \frac{i}{\Phi_0}\langle [\hat{H}, G] \rangle(t)$  will be used directly in Sec. 8 to derive the Ehrenfest equations.

## 7.2 Time-Translation Invariance and Energy Conservation

The first application of Theorem 7.1 is the conservation of energy under Schrödinger evolution. By Stone's Theorem 5.1, the Hamiltonian  $\hat{H}/\Phi_0$  is the generator of the time-translation group  $U(t)$ : it is the operator that controls the response of the system to infinitesimal time translations. The invariance of the static scalar-conformal transport system under time translation—guaranteed by the time-independence of the scalar capacity field  $\Lambda = \Lambda(x)$  in the static sector of Sec. 3.4—is reflected at the operator level by the trivial commutation  $[\hat{H}, \hat{H}] = 0$ .

**Corollary 7.3** (Energy conservation). *For any Schrödinger evolution  $\Psi(t) = U(t)\Psi_0$  with static Hamiltonian  $\hat{H}$  and  $\Psi_0 \in \mathcal{D}(\hat{H})$ ,*

$$\frac{d}{dt}\langle \hat{H} \rangle(t) = \frac{i}{\Phi_0} \langle [\hat{H}, \hat{H}] \rangle(t) = 0. \quad (30)$$

*The expectation value  $\langle \hat{H} \rangle(t)$  is constant for all  $t \in \mathbb{R}$ .*

*Proof.* Apply Theorem 7.1 with  $G = \hat{H}$ . The commutation condition  $[\hat{H}, \hat{H}]\Psi = 0$  holds trivially for all  $\Psi \in \mathcal{D}(\hat{H})$ .  $\square$

**Remark 7.4.** *The conservation of  $\langle \hat{H} \rangle(t)$  is the quantum-expectation-value counterpart of the classical energy conservation that follows, via the classical Noether theorem, from the time-translation invariance of the  $M$ -series scalar-conformal action in the static sector. In the NUVO framework, both results have the same geometric origin—the time-independence of the scalar capacity field  $\Lambda(x)$ —but are derived at different levels: the classical result from the action variational structure, the present result from the Schrödinger dynamics and the commutator  $[\hat{H}, \hat{H}] = 0$ . For an eigenstate  $\psi_n$  of  $\hat{H}$  with eigenvalue  $E_n$ , the expectation value  $\langle \hat{H} \rangle(t) = E_n$  is time-independent and constant for all  $t$ , consistent with the identification of the discrete spectrum with the bound holonomic closure modes of the  $Q$ -series.*

## 7.3 Spatial-Translation Invariance and Momentum Conservation

The momentum operators  $\hat{p}_j = -i\Phi_0 \partial_j$  are the generators of spatial translations in the transport closure system. Their commutation with  $\hat{H}$  is determined by the spatial uniformity or non-uniformity of the potential  $V(x)$ .

**Proposition 7.5** (Momentum commutator and conservation). *On the dense domain  $\mathcal{S}(\mathbb{R}^3) \subset \mathcal{H}$ , the commutator of  $\hat{H} = \hat{T} + \hat{V}$  with  $\hat{p}_j$  is*

$$[\hat{H}, \hat{p}_j] \Psi = i\Phi_0 \frac{\partial V}{\partial x^j}(x) \Psi(x) \quad (31)$$

*for all  $\Psi \in \mathcal{S}(\mathbb{R}^3)$ . Consequently:*

- (i) Free transport ( $V = 0$  or  $V = \text{const}$ ):  $[\hat{H}, \hat{p}_j] = 0$  on  $\mathcal{S}(\mathbb{R}^3)$ , and by Theorem 7.1,  $\langle \hat{p}_j \rangle(t)$  is conserved for all  $t \in \mathbb{R}$ .
- (ii) Non-uniform potential:  $[\hat{H}, \hat{p}_j] \neq 0$  in general, and the rate of change of mean momentum is

$$\frac{d}{dt}\langle \hat{p}_j \rangle(t) = \frac{i}{\Phi_0} \langle [\hat{H}, \hat{p}_j] \rangle(t) = - \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t). \quad (32)$$

*Proof.* Compute  $[\hat{H}, \hat{p}_j] = [\hat{T}, \hat{p}_j] + [\hat{V}, \hat{p}_j]$  on  $\mathcal{S}(\mathbb{R}^3)$ .

*First term:*  $\hat{T} = -(\Phi_0^2/2m)\Delta = \sum_k \hat{p}_k^2/(2m)$ . Since  $[\hat{p}_k^2, \hat{p}_j] = \hat{p}_k[\hat{p}_k, \hat{p}_j] + [\hat{p}_k, \hat{p}_j]\hat{p}_k$  and  $[\hat{p}_k, \hat{p}_j] = 0$  (both are  $-i\Phi_0$  times partial derivatives, which commute), we have  $[\hat{T}, \hat{p}_j] = 0$ .

*Second term:* For  $\Psi \in \mathcal{S}(\mathbb{R}^3)$ , compute  $[\hat{V}, \hat{p}_j]\Psi = V(\hat{p}_j\Psi) - \hat{p}_j(V\Psi)$ :

$$V(x)(-i\Phi_0\partial_j\Psi) - (-i\Phi_0)\partial_j(V(x)\Psi) = -i\Phi_0 V\partial_j\Psi + i\Phi_0((\partial_j V)\Psi + V\partial_j\Psi) = i\Phi_0(\partial_j V)\Psi.$$

Therefore  $[\hat{H}, \hat{p}_j]\Psi = i\Phi_0(\partial V/\partial x^j)\Psi$ , which is Eq. (31).

Part (i) follows immediately: if  $\partial V/\partial x^j = 0$  then  $[\hat{H}, \hat{p}_j] = 0$ , and Theorem 7.1 yields conservation of  $\langle \hat{p}_j \rangle(t)$ .

Part (ii) follows from the general identity of Remark 7.2:

$$\frac{d}{dt}\langle \hat{p}_j \rangle(t) = \frac{i}{\Phi_0}\langle [\hat{H}, \hat{p}_j] \rangle(t) = \frac{i}{\Phi_0} \cdot i\Phi_0 \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t) = - \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t).$$

□

**Remark 7.6.** Equation (32) is the quantum-mechanical analogue of Newton's second law: the rate of change of mean momentum equals the negative mean gradient of the potential, which is the mean force. In the scalar-conformal NUVO framework, this is not an independent physical law but a direct consequence of the commutator structure of the transport generators with the Hamiltonian, which is in turn a consequence of the geometry of the scalar capacity modulation. The force  $-\partial V/\partial x^j = \kappa_\Lambda \partial\delta\lambda/\partial x^j$  is determined entirely by the gradient of the scalar capacity modulation field. This result will appear again as the second Ehrenfest equation in Sec. 8.

## 7.4 Rotational Invariance and Angular Momentum Conservation

The angular momentum operators are the generators of spatial rotations in the transport closure system. Their full algebraic structure is developed in QM5; the present subsection establishes their commutation with  $\hat{H}$  for rotationally symmetric potentials, which is the content of angular momentum conservation needed for the QM-series.

**Proposition 7.7** (Angular momentum commutator and conservation). *Define the angular momentum operators by*

$$\hat{L}_j := \epsilon_{jkl} \hat{x}^k \hat{p}_l = \epsilon_{jkl} x^k (-i\Phi_0 \partial_l), \quad j = 1, 2, 3, \quad (33)$$

where  $\epsilon_{jkl}$  is the Levi-Civita symbol. Then on  $\mathcal{S}(\mathbb{R}^3)$ :

- (i)  $[\hat{T}, \hat{L}_j] = 0$  for all  $j$ .
- (ii) For a rotationally symmetric potential  $V(x) = V(|x|)$ ,  $[\hat{V}, \hat{L}_j] = 0$  for all  $j$ .
- (iii) Consequently, for  $\hat{H} = \hat{T} + \hat{V}$  with  $V = V(|x|)$ ,  $[\hat{H}, \hat{L}_j] = 0$ , and by Theorem 7.1,  $\langle \hat{L}_j \rangle(t)$  is conserved for all  $t \in \mathbb{R}$ .

*Proof. Part (i):* Write  $\hat{T} = \hat{p}^2/(2m) = \sum_k \hat{p}_k^2/(2m)$ . Since  $[\hat{p}_k, \hat{p}_l] = 0$  for all  $k, l$  (momentum operators commute, as their commutator is  $[\hat{p}_k, \hat{p}_l] = (-i\Phi_0)^2[\partial_k, \partial_l] = 0$ ), we have

$$[\hat{p}^2, \hat{L}_j] = \epsilon_{jkl} [\hat{p}^2, \hat{x}^k \hat{p}_l] = \epsilon_{jkl} ([\hat{p}^2, \hat{x}^k]\hat{p}_l + \hat{x}^k[\hat{p}^2, \hat{p}_l]).$$

The second term vanishes since  $[\hat{p}^2, \hat{p}_l] = 0$ . For the first:  $[\hat{p}^2, \hat{x}^k] = \sum_n [\hat{p}_n^2, \hat{x}^k] = \sum_n \hat{p}_n [\hat{p}_n, \hat{x}^k] + [\hat{p}_n, \hat{x}^k]\hat{p}_n$ . Using the CCR Eq. (6):  $[\hat{p}_n, \hat{x}^k] = -[\hat{x}^k, \hat{p}_n] = -i\Phi_0 \delta^k_n$ , so  $[\hat{p}^2, \hat{x}^k] = -2i\Phi_0 \hat{p}_k$ .

Therefore  $[\hat{p}^2, \hat{L}_j] = \epsilon_{jkl} (-2i\Phi_0) \hat{p}_k \hat{p}_l$ . Since  $\epsilon_{jkl}$  is antisymmetric in  $k, l$  and  $\hat{p}_k \hat{p}_l$  is symmetric (as  $[\hat{p}_k, \hat{p}_l] = 0$ ), the contraction  $\epsilon_{jkl} \hat{p}_k \hat{p}_l = 0$ . Hence  $[\hat{T}, \hat{L}_j] = 0$ .

*Part (ii):* For  $V(x) = V(|x|)$ , the operator  $\hat{V}$  depends only on  $|x|^2 = \sum_k (x^k)^2$ . The angular momentum operators generate rotations, and  $|x|^2$  is rotation-invariant. Formally, for  $\Psi \in \mathcal{S}(\mathbb{R}^3)$ :

$$[\hat{V}, \hat{L}_j] \Psi = \epsilon_{jkl} [V(|x|), x^k (-i\Phi_0 \partial_l)] \Psi.$$

Expanding:  $[V, x^k \partial_l] \Psi = V x^k \partial_l \Psi - x^k \partial_l (V \Psi) = -x^k (\partial_l V) \Psi$ . Since  $V = V(|x|)$ ,  $\partial_l V = V'(|x|) \cdot x^l / |x|$ , so the contribution is  $-x^k V'(|x|) x^l / |x|$ . Contracting with  $\epsilon_{jkl}$ :  $\epsilon_{jkl} x^k x^l = 0$  since  $\epsilon_{jkl}$  is antisymmetric and  $x^k x^l$  is symmetric in  $k, l$ . Hence  $[\hat{V}, \hat{L}_j] = 0$ .

*Part (iii):*  $[\hat{H}, \hat{L}_j] = [\hat{T}, \hat{L}_j] + [\hat{V}, \hat{L}_j] = 0 + 0 = 0$  by parts (i) and (ii). Theorem 7.1 with  $G = \hat{L}_j$  then yields  $\frac{d}{dt} \langle \hat{L}_j \rangle(t) = 0$ .  $\square$

**Remark 7.8.** *Proposition 7.7 establishes angular momentum conservation as a theorem of the scalar–conformal dynamics for rotationally symmetric potentials. The operators  $\hat{L}_j$  defined in Eq. (33) are introduced here in their minimal form, as products of position and momentum operators, sufficient for the commutation calculation. Their full algebraic structure—the commutation algebra  $[\hat{L}_j, \hat{L}_k] = i\Phi_0 \epsilon_{jkl} \hat{L}_l$ , the joint spectrum of  $\hat{L}^2$  and  $\hat{L}_3$ , the identification of spherical harmonics as stationary angular closure eigenstates, and the integer holonomy quantization of orbital quantum numbers—is developed in QM5 using the Schrödinger dynamics and the operator framework of the present paper as its foundation.*

## 7.5 The General Noether Identity

The three conservation laws established in Corollary 7.3 and Propositions 7.5 and 7.7 are special cases of a single general pattern, which constitutes the Noether bridge between the geometric transport symmetry of the M-series and the conservation of quantum-mechanical expectation values.

**Remark 7.9** (The Noether bridge). *The M-series established the scalar–conformal action to be invariant under three continuous symmetries in the static sector: time translation (invariance of  $\Lambda(x)$  under  $t \mapsto t + s$ ), spatial translation (invariance of  $\Lambda(x)$  under  $x \mapsto x + a$  for uniform  $\Lambda$ ), and spatial rotation (invariance of  $\Lambda(|x|)$  under  $x \mapsto Rx$  for  $R \in SO(3)$ ). At the level of the classical transport closure system, each symmetry generates a conserved Noether current. The present section has established the quantum-mechanical counterpart: each symmetry corresponds to a transport generator  $G$  that commutes with  $\hat{H}$  on a dense domain, and the associated expectation value  $\langle G \rangle(t)$  is conserved under Schrödinger evolution by Theorem 7.1. The correspondence is:*

$$\begin{aligned} \text{Time translation} &\longleftrightarrow G = \hat{H}, \quad [\hat{H}, \hat{H}] = 0, \quad \langle \hat{H} \rangle(t) = \text{const}, \\ \text{Spatial translation} &\longleftrightarrow G = \hat{p}_j (V = 0), \quad [\hat{H}, \hat{p}_j] = 0, \quad \langle \hat{p}_j \rangle(t) = \text{const}, \\ \text{Spatial rotation} &\longleftrightarrow G = \hat{L}_j (V = V(|x|)), \quad [\hat{H}, \hat{L}_j] = 0, \quad \langle \hat{L}_j \rangle(t) = \text{const}. \end{aligned}$$

*This is the Noether bridge: Gap (iv) of Sec. 2.5 is explicitly closed. The geometric transport symmetry established in the M-series, the self-adjoint operator structure derived from the scalar–conformal geometry in the QB-series and QM1, and the Schrödinger dynamics established in the present paper all combine to yield the quantum conservation laws without any classical-mechanics input. Every element of the derivation is internal to the NUVO program.*

## 8 The Ehrenfest Theorem

The conservation laws of Sec. 7 addressed observables that commute with the Hamiltonian. The Ehrenfest theorem addresses the complementary case: observables that do not commute with  $\hat{H}$ —specifically, position and momentum—and derives the equations governing the time evolution of their expectation values. The result is that these expectation values satisfy equations formally identical to the classical Hamilton equations of motion, with the potential gradient replacing the classical force. This is an exact result of the Schrödinger dynamics, not an approximation, and it holds for all normalized closure states and all times. In the NUVO framework it expresses the fact that the scalar–conformal transport geometry generates both the full quantum dynamics and the classical transport trajectories as a structural consequence, with the latter emerging as the evolution of mean transport quantities under the former.

### 8.1 Expectation Values as Transport Averages

For a Schrödinger evolution  $\Psi(t) = U(t)\Psi_0$ , the expectation values of position and momentum are defined as

$$\langle \hat{x}^j \rangle(t) := \langle \Psi(t), \hat{x}^j \Psi(t) \rangle_{\mathcal{H}} = \int_{\mathbb{R}^3} x^j |\Psi(x, t)|^2 d^3x, \quad (34)$$

$$\langle \hat{p}_j \rangle(t) := \langle \Psi(t), \hat{p}_j \Psi(t) \rangle_{\mathcal{H}} = \int_{\mathbb{R}^3} \overline{\Psi(x, t)} (-i\Phi_0 \partial_j \Psi(x, t)) d^3x. \quad (35)$$

These are the mean position and mean momentum of the closure state  $\Psi(t)$  at time  $t$ . In the NUVO framework,  $\langle \hat{x}^j \rangle(t)$  is the first spatial moment of the normalized closure density  $\tilde{\rho}(x, t) = |\Psi(x, t)|^2$ , identifying it as the centroid of the closure distribution. The mean momentum  $\langle \hat{p}_j \rangle(t)$  is the mean transport-phase gradient, weighted by the closure density, representing the average transport velocity of the closure configuration.

That the expectation values Eq. (34) and Eq. (35) are well-defined for all  $t$  follows from Theorem 5.5 (which ensures  $\Psi(t) \in \mathcal{D}(\hat{H}) \subset L^2(\mathbb{R}^3)$ ) together with the growth conditions on  $x^j$  and  $\partial_j$  for states in  $H^2(\mathbb{R}^3)$ . For initial data  $\Psi_0 \in \mathcal{S}(\mathbb{R}^3)$ , both expectation values are finite and smooth in  $t$ .

The rate of change of each expectation value is controlled by the general identity of Remark 7.2:

$$\frac{d}{dt} \langle A \rangle(t) = \frac{i}{\Phi_0} \langle [\hat{H}, A] \rangle(t) \quad (36)$$

for any self-adjoint  $A$  with  $\Psi(t) \in \mathcal{D}(\hat{H}) \cap \mathcal{D}(A)$ . The Ehrenfest theorem is the result of applying Eq. (36) to  $A = \hat{x}^j$  and  $A = \hat{p}_j$  in turn and computing the resulting commutators explicitly.

### 8.2 The Ehrenfest Equations

The two Ehrenfest equations are established as a single theorem, with each proved in turn from the general identity Eq. (36) and the commutator calculations carried out explicitly.

**Theorem 8.1** (Ehrenfest theorem). *Let  $\Psi(t) = U(t)\Psi_0$  be the Schrödinger evolution of Theorem 5.5 with  $\Psi_0 \in \mathcal{S}(\mathbb{R}^3)$ . Then the expectation values of position and momentum satisfy*

$$\frac{d}{dt} \langle \hat{x}^j \rangle(t) = \frac{\langle \hat{p}_j \rangle(t)}{m}, \quad (37)$$

$$\frac{d}{dt} \langle \hat{p}_j \rangle(t) = - \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t), \quad (38)$$

for all  $t \in \mathbb{R}$  and all  $j \in \{1, 2, 3\}$ .

*Proof. Equation (37): the position equation.*

Apply Eq. (36) with  $A = \hat{x}^j$ :

$$\frac{d}{dt} \langle \hat{x}^j \rangle(t) = \frac{i}{\Phi_0} \langle [\hat{H}, \hat{x}^j] \rangle(t).$$

Compute  $[\hat{H}, \hat{x}^j] = [\hat{T}, \hat{x}^j] + [\hat{V}, \hat{x}^j]$  on  $\mathcal{S}(\mathbb{R}^3)$ .

*Potential term:*  $[V(x), x^j] \Psi = V(x) x^j \Psi - x^j V(x) \Psi = 0$ , since multiplication operators commute pointwise. Therefore  $[\hat{V}, \hat{x}^j] = 0$ .

*Kinetic term:* Write  $\hat{T} = -(\Phi_0^2/2m)\Delta = -(\Phi_0^2/2m) \sum_k \partial_k^2$ . For each  $k$ , compute  $[\partial_k^2, x^j]$  on  $\Psi \in \mathcal{S}(\mathbb{R}^3)$ :

$$\partial_k^2(x^j \Psi) = \partial_k(\delta^j_k \Psi + x^j \partial_k \Psi) = \delta^j_k \partial_k \Psi + \delta^j_k \partial_k \Psi + x^j \partial_k^2 \Psi = 2\delta^j_k \partial_k \Psi + x^j \partial_k^2 \Psi,$$

so  $[\partial_k^2, x^j] \Psi = \partial_k^2(x^j \Psi) - x^j \partial_k^2 \Psi = 2\delta^j_k \partial_k \Psi$ . Summing over  $k$ :

$$[\Delta, x^j] \Psi = \sum_k [\partial_k^2, x^j] \Psi = 2\partial_j \Psi.$$

Therefore

$$[\hat{T}, \hat{x}^j] \Psi = -\frac{\Phi_0^2}{2m} [\Delta, x^j] \Psi = -\frac{\Phi_0^2}{2m} 2\partial_j \Psi = -\frac{\Phi_0^2}{m} \partial_j \Psi = \frac{i\Phi_0}{m} (-i\Phi_0 \partial_j \Psi) = \frac{i\Phi_0}{m} \hat{p}_j \Psi.$$

Combining:

$$[\hat{H}, \hat{x}^j] \Psi = \frac{i\Phi_0}{m} \hat{p}_j \Psi.$$

Substituting into Eq. (36):

$$\frac{d}{dt} \langle \hat{x}^j \rangle(t) = \frac{i}{\Phi_0} \cdot \frac{i\Phi_0}{m} \langle \hat{p}_j \rangle(t) = -\frac{1}{m} \langle \hat{p}_j \rangle(t) \cdot (-1) = \frac{\langle \hat{p}_j \rangle(t)}{m},$$

which is Eq. (37).

*Equation (38): the momentum equation.*

Apply Eq. (36) with  $A = \hat{p}_j$ :

$$\frac{d}{dt} \langle \hat{p}_j \rangle(t) = \frac{i}{\Phi_0} \langle [\hat{H}, \hat{p}_j] \rangle(t).$$

The commutator  $[\hat{H}, \hat{p}_j]$  was computed in the proof of Proposition 7.5:  $[\hat{H}, \hat{p}_j] \Psi = i\Phi_0 (\partial V / \partial x^j) \Psi$ .

Therefore

$$\frac{d}{dt} \langle \hat{p}_j \rangle(t) = \frac{i}{\Phi_0} \langle i\Phi_0 \partial_j V \rangle(t) = \frac{i}{\Phi_0} \cdot i\Phi_0 \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t) = - \left\langle \frac{\partial V}{\partial x^j} \right\rangle(t),$$

which is Eq. (38). □

**Remark 8.2.** *The second Ehrenfest equation Eq. (38) is the precise quantum-mechanical counterpart of Newton's second law in the scalar-conformal setting. The quantity  $-\partial V / \partial x^j = \kappa_\Lambda \partial \delta \lambda / \partial x^j$  is the restoring force per unit closure content generated by the gradient of the scalar capacity modulation field. Equation (38) states that the rate of change of mean momentum equals the mean restoring force. This is not a postulate imported from classical mechanics but a theorem derived from the commutator of  $\hat{H}$  with  $\hat{p}_j$ , which is in turn a consequence of the scalar-conformal transport geometry. The derivation chain is:  $\delta \lambda(x) \rightarrow V(x) \rightarrow [\hat{H}, \hat{p}_j] = i\Phi_0 \partial_j V \rightarrow \frac{d}{dt} \langle \hat{p}_j \rangle = -\langle \partial_j V \rangle$ .*

### 8.3 The Classical Limit as a Transport Correspondence

The Ehrenfest equations Eq. (37)–Eq. (38) are formally identical in structure to the classical Hamilton equations:

$$\frac{dx^j}{dt} = \frac{p_j}{m}, \quad \frac{dp_j}{dt} = -\frac{\partial V}{\partial x^j}. \quad (39)$$

The structural identity between Eq. (37)–Eq. (38) and Eq. (39) is exact: the Ehrenfest equations are not an approximation to the classical equations but the exact quantum equations for the expectation values, which happen to take the same form. The distinction between the quantum and classical descriptions lies not in the form of the equations but in their objects: the Ehrenfest equations govern expectation values  $\langle \hat{x}^j \rangle(t)$  and  $\langle \hat{p}_j \rangle(t)$ , which are spatial averages of the closure density and transport phase gradient, while the classical equations govern the individual trajectory coordinates of a point particle.

For a general closure state  $\Psi(t)$ , the expectation-value trajectory ( $\langle \hat{x}^j \rangle(t)$ ,  $\langle \hat{p}_j \rangle(t)$ ) is the centroid trajectory of the closure distribution. Individual fluctuations of  $\Psi$  around this centroid—encoded in the higher moments of  $|\Psi|^2$ —evolve according to the full Schrödinger dynamics and are not described by the Ehrenfest equations alone. In particular, the spatial spread  $\Delta x^j(t)$  of the closure distribution can grow in time even when the centroid follows a classical trajectory, a feature that will be studied in the context of coherent states in QM6.

**Remark 8.3.** *Within the NUVO framework, the Ehrenfest theorem carries the following precise interpretation. The scalar–conformal transport geometry generates a deterministic evolution for the full closure state  $\Psi(t)$  via the Schrödinger equation. This evolution encodes both the mean transport trajectory—the centroid dynamics governed by the Ehrenfest equations—and the fluctuation structure around that trajectory, encoded in the higher moments of  $|\Psi|^2$ . The mean transport trajectory follows the classical scalar–conformal transport law Eq. (39) exactly, without approximation. This is not a separate postulate or a statement about a classical limit: it is a theorem derived from the Schrödinger dynamics of Theorem 5.5 and the canonical commutation relation of QM1 Proposition 5.4.*

*The sense in which the classical transport trajectory is fully recovered by the quantum dynamics—beyond the expectation-value level—requires the notion of coherent states: minimum-uncertainty states for which the expectation-value trajectory and the full quantum trajectory coincide as closely as the uncertainty relations of QM3 permit. For the harmonic oscillator Hamiltonian of QM6, coherent states evolve as exact classical trajectories with no spreading of the closure distribution, providing the sharpest possible realization of the classical correspondence within the NUVO framework. The present theorem establishes the exact result; QM6 develops the optimal approximation.*

**Remark 8.4.** *A technical point deserves explicit statement. The Ehrenfest equations Eq. (37)–Eq. (38) involve  $\langle \partial_j V \rangle(t)$ , the expectation value of the potential gradient, which is in general not equal to the gradient of the potential evaluated at the mean position:*

$$\left\langle \frac{\partial V}{\partial x^j} \right\rangle(t) \neq \frac{\partial V}{\partial x^j}(\langle \hat{x}(t) \rangle) \quad (40)$$

*in general, with equality only when  $V$  is linear in  $x^j$  or when the closure state is sufficiently sharply localized that the nonlinearity of  $V$  is negligible over the spatial extent of  $|\Psi|^2$ . The departure from equality is controlled by higher moments of the closure distribution and the nonlinearity of  $V$ . In the NUVO framework, this departure is a measurable structural feature of the closure state: it quantifies the degree to which the closure distribution is sensitive to the curvature of the scalar capacity modulation field. The Ehrenfest theorem as stated in Theorem 8.1 is exact; the approximation*

$\langle \partial_j V \rangle \approx \partial_j V(\langle \hat{x} \rangle)$  is an additional step that is valid only in the semiclassical regime and is not assumed here.

## 9 Interpretive Clarifications and Scope

The present section maintains the interpretive discipline established in the Q-series and continued through the QB-series and QM1. Four items are addressed: the logical status of the Schrödinger equation within the NUVO framework, the derived character of the Hamiltonian, the preservation of determinism through the dynamical framework, and the scope of the present construction relative to the remainder of the QM-series.

### 9.1 The Schrödinger Equation as a Theorem, Not a Postulate

In the standard formulation of quantum mechanics, the time-dependent Schrödinger equation  $i\hbar \partial_t \Psi = \hat{H} \Psi$  is introduced as a fundamental postulate: a dynamical law asserted as a primitive constituent of the theory, with no derivation from more basic principles. Its justification is empirical—it produces predictions that agree with experiment—and its status as a postulate places it outside the scope of derivation within the standard framework.

In the present framework the same equation is a theorem. The derivation proceeds in four steps, each of which introduces no new postulate and relies only on results established in the prior series and the present paper.

Step (1): The scalar capacity modulation correspondence (Sec. 3). A spatial modulation  $\delta\lambda(x)$  of the baseline scalar capacity  $\Lambda_0$  generates a real-valued potential  $V(x) = -\kappa_\Lambda \delta\lambda(x)$  in the exchange-sector transport closure law at first order. This correspondence is derived from the exchange-sector action of the M-series and does not require any assumption about the form of the potential.

Step (2): Admissibility and the Kato class (Sec. 3.3). The scalar-conformal potential  $V$  arising from a localized modulation  $\delta\lambda \in L^2(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3)$  belongs to the admissible class  $\mathcal{K}$  of Definition 3.7, satisfying the relative bound  $\|V f\|_{\mathcal{H}} \leq a \|\hat{T} f\|_{\mathcal{H}} + b \|f\|_{\mathcal{H}}$  with  $a < 1$ .

Step (3): Self-adjointness of  $\hat{H}$  via Kato–Rellich (Sec. 4). With the kinetic operator  $\hat{T}$  self-adjoint on  $H^2(\mathbb{R}^3)$  and the potential  $\hat{V}$  admissible with relative bound  $a < 1$ , the Kato–Rellich Theorem 4.3 yields self-adjointness of  $\hat{H} = \hat{T} + \hat{V}$  on  $H^2(\mathbb{R}^3)$ . No extension choices or boundary conditions are required; the domain is uniquely determined.

Step (4): Stone’s theorem and the generator equation (Sec. 5). A self-adjoint operator on a Hilbert space uniquely determines a strongly continuous one-parameter unitary group via Stone’s Theorem 5.1. Applied to  $\hat{H}/\Phi_0$ , this yields the unitary group  $U(t) = e^{-i\hat{H}t/\Phi_0}$ , and the Schrödinger equation Eq. (22) is the generator equation of this group, not an independent assertion.

The logical status of the Schrödinger equation in the NUVO framework is therefore that of a theorem derived from the scalar–conformal geometry, the functional analysis of self-adjoint operators, and the algebraic structure of unitary groups. No dynamical postulate is introduced at any step. The equation is not less reliable for being derived rather than postulated; on the contrary, its derivation makes explicit the geometric and algebraic conditions under which it holds and the precise class of Hamiltonians to which it applies.

### 9.2 The Hamiltonian as a Derived Object

In the standard quantum-mechanical formalism, the Hamiltonian  $\hat{H}$  is a choice: given a physical system, one writes down a Hamiltonian that models its energy structure, guided by classical analo-

gies, experimental data, or symmetry arguments. The Hamiltonian is therefore a primitive input to the theory rather than a consequence of it.

In the present framework, the Hamiltonian is derived from the geometry. The kinetic operator  $\hat{T} = -(\Phi_0^2/2m)\Delta$  arises from the free exchange-sector transport closure system with uniform scalar capacity  $\Lambda = \Lambda_0$ : it is the transport energy of a closure state in the baseline geometric configuration. The potential operator  $\hat{V}$  arises from the spatial modulation  $\delta\lambda(x)$  of the scalar capacity field: it is the first-order energy correction generated by the departure of the scalar capacity from its baseline value. The full Hamiltonian  $\hat{H} = \hat{T} + \hat{V}$  is therefore not a choice but a structural consequence of the scalar–conformal geometry of the transport closure system.

This has a direct implication for the admissible class of potentials. Definition 3.7 identifies the admissible class  $\mathcal{K}$  as those potentials for which the Kato–Rellich theorem applies: potentials relatively bounded with respect to  $\hat{T}$  with relative bound strictly less than one. In the geometric picture, this class corresponds precisely to those scalar capacity modulations  $\delta\lambda$  that are localized and controlled enough that the departure from the baseline geometry does not dominate the free transport kinetic structure. Potentials outside  $\mathcal{K}$ —those with relative bound  $a \geq 1$  or those not relatively bounded at all—correspond to scalar capacity modulations so singular that the first-order expansion of Sec. 3.1 breaks down and the transport closure system in that background is not well-defined at the level of the present analysis. The admissible class is therefore not a technical convenience but a statement about which geometric configurations support well-defined transport closure at first order.

**Remark 9.1.** *The derivation chain  $\delta\lambda \rightarrow V \rightarrow \hat{H}$  is unique at each step: the correspondence of Definition 3.3 is determined by the M-series exchange-sector action and the first-order expansion, the Kato–Rellich theorem yields self-adjointness on a uniquely determined domain, and Stone’s theorem yields a unique unitary group. The entire dynamical framework—Hamiltonian, time evolution, conservation laws, Ehrenfest equations—is therefore uniquely determined by the scalar capacity field  $\Lambda(x)$ . There is no freedom of choice at any stage. In subsequent papers of the QM-series, each physical sector (harmonic oscillator, hydrogen atom, barrier potentials) corresponds to a specific scalar capacity configuration, and the Hamiltonian for that sector is determined by the configuration without additional input.*

### 9.3 Deterministic Transport and Schrödinger Dynamics

The Q-series established that the exchange-sector transport closure system—the coupled deterministic evolution of  $(\rho, \phi)$ —is fully determined by its initial conditions. Given  $(\rho(x, 0), \phi(x, 0))$ , the subsequent evolution is uniquely determined by the transport closure law. This deterministic character of the underlying transport geometry is reflected at the level of the Schrödinger dynamics.

Corollary 5.7 establishes that for each  $\Psi_0 \in \mathcal{D}(\hat{H})$ , the Schrödinger evolution  $\Psi(t) = U(t)\Psi_0$  is the unique strong solution of Eq. (22) with the given initial datum. No other evolution is consistent with the dynamics generated by  $\hat{H}$ . The determinism of the transport closure system is thus preserved in the Hilbert-space formulation: the closure state at any future time is uniquely determined by the closure state at the initial time.

The statistical character of quantum-mechanical predictions—the Born frequency law of QB6, the measurement correspondence of QB7—is not a consequence of indeterminism in the dynamics. It arises from the coherence-gated interaction structure of the transport system: the deterministic evolution of  $\Psi(t)$  generates deterministic closure density  $|\Psi(x, t)|^2$  at each time, and the Born frequency law assigns event frequencies to coherence-gated interaction channels based on this density. The statistical layer is logically separate from and does not alter the deterministic character of the dynamical layer.

**Remark 9.2.** *The logical separation between the deterministic Schrödinger dynamics and the statistical Born frequency law is a distinctive structural feature of the NUVO framework. In the standard formulation, the Schrödinger equation and the Born rule are two independent postulates, and their logical relationship is a matter of ongoing interpretive debate. In the NUVO framework, the deterministic Schrödinger dynamics is a theorem derived from the transport geometry (the present paper), and the Born frequency law is a theorem derived from the coherence-gated interaction structure (QB6). Both are results, and their logical independence from each other is explicit: neither derivation depends on the other. Their numerical consistency—both produce the same predictions for measurement frequencies—is a non-trivial structural feature of the scalar-conformal framework rather than an assumption.*

## 9.4 Scope of the Present Construction

The present paper establishes the dynamical foundation of the QM-series in a form that is both general and rigorous. It is equally important to record precisely what it does not establish, so that the logical dependencies of subsequent papers are transparent and no result is used before it has been derived.

The paper does not treat time-dependent potentials. The scalar capacity modulation  $\delta\lambda(x)$  is taken to be time-independent throughout Secs. 3–8, producing a static Hamiltonian  $\hat{H}$  that generates time-translation invariance and energy conservation. A time-dependent modulation  $\delta\lambda(x, t)$  would produce a time-dependent potential  $V(x, t)$ , generating a time-dependent Hamiltonian  $\hat{H}(t)$  for which Stone’s theorem does not directly apply and the energy conservation of Corollary 7.3 fails. The time-dependent sector—which is the natural setting for the electromagnetic interaction and the minimal coupling of charges to fields—is the domain of the RQM-series, where the full covariant treatment of time-dependent backgrounds is undertaken.

The paper does not develop the full angular momentum algebra. Proposition 7.7 introduces the angular momentum operators  $\hat{L}_j = \epsilon_{jkl}\hat{x}^k\hat{p}_l$  and establishes their commutation with rotationally symmetric Hamiltonians, but the commutation algebra  $[\hat{L}_j, \hat{L}_k] = i\Phi_0\epsilon_{jkl}\hat{L}_l$ , the joint spectrum of  $\hat{L}^2$  and  $\hat{L}_3$ , the identification of spherical harmonics as stationary angular closure eigenstates, and the integer holonomy quantization of orbital quantum numbers are all developed in QM5, which uses the Schrödinger dynamics and conservation structure of the present paper as its foundational input.

The paper does not treat the harmonic oscillator or coherent states. The harmonic potential  $V(x) = \frac{1}{2}k|x|^2$ , arising from a quadratically growing scalar capacity modulation, falls outside the  $L^2 \cap L^\infty$  class of the main admissibility argument and requires a separate self-adjointness treatment via the explicit Hermite-function eigenbasis, as noted in Remark 3.10. The algebraic ladder-operator structure of the harmonic oscillator, the energy spectrum  $E_n = (n + \frac{1}{2})\Phi_0\omega$ , and the identification of coherent states as minimum-uncertainty transport configurations are all developed in QM6.

The paper does not treat multi-particle Schrödinger dynamics. The single-particle Hamiltonian  $\hat{H}$  established here acts on the single-particle Hilbert space  $\mathcal{H} = L^2(\mathbb{R}^3, \mathbb{C})$ . The multi-particle generalization requires replacing  $\mathcal{H}$  with the  $N$ -particle configuration space  $L^2(\mathbb{R}^{3N}, \mathbb{C})$ , constructing the multi-particle Hamiltonian as a sum of single-particle kinetic terms and pair-interaction potentials, and imposing the exchange symmetry constraints (symmetric for bosonic, antisymmetric for fermionic closure structures) that arise from the topological identity of indistinguishable loop configurations. This construction, together with the derivation of the Pauli exclusion principle as a structural consequence of antisymmetry, is undertaken in QM7.

The paper does not treat spin dynamics. The single-particle Hamiltonian  $\hat{H}$  of the present paper acts on scalar-valued closure states in  $L^2(\mathbb{R}^3, \mathbb{C})$  and does not include a spin degree of freedom.

The spin- $\frac{1}{2}$  Hamiltonian requires the state space to be extended to  $L^2(\mathbb{R}^3, \mathbb{C}^2)$ , with the spin-space component of  $\hat{H}$  constructed from the Pauli matrices arising from the double-cover holonomy structure of QM8. The magnetic moment coupling and the spin-orbit interaction are part of the spin Hamiltonian developed in QM8, which uses the operator and dynamical framework of the present paper throughout.

In each case, the present paper provides the essential dynamical infrastructure—the Schrödinger equation, the unitary group, the conservation structure, and the Ehrenfest theorem—on which the subsequent development depends. No paper in QM5 through QM11 bypasses the framework established here.

## 10 Conclusion

### 10.1 Summary of Results

The present paper has established the complete dynamical framework for the scalar-conformal NUVO transport closure system on the Hilbert space  $\mathcal{H}$  of QM1, closing four structural gaps identified in the prior series and constructing the Schrödinger dynamics, conservation laws, and classical correspondence as theorems derived entirely from the scalar-conformal geometry. The principal results are as follows.

**The scalar-conformal potential correspondence** (Lemma 3.1, Definition 3.3, Propositions 3.5 and 3.8). A spatial modulation  $\delta\lambda(x) = \Lambda(x)/\Lambda_0 - 1$  of the scalar capacity field generates, at first order in  $\delta\lambda$ , a real-valued multiplicative potential  $V(x) = -\kappa_\Lambda \delta\lambda(x)$  in the exchange-sector transport closure equation. This is Gap (i) closed. The structural coupling constant  $\kappa_\Lambda$  is fixed by consistency with the hydrogenic sector, which recovers the Coulomb potential  $V_H(x) = -e^2/(4\pi\epsilon_0|x|)$  as a special case, providing an internal consistency check on the correspondence. Scalar-conformal potentials arising from localized modulations  $\delta\lambda \in L^2(\mathbb{R}^3) \cap L^\infty(\mathbb{R}^3)$  belong to the admissible class  $\mathcal{K}$  of Definition 3.7, satisfying the relative bound required by the Kato–Rellich theorem with  $a < 1$ .

**Self-adjointness of the Hamiltonian** (Theorem 4.5 and Corollary 4.7). The scalar-conformal Hamiltonian  $\hat{H} = \hat{T} + \hat{V} = -(\Phi_0^2/2m)\Delta + V(x)$  is self-adjoint on the second-order Sobolev domain  $H^2(\mathbb{R}^3)$  for all admissible potentials  $V \in \mathcal{K}$ , by the Kato–Rellich Theorem 4.3. This is Gap (ii) closed. The domain is unchanged from that of the free kinetic operator  $\hat{T}$ : the potential does not introduce additional regularity requirements. Essential self-adjointness on the Schwartz domain  $\mathcal{S}(\mathbb{R}^3)$  follows as a corollary, confirming that the scalar capacity geometry uniquely determines the Hamiltonian without freedom of boundary conditions or extension choices.

**The unitary time-evolution group and the Schrödinger equation** (Theorems 5.3 and 5.5, Corollary 5.7). Stone’s Theorem 5.1, applied to the self-adjoint operator  $\hat{H}/\Phi_0$  on  $\mathcal{H}$ , yields a unique strongly continuous one-parameter unitary group  $U(t) = \exp(-i\hat{H}t/\Phi_0)$  on  $\mathcal{H}$ . This is Gap (iii) closed. For each  $\Psi_0 \in \mathcal{D}(\hat{H})$ , the curve  $\Psi(t) = U(t)\Psi_0$  is the unique strong solution of the time-dependent Schrödinger equation  $i\Phi_0 \partial_t \Psi = \hat{H} \Psi$  with initial datum  $\Psi_0$ . The Schrödinger equation is thereby established as a theorem derived from the scalar-conformal geometry via the correspondence  $\delta\lambda \rightarrow V \rightarrow \hat{H} \rightarrow U(t) \rightarrow i\Phi_0 \partial_t \Psi = \hat{H} \Psi$ , with no dynamical postulate introduced at any stage. The Q-series Schrödinger-type compatibility result for the hydrogenic sector is recovered as Proposition 5.9, now a corollary of the general theorem.

**Unitarity and norm preservation** (Theorem 6.1 and Corollary 6.5). The time-evolution operator  $U(t)$  satisfies  $U(t)^*U(t) = \hat{\mathbf{1}}$  for all  $t \in \mathbb{R}$ , yielding norm preservation  $\|U(t)\Psi\|_{\mathcal{H}} = \|\Psi\|_{\mathcal{H}}$  and full inner product preservation  $\langle U(t)\Psi_1, U(t)\Psi_2 \rangle_{\mathcal{H}} = \langle \Psi_1, \Psi_2 \rangle_{\mathcal{H}}$  for all  $\Psi_1, \Psi_2 \in \mathcal{H}$ . These results are consistent with and logically independent of the geometric norm preservation of QM1

Corollary 3.4; their agreement constitutes a non-trivial internal consistency check on the NUVO framework. Orthogonality of distinct holonomic closure modes, established in QB3 and extended to  $\mathcal{H}$  in QM1, is preserved for all time under the Schrödinger evolution, and the Born frequency law of QB6 is time-stable for stationary states.

**Conservation laws from transport symmetry** (Theorem 7.1, Corollary 7.3, Propositions 7.5 and 7.7, Remark 7.9). The general conservation theorem establishes that  $\frac{d}{dt}\langle G \rangle(t) = \frac{i}{\Phi_0}\langle [\hat{H}, G] \rangle(t)$  for any self-adjoint observable  $G$ , yielding conservation of  $\langle G \rangle(t)$  whenever  $[\hat{H}, G] = 0$ . This is Gap (iv) closed. Applied to the three fundamental symmetries of the static scalar–conformal transport system: time-translation invariance yields  $\frac{d}{dt}\langle \hat{H} \rangle(t) = 0$ ; spatial-translation invariance yields  $\frac{d}{dt}\langle \hat{p}_j \rangle(t) = 0$  for free transport and  $-\langle \partial_j V \rangle(t)$  for non-uniform potentials; rotational invariance of  $V(|x|)$  yields  $\frac{d}{dt}\langle \hat{L}_j \rangle(t) = 0$ . Remark 7.9 records the Noether bridge explicitly: the geometric transport symmetry of the M-series is connected to the conservation of quantum-mechanical expectation values by the Schrödinger dynamics of the present paper, without classical mechanics input.

**The Ehrenfest theorem** (Theorem 8.1). The expectation values of position and momentum satisfy  $\frac{d}{dt}\langle \hat{x}^j \rangle(t) = \langle \hat{p}_j \rangle(t)/m$  and  $\frac{d}{dt}\langle \hat{p}_j \rangle(t) = -\langle \partial_j V \rangle(t)$ , equations structurally identical to the classical Hamilton equations. These are exact results holding for all normalized closure states and all times, derived from the Schrödinger dynamics and the canonical commutation relation of QM1 Proposition 5.4. The mean transport trajectory of any closure state follows the classical scalar–conformal transport law exactly, with departures from classical behavior encoded in the higher moments of the closure distribution.

## 10.2 Programmatic Significance

The present paper closes the four structural gaps that prevented the QM-series from proceeding beyond QM3 on a rigorous foundation. With the results now in place, the scalar–conformal transport geometry supports a complete derivation chain:

$$\delta\lambda(x) \xrightarrow{\text{Sec. 3}} V(x) \xrightarrow{\text{Sec. 4}} \hat{H} \xrightarrow{\text{Sec. 5}} U(t) \xrightarrow{\text{Sec. 5}} i\Phi_0 \partial_t \Psi = \hat{H} \Psi,$$

with conservation laws and the Ehrenfest theorem following as corollaries of the dynamics. Every element of this chain is a theorem derived from prior NUVO results. No dynamical postulate, no Hamiltonian chosen by analogy with classical mechanics, and no conservation law imported from classical Noether theory enters the framework. The scalar–conformal geometry determines the dynamics completely and uniquely.

The dynamical infrastructure established in the present paper propagates forward through every subsequent paper in the QM-series without exception. QM5 uses the conservation structure of Sec. 7 and the commutation algebra of the angular momentum operators established in Proposition 7.7 as the starting point for the full rotational transport analysis. QM6 uses the Schrödinger equation of Theorem 5.5 with a quadratic scalar capacity modulation to establish the harmonic oscillator spectrum and to identify coherent states as the closure configurations that most closely realize the classical transport trajectory. QM7 extends the single-particle Hamiltonian framework of the present paper to the  $N$ -particle configuration space  $L^2(\mathbb{R}^{3N}, \mathbb{C})$ , with multi-particle Hamiltonians constructed as sums of single-particle kinetic terms and pair-interaction scalar-conformal potentials. QM8 extends the state space to  $L^2(\mathbb{R}^3, \mathbb{C}^2)$  and equips the Hamiltonian with a spin-space component constructed from the Pauli matrices arising from double-cover holonomy. QM9 constructs entangled states as non-factorizable elements of the multi-particle state space of QM7, using the inner product preservation of Corollary 6.5 to track correlations. QM10 uses the continuous-spectrum structure of  $\hat{H}$  established in Proposition 4.9 and the time-evolution group  $U(t)$  to construct the S-matrix

and analyze scattering and tunneling. QM11 reformulates the transport generators as Lorentz-covariant objects using the SR-series kinematics, and the covariant Schrödinger dynamics that results prepares the transition to the Klein-Gordon and Dirac equations of the RQM-series.

The Noether bridge of Remark 7.9 closes a conceptual gap that has been present in the NUVO program since the M-series established the geometric transport symmetry structure. The M-series showed that the scalar–conformal action is invariant under time translation, spatial translation, and spatial rotation in the static sector, and derived the associated Noether currents at the classical geometric level. The present paper shows that these same symmetries, when the transport closure system is represented on the Hilbert space  $\mathcal{H}$  and governed by the Schrödinger dynamics, imply the conservation of the corresponding quantum-mechanical expectation values via the commutation structure of Theorem 7.1. The derivation chain from the scalar–conformal geometry to all three standard conservation laws of quantum mechanics is now complete, without any step that appeals to classical mechanics, external symmetry principles, or independent postulate.

### 10.3 Transition to QM5

The conservation of angular momentum established in Proposition 7.7 is the point of departure for QM5. That paper develops the complete rotational transport algebra from the operators  $\hat{L}_j = \epsilon_{jkl} \hat{x}^k \hat{p}_l$  introduced in Definition Eq. (33) of the present paper. The commutation algebra  $[\hat{L}_j, \hat{L}_k] = i\Phi_0 \epsilon_{jkl} \hat{L}_l$  is derived from the canonical commutation relation of QM1 using the Schrödinger dynamics of the present paper to identify the physical significance of the algebraic structure. The joint spectrum of  $\hat{L}^2$  and  $\hat{L}_3$  is then derived from the integer holonomy quantization condition on rotationally closed transport paths, producing the angular quantum numbers  $\ell$  and  $m$  as structural outputs rather than assumed labels. The spherical harmonics  $Y_\ell^m(\theta, \varphi)$  emerge as the stationary angular closure eigenstates in the rotationally symmetric sector, reconnecting to the hydrogenic structure of the Q-series and completing the angular part of the hydrogen atom solution on the basis of the dynamical framework established in the present paper and the Hilbert space framework of QM1.

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tial self-adjointness of differential operators (Ch. X), the Kato–Rellich theorem, Sobolev space domains, and the Fourier-transform characterization of momentum operators on  $L^2(\mathbb{R}^3)$ .